3) Il y a un assez grand désaccord entre la valeur théorique de $3\hbar^2/\mathscr{I}$ et la valeur expérimentale. Ceci semble un fait assez général dans les noyaux impairs qui présentent la bande de rotation $K = 0$. On constate que plus la déformation du noyau est importante, plus la valeur théorique de $3\hbar^2/\mathscr{I}$ est voisine de la valeur expérimentale.

Pour Ho^{166} (déformation $\delta = 0.3$) le rapport $\mathscr{I}_{\text{théorique}}/\mathscr{I}_{\text{expérimental}}$ est égal à 1.02. Pour Lu^{172}, pour lequel $\delta = 0.26$, ce rapport est égal à 1.14. Dans notre cas, ce rapport a pour valeur 1.4 et la déformation est de l'ordre de 0.23.

Il y a lieu de noter toutefois que le niveau $K = 0$ est un niveau excité dans le cas de Lu^{172} et Ta^{176}, tandis qu'il est niveau fondamental de Ho^{166}. La divergence entre le résultat théorique et le résultat expérimental s'expliquerait peut être par ce fait, plutôt que par la variation de la déformation [29]).

Nous tenons à remercier M. M. Valadares, directeur du Centre de Spectrométrie nucléaire et de Spectrométrie de masse (C.N.R.S.) qui nous a aimablement permis d'utiliser les spectrographes de son laboratoire.

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Nuclear Physics **47** (1963) 313—327; © *North-Holland Publishing Co., Amsterdam*

ELECTRON-ELECTRON SCATTERING CROSS-SECTION TAKING INTO ACCOUNT HARD-PHOTON RADIATION

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Received 22 January 1963

Abstract: Radiation corrections for the electron-electron scattering cross-section are calculated, taking into account hard photon radiation, by a numerical integration of the precise formula for bremsstrahlung in an electron-electron collision. The corrections are found to be 15.8 % and 10.6 % for a scattering angle of 40°, and 19.6 % and 14.4 % for that of 90° for electrons with energies of 100 MeV and 500 MeV in the c.m.s., respectively. Analytical expressions are obtained for the asymptotic behaviour of hard photon contribution in the limit of rather hard photons and relatively soft photons.

1. Introduction

Electron-electron colliding beam experiments with two intersecting electron beams with energies 100 to 500 MeV are now planned to check the applicability of quantum electrodynamics at small distances (see refs. [1,2])). Since quantum electrodynamics is a quantitative theory any deviation of experimental cross-sections from those calculated theoretically will point to the breakdown of quantum electrodynamics at small distances. In this connection it is especially important to estimate correctly all theoretical contributions to the electrodynamic cross-sections.

In the lowest order of perturbation theory the electron-electron scattering cross-section is known to be given by the Møller formula †

$$d\sigma_0(\vartheta) = \frac{r_0^2}{8\gamma^2}\left[\frac{s^4+q'^4}{q^4}+\frac{2s^4}{q^2q'^2}+\frac{s^4+q^4}{q'^4}\right]d\Omega. \tag{1.1}$$

Here r_0 is the classical electron radius, $\gamma = E/m$, E the electron energy and $s = p+p'$, $q = p-p_0$, $q' = p-p'_0$. In the c.m.s. we have

$$q^2 = -2p^2(1-\cos\vartheta), \quad q'^2 = -2p^2(1+\cos\vartheta), \quad s^2 = 4E^2. \tag{1.2}$$

For ultra-relativistic electrons in the c.m.s. the Møller formula assumes the form

$$d\sigma_0(\vartheta) = \frac{r_0^2}{4\gamma^2}\left(\frac{3+\cos^2\vartheta}{\sin^2\vartheta}\right)^2 d\Omega. \tag{1.3}$$

To estimate the accuracy of this formula it is necessary to calculate the contributions of the subsequent terms of the perturbation theory series in the coupling

† We use the system of units $\hbar = c = 1$, metrics $(ab) = a_0b_0 - \mathbf{a}\cdot\mathbf{b}$, $e^2 = 1/137$.

constant e, i.e. to obtain the radiation corrections. The latter were calculated to the order e^6 in refs. [3-9]). In this case it is necessary to consider the graphs given in fig. 1,

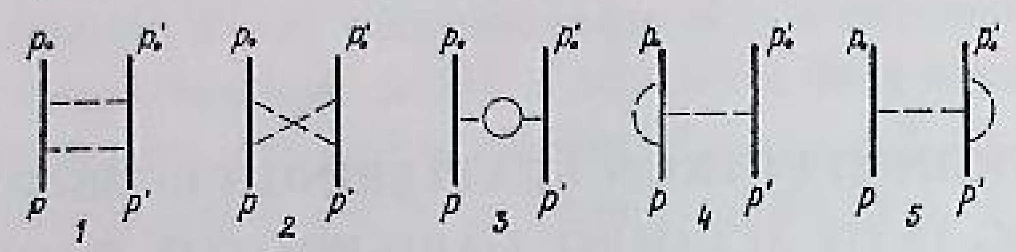

Fig. 1. Graphs of electron-electron scattering in e^4 order.

adding to them the exchange $p_0 \leftrightarrow p_0'$. The graph containing the proper electron self-energy are not given in fig. 1 since they are eliminated in the regularization performed by the standard methods [10,11]). The matrix elements of graphs 1, 2, 4 and 5 also diverge in the integration in the region of small momenta of virtual photons (infrared catastrophe). The fact is that the very concept of elastic process is purely conventional in quantum electrodynamics since soft quanta are radiated in each scattering event; the cross-section of their radiation also diverges in the region of small frequencies but the total elastic and inelastic scattering cross-section contains no divergence. Thus, to eliminate the infrared divergence it is necessary to take into account the graphs with the radiation of real photons (exchange graphs should be added to these graphs, fig. 2) in order to obtain the total elastic and inelastic cross-section containing no infrared divergence (however, this cross-section will depend on the maximum energy of the radiated photons ε). The quantity ε is determined by the parameters of the scattered particle detectors.

