EFFECTS OF ELECTROMAGNETIC INTERACTION BETWEEN PARTICLES AND COLLIDING BUNCH

Ja. S. Derbenev, S. I. Mishnev and A. N. Skrinsky

Institute of Nuclear Physics, Siberian Division of the USSR Academy of Sciences, Novosibirsk (USSR)

1) In the study of the influence of electromagnetic field of the colliding bunch on betatron oscillations of particles from the low intensity bunch, one may restrict oneself, for simplicity, to azymuthally symmetric type of weak focusing electron-electron storage ring (the influence of "small" bunch on the colliding bunch particles is negligible). The equations for the one-dimensional particle oscillations periodically perturbed by the colliding bunch of the given configuration are to be

$$\frac{1}{2}\frac{da^2}{d\theta} = \frac{R^2}{\nu E} f_x(x,\theta)\, a\cos\psi \equiv W(a,\psi,\theta) \qquad [1]$$

$$\frac{d\varphi}{d\theta} = -\frac{R^2}{\nu E} f_x(x,\theta)\frac{1}{a}\sin\psi \equiv \Delta\nu(a,\psi,\theta) \qquad [2]$$

Here $x = a\sin\psi = a\sin(\nu\theta + \varphi)$, ν is betatron frequency, θ is azymuthal angle, R is orbit's radius, E is particle energy, f_x is the x-component of the transverse part of the force of electromagnetic particle interaction with bunch periodic with respect to ψ and θ, with 2π period. For the simplicity of presentation one assumes in that follows, that $f_x(x,\theta)$ is an even function of θ and the odd one of x.

The right-hand sides of [1] and [2] are, therefore, instantenous power W and frequency shift $\Delta\nu$ due to the interaction with colliding bunch being also periodic with respect to ψ and θ. The frequency shift $\Delta\nu$ contains the amplitude dependent constant component $\Delta\nu_0(a^2)$ whose asymptotic behaviour at $a \to \infty$ is $N r_e R/\pi\nu\gamma a^2$ where N is the number of particles in the bunch, $r_e = 3\cdot 10^{-13}$ cm is the classical electron radius, an γ is a relativistic factor.

With $\Delta\nu_0(a^2)$ taken into account the mean effective frequency $\nu(a^2)$ depends on the amplitude, as shown in Fig. 1. So, in principle, one may expect the appearance of any resonance $q\nu(a^2) = p$, where p and q are the prime numbers satisfying the inequality.

$$\nu + \Delta\nu_0(0) < \frac{p}{q} < \nu \qquad [3]$$

$\Delta\nu_0(0)$ in the order of the magnitude is $N r_e R/2\gamma\nu s$ where s is the effective cross-section of the bunch.

For the description of the effects corresponding to this resonance we used the first approximation equations making use of the method of averaging (1, 2, 3) in term of a^2 and the phase difference $\Phi = \psi - (p/q)\theta$:

$$\frac{da^2}{d\theta} = -\frac{\partial H}{\partial\Phi} = B\sin q\Phi \qquad [4]$$

$$\frac{\partial\Phi}{d\theta} = \frac{\partial H}{\partial a^2} = \nu - \frac{p}{q} + \Delta\nu_0(a^2) + \frac{1}{q}\frac{dB}{da^2}\cos q\Phi \qquad [5]$$

where

$$H(a^2,\Phi) = \left(\nu - \frac{p}{q}\right)a^2 + \int_0^{a^2}\Delta\nu_0(a^2)\,da^2 + \frac{1}{q}B(a^2)\cos q\Phi$$

$$B(a^2) = \frac{1}{\pi^2}\int_0^{2\pi}d\psi\int_0^{2\pi}d\theta\, W(a^2,\psi,\theta)\sin(q\psi - p\theta)$$

In Fig. 2 are shown the curves $H(a^2,\Phi)$ for the values $\Phi = \pi/q$ and $\Phi = 0$; others are placed between them. In region I $d\Phi/d\theta < 0$, i.e. the mean frequency is less than the resonant one; in region III, on the contrary, $d\Phi/d\theta > 0$. Finally, the region II is just the resonant region. With the movement in this region the point $\partial H/\partial a^2 = d\Phi/d\theta = 0$ is crossed periodically i.e. the mean frequency is equal to the resonance value p/q. Phase Φ undergoes here limited oscillations near $\Phi_s = (2K+1)\pi/q$, K = 0, 1, 2, 3... The bottom point of the region II defined by the equation $\partial H(a^2,\Phi)/\partial a^2 = 0$ (the point A of the Fig. 2) corresponds to the closed stable orbits whose number is q. The stability of these new equilibrium orbits is provided by a peculiar phase stability mechanism. The main characteristics of the motion here are the equilibrium amplitude a_s and the phase Φ_s determining the closed orbit.

