At the present time the first results on electron beam influence upon positron beam are obtained at electron-positron storage ring VEPP-I. At 100 MeV energy electron current up to 400 mA (10'' particles) and positron current up to 0,5 mA (10^8 particles) can be obtained being stored separatly.

But at the presence of positron beam the effects of beam-beam interaction are already essential while storing electron current of the order of several mAs.

Every storing cycle exites the vertical betatron oscillations of electron bunch which gives growth of vertical positron beam dimension and that gives positron losses, it electron current is sufficiently high.

The effect can be avoided by means of non-uniform electric field, which splits the betatron frequencies of electrons and positrons. Besides this for reducing the growth of transverse dimensions of positron bunch and for increasing its lifetime the electron and positron orbits are also separated at the injection.

In this way electron current up to 50-60 mA can be stored without positron losses.

While the beams being intersected the transverse dimensions and lifetime of positron beam strongly depend upon electron current, upon frequency of betatron oscillations of positrons and upon the energy at wish the interaction is performed.

These effects are analogous to the effects observed in the machine VEP-1.

The main particularity in the observed effects is the ever present increasing of the radial dimension of the positron beam when there is a collision. This rise is of the order of a factor at the energy of 200 MeV and it is correlated, may be, with a presence of small coherent radial oscillations in the electron bunch (Fig. 8).

At the present time an intersection can be performed at an electron current up to 10 mA at 100 MeV energy and up to 50 mA at 200 MeV energy without essential roth of axial dimension of the positron beam and without essential lifetime reduction.

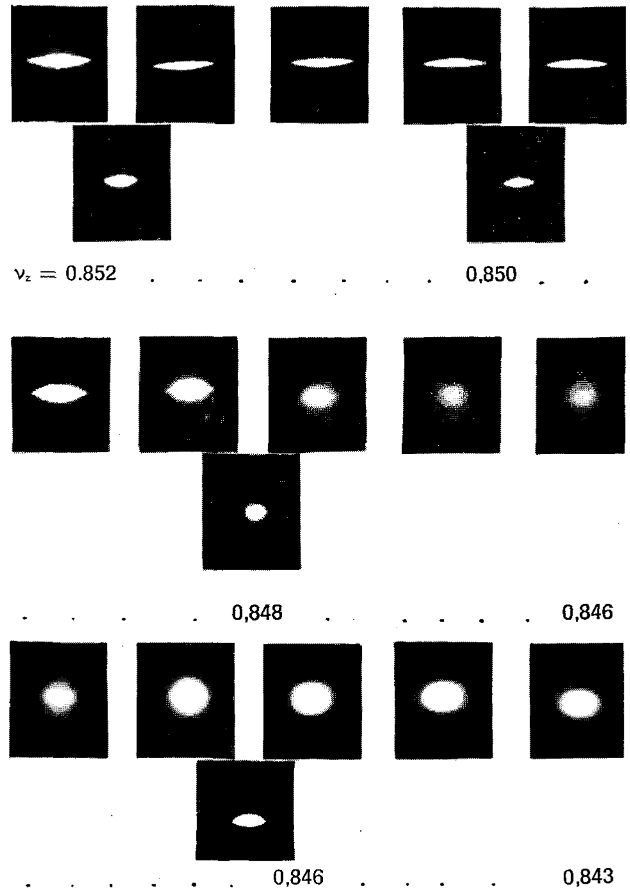

Fig. 8

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PHASE INSTABILITY OF INTENSE ELECTRON BEAM IN A STORAGE RING

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I. In working out the program of creating storage rings for carrying out colliding beam experiments a supposition was made by G. I. Budker and A. A. Naumov in 1959 that at high intensities phenomena of phase intability are possible due to the interaction between the beam and the accelerating cavities and other elements of vacuum chamber.