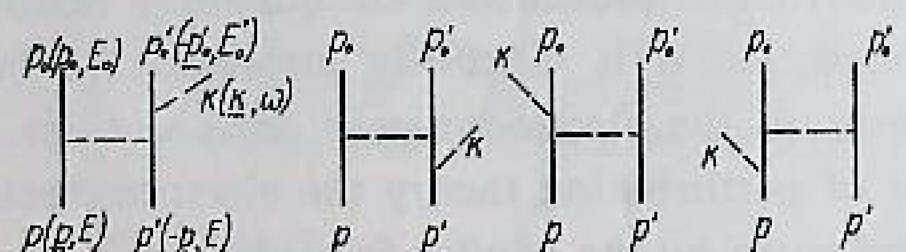

Fig. 2. Graphs of photon bremsstrahlung in electron-electron collisions.

It should be noted that for high energies the expansion parameter in quantum electrodynamics is (after the infrared divergence has been elininated) not e^2 but $e^2 \ln^q(E/m) \ln^p(E/\varepsilon)$, where q and p are 0 or 1. The "twice-logarithmic" terms $e^2 \ln(E/m) \ln(E/\varepsilon)$ prove to be especially essential since for sufficiently small ε the expansion parameter may be of the order of unity; at the same time for the energies attainable at present $e^2 \ln(E/m) \ll 1$. Consequently, to obtain the contributions from the twice-logarithmic terms, it is necessary to sum the perturbation theory series, while the other terms can be taken into account in the e^6 order of perturbation theory. If only twice-logarithmic terms are taken into account the total elastic and inelastic scattering cross-section [12]) is of the form

$$d\sigma(\vartheta) = d\sigma_0(\vartheta) \exp\left[-\frac{8e^2}{\pi} \ln\frac{E}{m} \ln\frac{E}{\varepsilon}\right]. \tag{1.4}$$

Thus, two different situations are possible depending on the quantity ε: (1) if ε is small ($\varepsilon \ll E$), the main contribution to the radiation corrections comes from the twice-logarithmic terms, the energies of the radiated photons are much less than the electron energy and the quantum radiation process is classical; (2) if the quantity ε is comparable with the electron energy, the contribution from the twice-logarithmic terms is small, but at the same time processes with hard photon radiation are essential. It is the latter case that occurs in the contemplated experiments, in which scattered electrons are detected by pairs of counters arranged for coincidence.

Taking into account the hard photon radiation complicates a great deal the problem of calculating the radiation corrections and the result obtained depends rather essentially on the specific conditions of the experiment. The attempt to take into account the hard photon radiation when detecting scattered electrons with a pair of coincidence-circuit counters was undertaken in ref. [7]). However, this investigation contains in this part several unwarranted neglects and inaccuracies. In this paper the contribution of hard photons to the radiation corrections for the electron-electron scattering cross-section is calculated correctly for high energies under the same experimental conditions.

Sect. 2 gives the precise formula for the bremsstrahlung cross-section in the electron-electron collision. This formula is analysed in sect. 3. Sect. 4 gives the numerical calculation of the contribution of hard photons to the radiation corrections. The dependence of the radiation corrections on the characteristic parameters of the problem is considered in sect. 5. Sect. 6 analyses the limiting cases in which an analytical expression can be obtained for the hard photon contribution; the details of the calculation are given in appendix 1. Appendix 2 presents the estimate of the bremsstrahlung cross-section values at the maxima. Appendix 3 shows the equivalence of the use of negatons and positons to check the applicability of quantum electrodynamics at small distances.

2. Garibyan's Formula

The precise formula for the radiation electron collision cross-section (e+e → e+e+γ) was obtained by Garibyan [13]) (see fig. 2). If it is assumed that the electron momenta before the collision are $p(\boldsymbol{p}, E)$, $p'(-\boldsymbol{p}, E)$ after the collision $p_0(\boldsymbol{p}_0, E_0)$, $p_0'(-\boldsymbol{p}_0', E_0')$ and the radiated quantum momentum is $k(\boldsymbol{k}, \omega)$, the conservation laws in the c.m.s. are of the form

$$2E = E_0' + E_0 + \omega, \quad \boldsymbol{p}_0 - \boldsymbol{p}_0' + \boldsymbol{k} = 0. \tag{2.1}$$

The radiation collision cross section is †

$$d\sigma = \int k dk d\Omega_k R\rho, \tag{2.2}$$

† The factor 2 omitted in ref. [13]) is restored.

where

$$\rho = \frac{e^6 d\Omega}{2(2\pi)^2}\,\varphi, \quad \varphi = \frac{p_0^2}{E^2 E_0 E_0' \left[\frac{|p_0|}{E_0} + \frac{|p_0| + |k|\cos\gamma}{E_0'}\right]}, \tag{2.3}$$

$$\cos\gamma = \frac{p_0 \cdot k}{|p_0||k|}.$$

If we discard the terms containing m/E, m/E_0, m/E_0' (i.e. regard both initial and final electrons as ultra-relativistic) we have

$$\begin{aligned}
R = 2\Big[&\left(\frac{1}{g_1c_1} - \frac{1}{g_2c_3}\right)^2 c_1c_3(p_0p_0') + \left(\frac{1}{g_1c_2} - \frac{1}{g_2c_4}\right)^2 c_2c_4(pp') \\
&+ \left(\frac{1}{g_1c_1} - \frac{1}{g_2c_4}\right)^2 c_1c_4(pp_0) + \left(\frac{1}{g_1c_2} - \frac{1}{g_2c_3}\right)^2 c_2c_3(p'p_0')\Big] - 2a^2[(p_0p_0')(pp') \\
&+ (pp_0)(p'p_0')] - 2a_\mu\Big[(c_4p_\mu' - c_1p_{0\mu}')(pp_0)\left(\frac{1}{g_1c_1} - \frac{1}{g_2c_4}\right) + (c_1p_\mu - c_3p_\mu')(p_0p_0') \\
&\times \left(\frac{1}{g_2c_3} - \frac{1}{g_1c_1}\right) + (c_4p_{0\mu} - c_2p_{0\mu}')(pp')\left(\frac{1}{g_1c_2} - \frac{1}{g_2c_4}\right) + (c_2p_\mu - c_3p_{0\mu})(p'p_0') \\
&\times \left(\frac{1}{g_2c_3} - \frac{1}{g_1c_2}\right) + (c_3p_\mu' - c_1p_\mu)(p_0p_0')\left(\frac{1}{g_3c_1} - \frac{1}{g_4c_3}\right) \\
&+ (c_2p_{0\mu}' - c_4p_{0\mu})(pp')\left(\frac{1}{g_3c_4} - \frac{1}{g_4c_2}\right)\Big] \\
&+ \text{exchange terms } (p_0 \leftrightarrow p_0') + 4(p_0p_0')\left(\frac{c_3}{g_1g_3c_1} + \frac{c_1}{g_2g_4c_3} - \frac{1}{g_1g_4} - \frac{1}{g_2g_3}\right) \\
&+ 4(pp')\left(\frac{c_4}{g_1g_3c_2} + \frac{c_2}{g_2g_4c_4} - \frac{1}{g_1g_3} - \frac{1}{g_2g_4}\right) - 4(ab)(pp')(p_0p_0'),
\end{aligned} \tag{2.4}$$