This topic was theoretically investigated by a number of authors. According to the stability conditions obtained in (3, 4, 6) it is sufficient to tune the accelerating cavities at a frequency

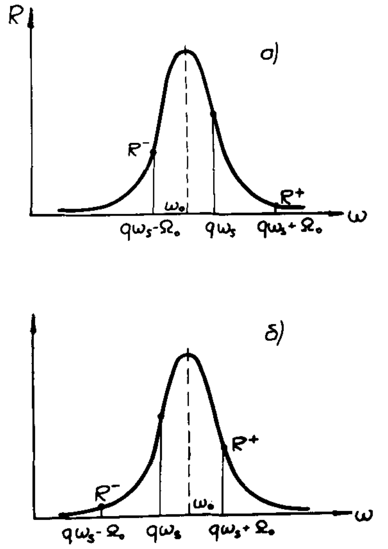

Fig. 1 - Resonator frequency response.

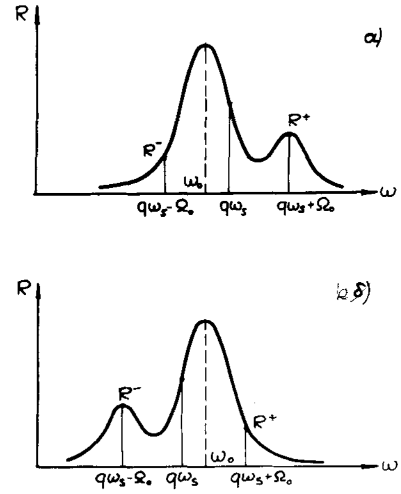

Fig. 2 - Resonator frequency response with "parasitic" resonance.

somewhat lower than the frequency of r.f. generator to prevent the possibility of excitation of phase autoscillations at an arbitrary high beam intensity.

In the process of experimental investigations of storing high intensity currents in electron-electron storage rings (1) (VEP-1) and in electron-positron storage ring (2) (VEPP-2) it was found that stability conditions obtained in (3, 4, 6) were insufficient. A great detuning in a proper direction did not suppress oscillations at high current intensities.

A more careful investigation of stability make it necessary to take into account the following factors:

a) The real frequency response curve of the accelerating system differs from the ideal single resonant circuit.
b) There is a possibility of resonant excitation of the accelerating system and other chamber elements by beam current at circulation frequency harmonics.
c) There is a spread in the natural radial-phase oscillation frequencies of different electrons.

II. As it is known, the equation of the radial-phase oscillations of electrons in the storage ring may be written as:

$$\ddot{\varphi} + \frac{\omega_s}{2\pi}\frac{dW}{dE}\dot{\varphi} + \Omega_0^2\,\varphi = 0 \qquad [1]$$

$$\Omega^2_0 = \frac{U_m \mid \cos\varphi_s \mid \, q\,\omega_s^2\,\alpha}{2\,\pi\,E_s} \qquad [2]$$

where q is a harmonic number of r.f., ω_s is the angular velocity of equilibrium particle, α is a momentum compaction factor, W is a radiation loss of energy per revolution. Radiation loss consists of the coherent and incoherent losses. For coherent losses, provided $\Omega \ll 1/r$, where τ is the transition time of the accelerating system, it is possible towrite down:

$$\frac{dW^{coh}}{dE} = -I_q \cdot q\frac{dR}{d\omega_r}\cdot\frac{d\omega}{dE} = -I_q \cdot q\frac{\omega_s\,\alpha}{E_s}\cdot\frac{dR}{d\omega_r} \qquad [3]$$

where R is the resistance of the accelerating system for beam, ω_r is the angular frequency of the generator ($\omega_r = q\ \omega_s$), I_q is the magnitude of beam currents q harmonic. The stability condition lies in the positiveness of the coefficient of the second term of the equation [1], so that

$$I_q \cdot \frac{\Omega_0^2\tau_c}{2\,U_m \mid \cos\varphi_s \mid}\cdot\frac{dR}{d\omega_r} < 1 \qquad [4]$$

where τ_c is the radiation damping time of oscillations.

Hence at

$$\frac{dR}{d\omega_r} < 0 \qquad [5]$$

the system is stable at any currents. Thus, for a single ideal resonant circuit the inequality [5] is satisfied when its natural frequency lies below the frequency of the generator.

III. For more general case the stability condition may be obtained by means of the following consideration. Let us consider the stability of the system consisting of the accelerating device and a bunch of electrons, phase oscillation of which is possible.

If in the frequency spectrum of the accelerating voltage there are frequencies which are at $\pm\Omega$ away from the generator frequency, then the bunch performs oscillations described by the equation

$$\ddot{\varphi} + \frac{2}{\tau_c}\dot{\varphi} + \Omega_0^2\,\varphi = \frac{\Omega_0^2}{U_m\,|\cos\varphi_s|}\,U_{sb\,m}\sin\Omega t \qquad [6]$$

where U_{sbm} is the voltage amplitude of the sideband frequency.

The induced phase oscillations of the bunch are described by the following expression:

$$\varphi(t) = \Phi_m \sin(\Omega t + \psi_\varphi) \qquad [7]$$

where

$$\Phi_m = \frac{Q_c}{\sqrt{1+Q_c^2 x_c^2}}\cdot\frac{U_{sb\,m}}{U_m\,|\cos\varphi_s|}; \qquad [8]$$

$$\psi_\varphi = \frac{\pi}{2} - \operatorname{arctg} Q_c x_c; \qquad [8]$$

$$Q_c = \frac{\Omega_0\tau_c}{2}; \quad x_c = \frac{\Omega}{\Omega_0} - \frac{\Omega_0}{\Omega} \approx \frac{2(\Omega-\Omega_0)}{\Omega_0} \qquad [9]$$

The bunch current i_q is the phase modulated current:

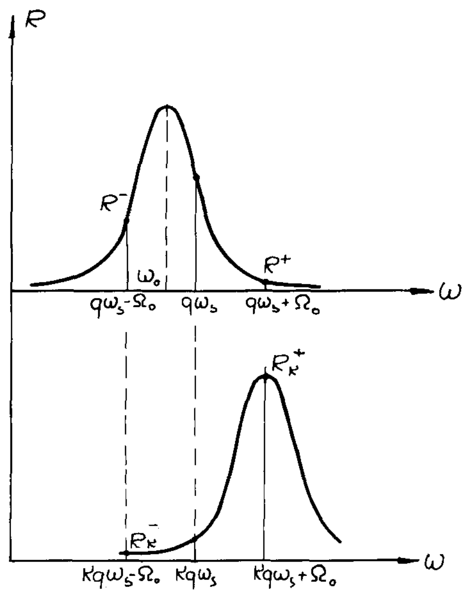

Fig. 3 - « Parasidic » resonance at high harmonic.

$$i_q = -I_q\cos[q\,\omega_s t - \varphi(t)] \qquad [10]$$

Such a current also contains sideband components which produce sideband frequency voltage on the accelerating system.

Acting on the bunch, the instantaneous sideband voltage is equal to (provided $\varphi \ll 1$):

$$U_{sb} = \frac{I_q\Phi_m}{2}\{[(R^+ - R^-)\cos\psi_\varphi - (X^+ + X^-)\sin\psi_\varphi]\cos\Omega t -$$
$$- [(R^+ - R^-)\sin\psi_\varphi + (X^+ + X^-)\cos\varphi_\varphi]\sin\Omega t\} \quad [11]$$

where R^+, R^- are active components of impedances for the lower and upper sideband frequencies, X^+, X^- are reactive components of impedances.

The stability criterion of this locked bunch-cavity system can be found by means of Nyguist's criterion which in this case is:

$$(R^+ - R^-)\cos\psi_\varphi - (X^+ + X^-)\sin\psi_\varphi = 0$$

$$\frac{I_q\Phi_m}{2}[(R^+ - R^-)\sin\psi_\varphi + (X^+ + X^-)\cos\psi_\varphi] < U_{sb\,m} \qquad [12]$$

From the first equation with [9] taken into account we have

$$Q_c x_c = \frac{X^+ + X^-}{R^+ - R^-} \qquad [13]$$

The second inequality together with [8], [9] and [13] give the stability criterion

$$\frac{I_q Q_c}{2\,U_m\,|\cos\varphi_s|}(R^+ - R^-) < 1 \qquad [14]$$

We note that the equation [13] defines the frequency of the small amplitude auto-oscillations which take place when inequality [14] is not satisfied.