where

$$a_\mu = \frac{p_{0\mu}}{g_1c_2} + \frac{p_{0\mu}'}{g_2c_4} - \frac{p_\mu}{g_2c_3} - \frac{p_\mu'}{g_1c_1},$$

$$b_\mu = a_\mu(p_0 \leftrightarrow p_0'),$$

$$c_1 = (p'k),\ c_2 = (p_0k),\ c_3 = (pk),\ c_4 = (p_0'k),$$

$$g_1 = (p - p_0')^2,\ g_2 = (p' - p_0)^2,\ g_3 = (p - p_0)^2,\ g_4 = (p' - p_0')^2. \tag{2.5}$$

Since we are interested in the cross-section of the process in which both electrons get into the counters, the integration limits in eq. (2.2) are determined from this condition. This problem is dealt with in the next section.

3. Analysis of the Garibyan Formula

We consider the following experimental set-up for the colliding beam experiment (fig. 3): the scattered electrons are registered by pairs of counters with the angular aperture α and threshold resolution E_1 (electrons with energies $E < E_1$ are not registered). It is assumed that we have a point source of scattered electrons. This condition is adequate to the experiment at least in the case when the beams cross at a small angle and the counters are located in the plane of the beams.

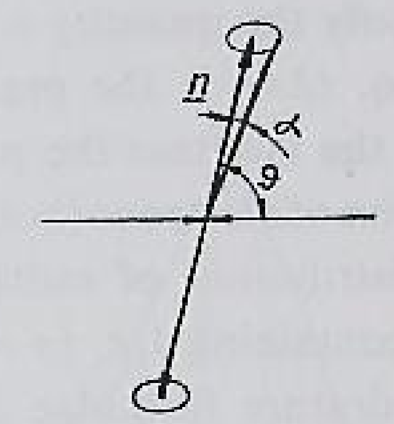

Fig. 3. Scheme of colliding beam experiment.

Registered under this experimental scheme are the radiation collisions for which the following conditions are fulfilled:

$$E_0 > E_1,\ E_0' > E_1,\ \frac{p_0' \cdot n}{|p_0'|} > \cos\alpha, \tag{3.1}$$

if it is assumed that an electron with momentum p_0' has penetrated into one of the counters and the vector n determines the direction to the centre of this counter. These conditions implicitly contain restrictions which determine the integration limits in eq. (2.2). To obtain the cross-section of interest it is necessary to perform, besides the integration over the final photon states d^3k, the integration over the solid angle $d\Omega$ of one of the scattered electrons within the counter.

Since the integration is quite complicated let us first discuss the integrand function. The integral over dk diverges at $k \to 0$ (infrared catastrophe). However, in the total elastic and radiation scattering cross-section this divergence is cancelled, as has been indicated, by the divergence arising the elastic scattering cross-section when the radiation corrections are taken into account. Therefore, it is convenient to divide the integral over radiated photon momentum into two parts: the integral over soft photons up to $k = \Delta E \ll E$ and the integral over k from ΔE to ε. The former integral can be explicitly calculated since in eq. (2.4) we can keep merely the terms $\propto 1/k^2$ and the other expression can be taken for $k = 0$. The sum of this expression with the elastic scattering cross-section does not contain divergences, but depends on the limit parameter ΔE. This sum was calculated in several papers (see ref. [7]), for example) and is of the form

$$d\sigma_{el}+d\sigma_{soft} = \frac{r_0^2}{8\gamma^2}\,d\Omega\left[\frac{s^4+q'^4}{q^4}+\frac{s^4}{q^2q'^2}\right]$$
$$\times\left\{1-\frac{4e^2}{\pi}\left[\tfrac{23}{18}-\tfrac{11}{12}\ln\left(\frac{-q^2}{m^2}\right)+\ln\frac{E}{\Delta E}\left(\ln\frac{q^2q'^2}{s^2m^2}-1\right)\right]\right\} \quad (3.2)$$
$$+\text{ terms with } q^2 \leftrightarrow q'^2.$$

It is necessary to add to this sum the integral over hard photons from $k = \Delta E$ to $k = \varepsilon$. If ΔE is chosen sufficiently small for eq. (3.2) to hold, the result will not depend on ΔE but will contain only the quantity ε.

An important peculiarity of eq. (2.4) is the presence of four high and narrow peaks (see appendix 2) reflecting the fact that the probability for the radiation of a photon along the direction of motion of electrons (both initial and final) is a maximum. The qualitative picture of the distribution of radiated photons is given in fig. 4. These peaks, given by the terms containing $1/c_n$ ($n = 1\ldots4$), make the integration over d^3k by the conventional quadrature formulae difficult. On the other hand, the evaluation of the contribution from very hard photons is facilitated for the same reason (see sect. 6). Note also that the dependences on the angles determining the directions of scattered electrons are very smooth.

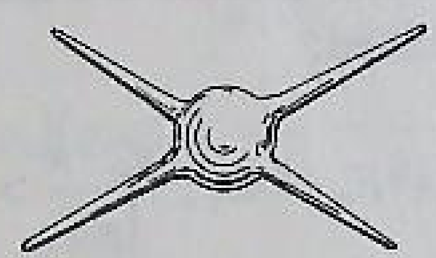

Fig. 4. Qualitative picture of the distribution of hard photons radiated in electron-electron collision.

Fig. 5 represents the graphs of the function $\frac{1}{2}Rk\varphi E^3$ (eqs. (2.3) and (2.4)) versus the polar angle of the photon emission ϑ_k for different azimuthal angles of the photon emission φ_k and different photon energies k, for 500 MeV initial electron energy and a scattering angle of 50° for one of the electrons. These graphs represent different cross-sections of fig. 4 for a certain photon energy. The polar axis is directed along the momentum of a scattered electron and the plane $\varphi_k = 0$ coincides with the electron scattering plane. The graph shows narrow maxima along the direction of the momenta of the initial and final electrons, the peaks being narrower the higher the energy of the radiated photon. Curve 1 gives the graph for $k/E = 0.3$, $\varphi_k = 0$, and the peak half-width is $\vartheta_r \approx 0.5°$. Curves 2 and 3 are plotted for $k/E = 0.1$ and the angles $\varphi_k = 0°$ and $\varphi_k = 90°$ respectively. It is clear that the peak width increases as the photon energy decreases ($\vartheta_r \approx 1.5°$). At the angle $\varphi_k = 90°$ the cross-section is essentially less and has no peak, which reflects the fact that the peaks are located in the scattering plane. Curves 4-6 are plotted for $k/E = 0.02$ and the angles $\varphi_k = 0°$, 45° and 90° respectively; the peak half-width continues to increase ($\vartheta_r \approx 3.5°$); here the peaks are so wide that their mutual influence is evident.

It is shown in appendix 2 that for $k/E \ll 1$ the peak half-width is $\vartheta_r = 2m/k$ when $\varphi_k = 0$ and the quantity R at the peak

$$R_{max} \approx \left(\frac{3+\cos^2\vartheta}{\sin^2\vartheta}\right)^2\frac{2(E-k)}{Em^2}.$$

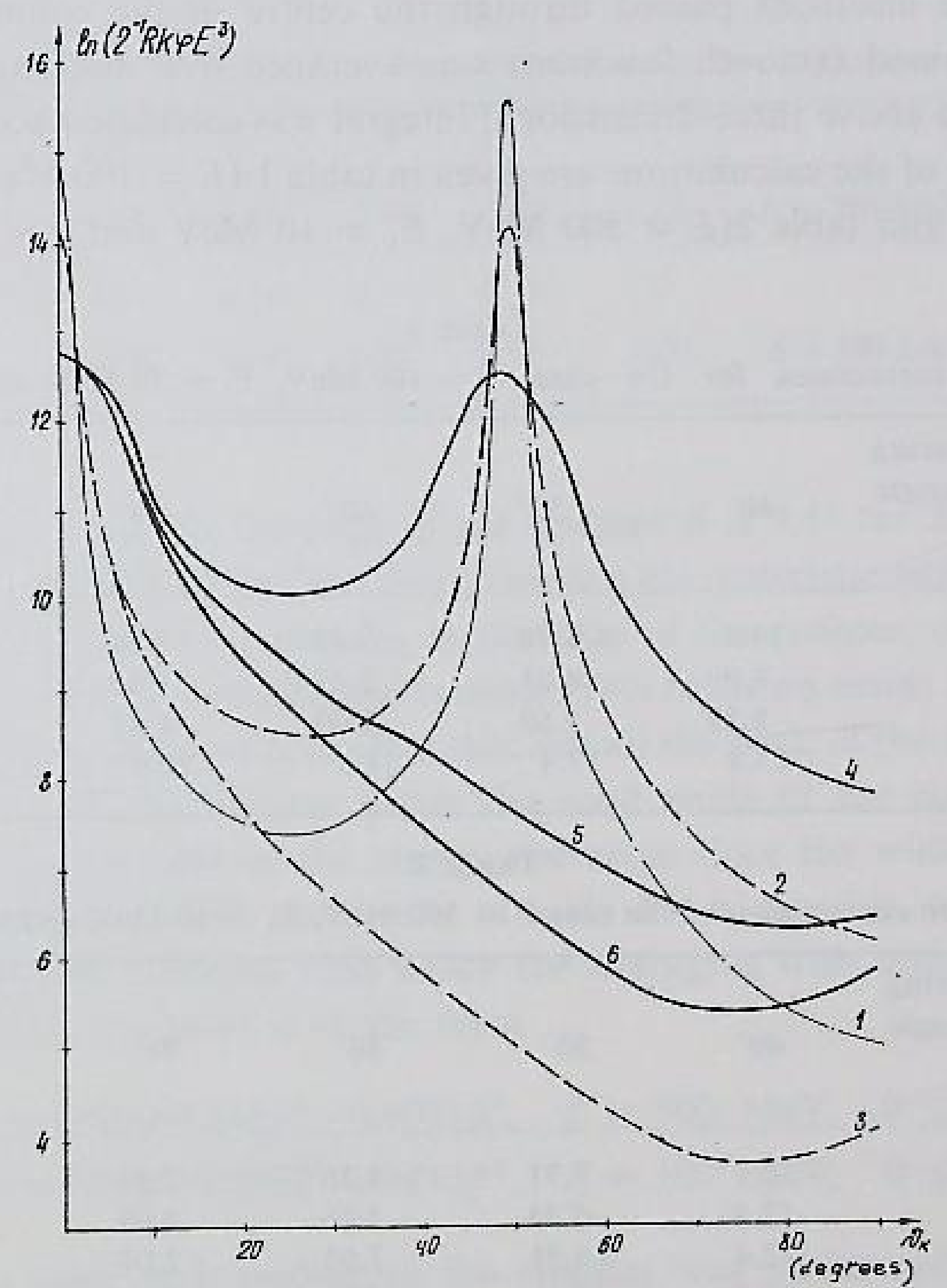

Fig. 5. Function $\frac{1}{2}Rk\varphi E^3$ (eqs. (2.3)-(2.4)).

In this case the angle between scattered electrons is $\vartheta_p = (k/E)\sin\vartheta_k$ so that only those inelastic processes for which $(k/E)\sin\vartheta_k < \alpha$ can be registered. For example, for $\alpha = 3.5°$ and $k/E = 0.02$, $k/E = 0.1$, $k/E = 0.3$ the processes for which ϑ_k are arbitrary, $\vartheta_k < 36°$ and $\vartheta_k < 10°$ are registered. Thus, the processes with the hard photon radiation yield contributions only at small angles of photon emission with respect to the direction of a scattered electron momentum.