To make the stability criterion more precise, it is necessary to take into account the fact that electrons undergo random oscillations initiated by the quantum fluctuations with statistical distribution of amplitudes. The problem can be solved in the supposition that the system is linear and the action of fluctuations only changes the natural frequencies of oscillations.

The stability criterion, the spread in the natural frequencies taken into account are the same as in [14], but instead of Q_c there is $Q_s' \leqslant Q_c$. The quantity Q_c' depends upon the accelerating system properties in contrast to Q_c. The interaction of the bunch with the accelerating system and other chamber elements can take place not only on q harmonic, but on other

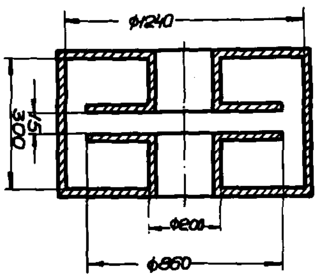

Fig. 4 - Schematic view of resonator.

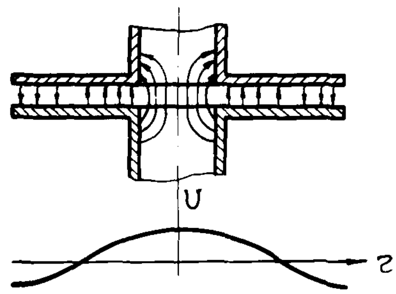

Fig. 5 - « Parasitic » resonator.

harmonics if the accelerating system has parasitic resonances of frequencies near the harmonic frequencies. The stability condition with the interaction on harmonics taken into account for q identical bunches is

$$\frac{I_q \cdot Q_c'}{2 U_m | \cos \varphi_s |} \sum_{k=1} K C_{kq} \cdot D^2_{kq} (R^+_{kq} - R^-_{kq}) < 1 \qquad [15]$$

where C_{kq} is the relative value of the kq current harmonic ($C_{1q} = 1$), D_{kq} is the coefficient taking into account the electron transit angle in the accelerating gap, R^+_{kq} and R^-_{kq} are active components of the accelerating system impedance at kq harmonic sideband frequencies ($\omega_{kq} + \Omega_o$ and $\omega_{kq} - \Omega_o$).

The active component of the impedance of the ideal and real resonant circuit at different detuning is shown in Fig. 1a, b and 2a, b. The systems 1a and 2b are stable, the systems 1b and 2a can be unstable although according to (3, 4, 6) the sistem 2a must be stable and the system 2b must be unstable. Fig. 3 shows the example of the frequency response curve with the additional resonance at one of the harmonics of the circulating frequency.

IV. The experimental data obtained in VEPP-2 are listed below. Fig. 4 shows the arrangement of the machine VEPP-2 resonator. Its natural frequency is about 25,1 Mc/s. Phase oscillations were observed by means of frequency detector. While testing the storage ring VEPP-2 it was found that when keeping the generator frequency constant near 25,1 Mc the stored current strongly depended on the cavity tuning. For storing the intense current, the cavity should be detuned so that its natural frequency lie below the generator frequency for about 4 - 5 frequency bands. If the limiting current was stored when cavity was strongly detuned, then while retuning the cavity, phase oscillations rose and the fraction of the stored particles was lost.

It has been found later that the section of the radial line formed by cavity discs gives a resonance with a wave-type E_{o10}. This type of oscillations has a maximum voltage at resonator axes (Fig. 5). The resonance frequency of the parasitic resonance is about 452 Mc exceeding the frequency of the 18th harmonic by 200 kc. This corresponds to Fig. 3 and brings the phase instability of the beam. To remove this instability a special element is introduced into the cavity. This element makes it possible to tune the parasitic resonator so that its natural frequency comes below the 18th harmonic of the particle circulation frequency. Such returning made it postible to remove the observed phase instability.

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