4. Radiation Collision Cross-Section for $k > \Delta E$

To obtain the total radiation corrections we must add to eq. (3.2) the integral over hard photons taking into account the restrictions (3.1). The calculation of this multidimensional integral of a highly cumbersome function (2.4) cannot be made with sufficient accuracy considering the rate of present electronic computers. Hence, the problem of decreasing the number of integrations. The following integration scheme was chosen on the basis of the above analysis: integration over d^3k was performed (since the integrand function has sharp peaks) under the assumption that one of the electrons passed through the centre of the counter and then the expression obtained (smooth function) was averaged over the angular aperture of the counter. The above three-dimensional integral was calculated accurately to within 2%. The results of the calculations are given in table 1 (E = 100 MeV, E_1 = 50 MeV and $\alpha = 1.25°$) and table 2(E = 500 MeV, E_1 = 10 MeV and $\alpha = 3.5°$). Note that

TABLE 1

Radiation corrections for the case E = 100 MeV, E_1 = 50 MeV and $\alpha = 1.25°$

Cross-section \ Scattering angle	40°	50°	60°	70°	80°	90°
$d\sigma_{el}+d\sigma_{soft}-d\sigma_0$	−14.5	−6.76	−3.85	−2.59	−2.05	−1.89
$d\sigma_h$	8.99	4.01	2.22	1.46	1.14	1.05
$d\bar{\sigma}_h = \eta d\sigma_h$	8.10	3.60	2.00	1.32	1.03	0.95
δ(%)	15.8	17.4	18.3	18.9	19.4	19.6

TABLE 2

Radiation corrections for the case E = 500 MeV, E_1 = 10 MeV and $\alpha = 3.5°$

Cross-section \ Scattering angle	40°	50°	60°	70°	80°	90°
$d\sigma_{el}+d\sigma_{soft}-d\sigma_0$	−16.7	−7.71	−4.36	−2.93	−2.31	−2.13
$d\sigma_h$	12.8	5.73	3.16	2.09	1.63	1.49
$d\bar{\sigma}_h = \eta d\sigma_h$	12.4	5.51	3.03	2.01	1.57	1.43
δ(%)	10.6	12.1	13.2	13.7	14.1	14.4

the cross-sections in tables 1-3 are given in the units $10^4\, e^6/16\pi^2E^2$. The first line gives the quantity $d\sigma_{el}+d\sigma_{soft}-d\sigma_0$ (eq. (3.2)) and the second line the hard photon radiation cross-section $d\sigma_h$ at ΔE = 1.0 eV in table 1 and ΔE = 10 eV in table 2. The sum of these quantities must not, naturally, depend on the quantity ΔE at all; this was checked at one of the points by changing ΔE by a factor 10 or 100, the total result remaining the same. The actual choice of ΔE was determined mainly by considerations of convenience of integration.

Above it is indicated that one of the electrons was assumed to pass through the centre of the counter. The averaging with respect to the counter was performed as follows: for the scattering angle $\vartheta = 90°$ the dependence of the quantity $d\sigma_h$ on the spot where one of the electrons enters the counter was calculated. If the origin of

the polar system of coordinates is placed at the centre of the counter it appears that within the accuracy of the calculations the cross-section $d\sigma_h$ does not depend on the polar angle but depends appreciably on the radius. For the scattering angle 90° this dependence is given in table 3; the distance from the centre of the counter is

TABLE 3

Cross-section $d\sigma_h$ as function of the point where the electron enters the counter

Cross-section $d\sigma_h$ \ angle β	0°	1°	2°	2.5°	3	Note
Cross-section $d\sigma_h$	1.49	1.46	1.42	1.32	1.22	$E \equiv 500$ MeV, $\alpha \equiv 3.5°$, $\vartheta = 90°$
angle β	0°	0.25°	0.5°	0.75°	1°	
Cross-section $d\sigma_h$	1.05	1.02	1.00	0.05	0.83	$E \equiv 100$ MeV, $\alpha = 1.25°$, $\vartheta = 90°$

given in angular units: at the edge of the counter β is 3.5° for E = 500 MeV and 1.25° for E = 100 MeV. It is clear that there is a characteristic edge effect leading to the decrease of the cross-section $d\sigma_h$ at the edge of the counter, this decrease being the more appreciable the smaller the counter. This situation seems natural and is connected, roughly speaking, with the fact that not all the peak of the integrand function reaches the edge of the counter inside the solid angle of the counter. This effect becomes more appreciable as the energy decreases since the width of the peak increases in the process. With the results of table 3 it is possible, e.g. by cubic interpolation, to plot the function with which the averaging with respect to the counter is performed. This function is of the form

$$\begin{aligned} f(\beta) &= 1-0.016\beta+0.016\beta^2-0.0071\beta^3, \quad E = 500 \text{ MeV}, \quad 0 \leqq \beta \leqq 3.5, \\ f(\beta) &= 1-0.0526\beta-0.068\beta^2-0.213\beta^3, \quad E = 100 \text{ MeV}, \quad 0 \leqq \beta \leqq 1.25. \end{aligned} \tag{4.1}$$

The averaging itself with respect to the counter was performed in this way: the averaging coefficient $\eta = (1/\beta_0)\int_0^{\beta_0} f(\beta)d\beta$ was obtained for β_0 = 3.5 and β_0 = 1.25; this coefficient depends on energy and the chosen size of the counters, but does not depend, within the accuracy of the calculations, on the electron scattering angle. Therefore the value $d\sigma_h$ obtained under the assumption that an electron gets at the centre of the counter was then multiplied by η. The following values of the averaging coefficient were obtained: η = 0.962 for E = 500 MeV and $\alpha = 3.5°$ and η = 0.90 for E = 100 MeV and $\alpha = 1.25°$. The cross section thus averaged is given in the third line of tables 1 and 2. The total cross-section of the process may be represented as

$$d\sigma = d\sigma_{el}+d\sigma_{soft}+d\bar{\sigma}_h = d\sigma_0(1-\delta). \tag{4.2}$$

In the fourth line the radiation corrections δ to the Møller formula are given in percent.

Note that the calculation of the cross-section and averaging with respect to the counter were performed accurately to within $\approx 2\%$. However, in the expression for the total cross-section $d\sigma$ there arises a difference of two large numbers and the final result is obtained with a lower accuracy which may be estimated $\approx 5\%$.

Eqs. (4.1) are of independent interest since they characterise the averaging by the counter of the radiation collision cross-section.

5. Dependence of the Radiation Collision Cross-Section on Threshold Energy

The condition $E_0 > E_1$, $E_0' > E_1$ may become essential if the quantity E_1 is sufficiently large. This problem has not been considered so far. Fig. 6 gives the dependence of the total cross-section $(d\sigma_0 - d\sigma)$ on E_1 for $E = 100$ MeV, $\alpha = 1.25°$, $\vartheta = 90°$, when one of the electrons passes through the centre of the counter.

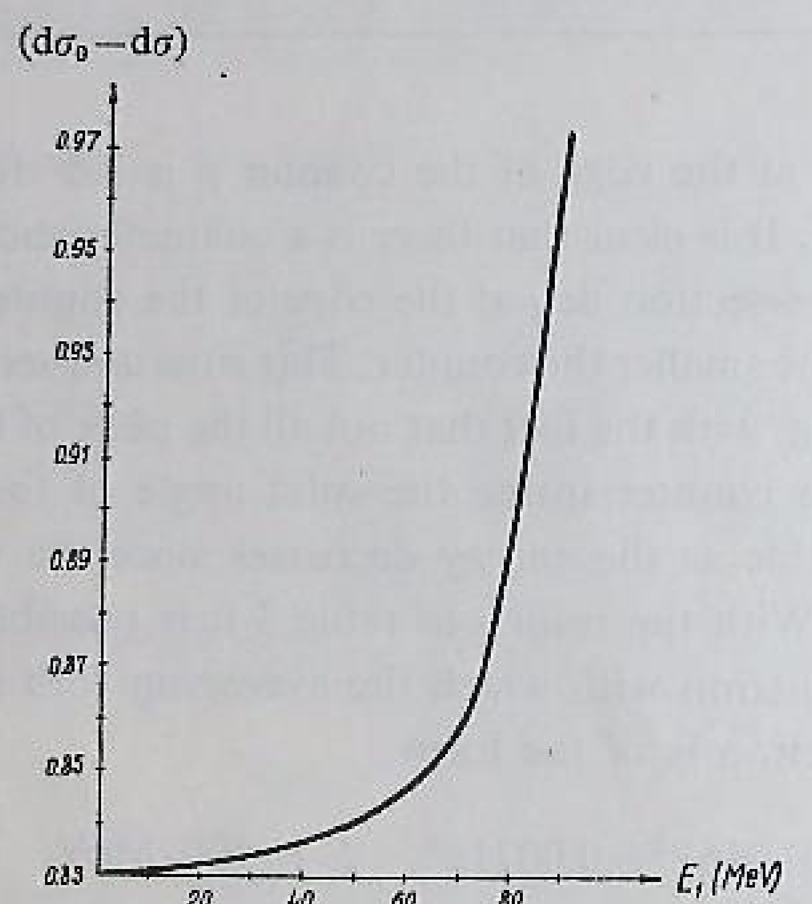

Fig. 6. Cross-section $(d\sigma_0 - d\sigma)$ as function of registration threshold energy.

Of considerable interest is the variation of the radiation corrections with the characteristic parameters of the problem, the electron energy, the dimensions of the counters and the registration threshold energy. The dependence is illustrated by table 4.

TABLE 4

Radiation correction as function of the parameters of the problem (without averaging over the counter)

Parameters			Radiation correction δ in % for scattering angles ϑ	
E_1(MeV)	α	E(MeV)	40°	50°
50	1.25°	100	13.8	15.1
5	1.25°	100	13.7	15.0
5	3.5°	100	7.9	9.0
5	3.5°	500	9.5	10.9

6. Limiting Cases

In this section we consider situations in which the analytical expressions for $d\sigma_h$ can be calculated. This proves to be possible in the limit of soft quanta as well as in that of very hard photons.

Suppose we are interested in the cross-section for radiation collision with emission of relatively soft photons $k \ll E$. If $k < E\alpha$ the condition $\boldsymbol{p}_0' \cdot \boldsymbol{p}_0 / |\boldsymbol{p}_0'||\boldsymbol{p}_0| > \cos\alpha$ imposes no restriction on the photon emission angles and there remains only the condition $\Delta E < k < \varepsilon$, where $\varepsilon < E\alpha$ and $\varepsilon \ll E$. In this case the integral over $d\Omega_k$ can be taken with respect to all directions, and terms of the type $(1/k^2)\psi(E, k)$ only kept in eq. (2.4); we can take $\psi(E, k = 0)$. As a result we obtain

$$d\sigma_h(\vartheta) = d\sigma_0(\vartheta)\frac{4e^2}{\pi}\ln\frac{\varepsilon}{\Delta E}\left(2\ln\frac{E\sin\vartheta}{m} - 1\right). \tag{6.1}$$

In the other limiting case, that of very hard photons, an analytical expression can be obtained by assuming the counters sufficiently narrow. Then the condition $\boldsymbol{p}_0' \cdot \boldsymbol{p}_0 / |\boldsymbol{p}_0'||\boldsymbol{p}_0| > \cos\alpha$ for the case when one of the electrons passes through the centre of the counter can approximately be written as $k \sin\vartheta_k < E\alpha$ and for $k > \Delta E_1$, we get $\sin\vartheta_k < E\alpha/\Delta E_1$. For narrow counters, such that $E\alpha/\Delta E_1 \ll 1$, the quantity ϑ_k becomes very small. Consequently the integration over $d\Omega_k$ must be performed in this case in the narrow cones around the directions $\boldsymbol{p}_0'$, $\boldsymbol{p}_0$ so that the main contribution to the integral comes from the terms containing

$$\frac{1}{c_2} \propto \frac{1}{1 - \dfrac{|\boldsymbol{p}_0|}{E_0}\cos\gamma}, \quad \frac{1}{c_4} \propto \frac{1}{1 + \dfrac{|\boldsymbol{p}_0'|}{E_0'}\cos\gamma'}.$$

The integrand can be represented as a series in powers of $(1-\cos\gamma)(1+\cos\gamma')$:

$$R = \frac{\psi(\cos\gamma = 1)}{1 - \dfrac{|\boldsymbol{p}_0|}{E_0}\cos\gamma} + \frac{\psi(\cos\gamma' = -1)}{1 + \dfrac{|\boldsymbol{p}_0'|}{E_0'}\cos\gamma'} + \text{const} + O[(1-\cos\gamma)(1+\cos\gamma')], \tag{6.2}$$

$$\cos\gamma = \frac{\boldsymbol{p}_0 \cdot \boldsymbol{k}}{|\boldsymbol{p}_0||\boldsymbol{k}|}, \quad \cos\gamma' = \frac{\boldsymbol{p}_0' \cdot \boldsymbol{k}}{|\boldsymbol{p}_0'||\boldsymbol{k}|}.$$

In integrating over $d\Omega_k$ the first two terms yield large logarithms of the type $\ln(E^2/m^2)$ and the subsequent terms are of the order $E\alpha/\Delta E_1$. If we confine ourselves to the first two terms we obtain an expression with relative accuracy of the order $(E\alpha/\Delta E_1)\ln^{-1}(E^2/m^2)$. Estimates show that accuracy of the order of several per cent can be reached for $\Delta E_1 \approx 100$ MeV (at $E = 500$ MeV).

Proceeding from the above consideration we obtain (see appendix 1) the asymptotic formula

$$d\sigma_h(\Delta E_1 < k < \varepsilon_1) = d\sigma_0\frac{2e^2}{\pi}J(k_1, k_0), \tag{6.3}$$

where

$$I(k_1, k_0) = \ln \frac{k_1}{k_0}\left(\ln \frac{\lambda_0^2}{k_0 k_1} - 1\right) + \tfrac{1}{2}k_1^2\left(\tfrac{1}{2} + \ln \frac{\lambda_0}{k_1}\right) - \tfrac{1}{2}k_0^2\left(\tfrac{1}{2} + \ln \frac{\lambda_0}{k_0}\right) - 2k_1\left(1 + \ln \frac{\lambda_0}{k_1}\right)$$
$$+ 2k_0\left(1 + \ln \frac{\lambda_0}{k_0}\right) + \tfrac{1}{2}(1-k_0)^2[\tfrac{1}{2} - \ln(1-k_0)] - \tfrac{1}{2}(1-k_1)^2[\tfrac{1}{2} - \ln(1-k_1)]$$
$$+ (1-k_0)[2 - \ln(1-k_0)] - (1-k_1)[2 - \ln(1-k_1)] + 2\int_{1-k_1}^{1-k_0} \frac{\mathrm{d}\xi \ln\xi}{1-\xi}. \tag{6.4}$$

Here we have

$$k_1 = \min\left\{\frac{\varepsilon_1}{E}, \frac{E\alpha}{m+E\alpha}\right\}, \quad k_0 = \frac{\Delta E_1}{E}, \quad \lambda_0 = \frac{E\alpha}{m}, \quad \varepsilon_1 = E - E_1. \tag{6.5}$$

Eq. (6.3) gives the cross-section for electron-electron scattering with emission of very hard photons whose energies lie in the interval $\Delta E_1 < k < \varepsilon_1$ and $\Delta E_1 \gg E\alpha$. It should be noted that the entire angle dependence in the expression for $\mathrm{d}\sigma_\mathrm{h}$ (eq. (6.3)) is contained in the factor $\mathrm{d}\sigma_0$ (Møller cross-section) since $I(k_1, k_0)$ (eq. (6.4)) does not depend on the scattering angle. From comparison with the numerical result it is clear that eq. (6.3) approximates the integration result sufficiently well: thus for $E = 500$ MeV, $\alpha = 3.5°$ and $\Delta E_1 = 150$ MeV, the numerical result is $I(k_1, k_0) = 5.05$ for the scattering angle $\vartheta = 40°$ and $I(k_1, k_0) = 5.01$ for the scattering angle $\vartheta = 90°$; from eq. (6.4) we obtain $I(k_1, k_0) = 5.07$.

The authors wish to thank V. Synakh for valuable advice, V. Sidorov for discussions and O. Kapranova, L. Sekerina and B. Butenko for their encouragement and assistance in the calculations.

Appendix 1

DERIVATION OF THE RADIATION COLLISION CROSS-SECTION IN THE CASE OF HARD PHOTON RADIATION

On the basis of the analysis performed in sect. 6 let us keep in eq. (2.4) only the terms proportional to $1/c_2$ (the terms $\propto 1/c_4$ give the same contribution and therefore it is sufficient to calculate the contribution from the terms with $1/c_2$ and then double the result). As a result we have

$$R \approx \frac{2}{c_2 g_1}\left\{\frac{1}{g_1}[c_4(pp') + c_3(p'p_0')] - \left[\frac{m^2}{c_2} - \frac{2(p_0 p')}{c_1}\right]\left[\frac{(pp')}{g_1}(c_4 + (p_0 p_0'))\right.\right.$$
$$\left. + \frac{(p'p_0')}{g_1}(c_3 + (pp_0)) + \frac{(pp')}{g_4}c_4\right] + \text{exchange terms } (p_0 \leftrightarrow p_0')$$
$$\left.+ \frac{2(pp')}{g_4}\left[c_4 - (p_0 p_0')\left(\frac{m^2}{c_2} - \frac{2(p_0 p')}{c_1}\right)\right]\right\}. \tag{A.1}$$

For the sake of simplicity let us consider the case when one of the electrons passes through the centre of the counter ($\gamma = \vartheta_k$). In this case we have

$$R \approx \frac{1}{c_2}\left(\frac{3+\cos^2\vartheta}{\sin^2\vartheta}\right)^2\left(\frac{2E}{k} + \frac{k}{E} - \frac{m^2}{c_2} - 2\right). \tag{A.2}$$

We must calculate the integral (2.2):

$$\mathrm{d}\sigma_\mathrm{h}(\Delta E_1 < k < \varepsilon_1) = \frac{e^6 \mathrm{d}\Omega}{16\pi^2 E^2}\, 2\int k\mathrm{d}k\mathrm{d}\Omega k\, \frac{E-k}{E} R, \tag{A.3}$$

the integral being taken over the solid angle of one of the counters. As was indicated the integration limits are

$$\Delta E_1 < k < \varepsilon_1, \quad \Delta E_1 \gg E\alpha. \tag{A.4}$$

The condition $\boldsymbol{p}_0 \cdot \boldsymbol{p}_0'/|\boldsymbol{p}_0||\boldsymbol{p}_0'| > \cos\alpha$ (for $\gamma = \vartheta_k$) can be written as

$$\cos\vartheta_k > 1 - \frac{\alpha^2 E^2}{2k^2}, \quad \cos\vartheta_k < -1 + \frac{\alpha^2 E^2}{2k^2}. \tag{A.5}$$

In the integration over one of the counters only the first of these conditions remains.

Calculating the integral

$$\mathrm{d}\sigma_\mathrm{h}(\Delta E_1 < k < \varepsilon_1) = \mathrm{d}\sigma_0 \frac{e^2}{\pi}\int_{\Delta E_1}^{\varepsilon_1} \mathrm{d}k\, \frac{k(E-k)}{E^2}\int_0^{E^2\alpha^2/2k^2} \mathrm{d}x$$
$$\times\left[\frac{1}{E_0 - |\boldsymbol{p}_0|(1-x)} + \frac{E\boldsymbol{p}_0^2 x(2-x)}{k^2(E_0 - |\boldsymbol{p}_0|(1-x))^2}\right] \tag{A.6}$$
$$= \mathrm{d}\sigma_0 \frac{e^2}{\pi}\frac{1}{E^2}\int_{\Delta E_1}^{k_1 E} k\mathrm{d}k\left[\frac{E^2 + (E-k)^2}{k^2}\ln\frac{E^2(E-k)^2\alpha^2}{m^2k^2} - \frac{2E(E-k)}{k^2}\right]$$

we obtain eq. (6.3).

Appendix 2

BEHAVIOUR OF THE FUNCTION R NEAR MAXIMA

If we put the angle $\vartheta_k = 0$ in eq. (A.2), we obtain

$$R_\mathrm{max} \approx \left(\frac{3+\cos^2\vartheta}{\sin^2\vartheta}\right)^2 \frac{2(E-k)}{Em^2}, \tag{A.7}$$

the half-width of the peak being given by the formula

$$\vartheta_r^2 = \frac{m^2}{(E-k)^2}\left[\frac{2E(E-k)}{k^2} + \sqrt{\frac{4E^2(E-k)^2}{k^4} + 1}\right]. \tag{A.8}$$

For the case $k \ll E$ we have

$$\vartheta_r^2 \approx \frac{4m^2}{k^2}. \tag{A.9}$$

Appendix 3

REMARKS CONCERNING A CHECK OF THE APPLICABILITY OF QUANTUM ELECTRODYNAMICS AT SMALL DISTANCES IN ELECTRON SCATTERING EXPERIMENTS

If we assume that quantum electrodynamics does not apply at small distances, in the one-photon electron-exchange approximation (the terms containing the anomalous electron magnetic moment being neglected) we obtain the following expression for the negaton-negaton scattering cross-section:

$$\sigma^-(\vartheta) = \frac{r_0^2}{8\gamma^2}\left[\frac{s^4+q'^4}{q^4}f^4(q^2)+\frac{2s^4}{q^2q'^2}f^2(q^2)f^2(q'^2)+\frac{s^4+q^4}{q'^4}f^4(q'^2)\right]. \qquad (A.10)$$

The function $f(q^2)$ can be connected with the change of the electron-photon interaction vertex as well as with the modification of the photon distribution function at high energies (for more detail see ref. [14]). The restriction to the one-photon exchange is justified since graphs with exchange of two or more photons do not yield large logarithmic contributions. If we assume that the form factor $f(q^2)$ can be expanded in series:

$$f(q^2) = 1-a^2q^2, \qquad (A.11)$$

the negaton-negaton scattering cross-section can be represented as

$$\sigma^-(\vartheta) = \sigma_{0M}(\vartheta)-\Delta\sigma_M(\vartheta). \qquad (A.12)$$

It appears that the relative change of the cross-section (c.m.s.) is

$$\left(\frac{\Delta\sigma(\vartheta)}{\sigma_0(\vartheta)}\right)_M = \frac{24p^2a^2\sin^2\vartheta}{3+\cos^2\vartheta}. \qquad (A.13)$$

In the case of negaton-positon scattering we obtain

$$\sigma^+(\vartheta) = \frac{r_0^2}{8\gamma^2}\left[\frac{s^4+q'^4}{q^4}f^4(q^2)+\frac{2q'^4}{s^2q^2}f^2(q^2)f^2(s^2)+\frac{q'^4+q^4}{s^4}f^4(s^2)\right]$$
$$= \sigma_{0B}(\vartheta)-\Delta\sigma_B(\vartheta). \qquad (A.14)$$

For ultra-relativistic electrons the formula for negaton-negaton scattering (Bhabha formula) has the simpler form (in the c.m.s.)

$$\sigma_{0B}(\vartheta) = \frac{r_0^2}{16\gamma^2}\left(\frac{3+\cos^2\vartheta}{1-\cos\vartheta}\right)^2. \qquad (A.15)$$

If it is assumed that the expansion of the form factor for time-like and space-like momentum transfers is the same and is of the form (A.11) it appears that the relative change of the cross-sections is the same

$$\left(\frac{\Delta\sigma(\vartheta)}{\sigma_0(\vartheta)}\right)_M = \left(\frac{\Delta\sigma(\vartheta)}{\sigma_0(\vartheta)}\right)_B. \qquad (A.16)$$

Thus, under the assumption made, despite the fact that for the case of negaton-positon scattering the scattering can be measured through an angle larger than 90°, where large momentum transfers occur, the maximum deviation from the electrodynamic cross-sections takes place for the scattering angle 90° just as in the case of negaton-negaton scattering. In this sense the possibilities for e^--e^- scattering and e^--e^+ scattering experiments prove equal.

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