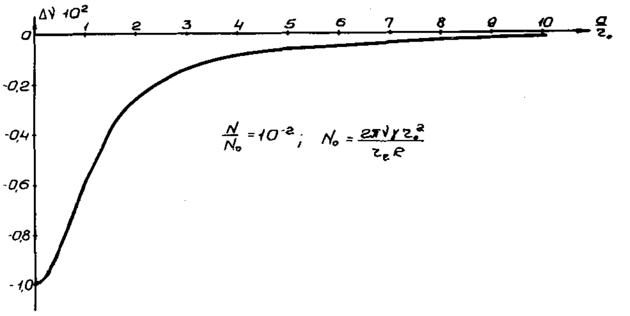

Fig. 1 - Dependence of frequency on the amplitude for the bunch with the density distribution being

$$\sigma(z, r) = (\pi r_o^2)^{-1}\left(1 + \frac{z^2 + r^2}{r_o^2}\right)^{-2}.$$

The amplitude a_s is found from the equation:

$$\nu - \frac{p}{q} \simeq -\Delta \nu_o (a_s^2). \qquad [6]$$

The frequency of small oscillations of the phase Φ near Φ_s by the order of the magnitude is

$$\Omega_s \simeq \left(qB\frac{d\nu}{da^2}\right)_A^{1/2} \qquad [7]$$

and the width of the phase stability region with respect to the amplitudes is:

$$\Delta a \simeq \frac{1}{a_s}\left(\frac{B}{q\frac{d\nu}{da^2}}\right)_A^{1/2} \qquad [8]$$

The whole consideration is carried over the electron-positron case by replacing $p \to mp$ (m is the number of collisions per turn) with the difference that p and q must satisfy the condition

$$\nu < \frac{mp}{q} < \nu + \Delta\nu_o(0)$$

2) Due to the small angular dimension of the interaction region θ_0 and highly nonlinear behaviour of the perturbation force ($f_x \sim 1/x$ beyond the transverse dimensions of the bunch) in the power expansion over Fourier series there are harmonics with very high p and q. We shall not touch, for the moment, the question of applicability of the method of averaging in such a situation and, get the restraint on the number of acting resonances by introducing the radiation damping into the equations [4], [5]. The equations will *then be as:

$$\frac{da^2}{d\theta} = B \sin q\,\Phi - 2\lambda a^2 \qquad [9]$$

$$\frac{d\Phi}{d\theta} = \nu - \frac{P}{q} + \Delta\nu_o(a^2) + \frac{1}{q}\frac{dB}{da^2}\cos q\,\Phi \qquad [10]$$

where λ is the decrement of damping in $(\mathrm{rad})^{-1}$. From [9] it follows that the parasitic equilibrium orbits are absent at

$$2\lambda a_s^2 \geqslant |B(a_s^2)| \qquad [11]$$

which determines the boundary of the parameters $N, \nu - p/q, \lambda$), beyond which the resonance ceases to act.

In Fig. 3 are shown zones of one-dimensional resonance action in the working region of frequencies ν_z and ν_r in the electron-positron storage ring provided there are two collisions per turn. As it can be seen from the picture, zones of "low" resonances are almost undisturbed while the zone of "high" resonances rise and become narrower. Whit the sufficiently small $Nr_eR/2\gamma\nu s$ there are regions of frequencies free of resonances of the one-dimensional oscillations.

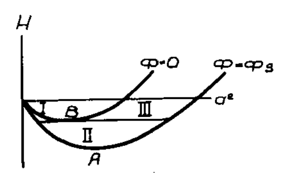

Fig. 2

Taking into account the radiational friction allows one to classify resonances as "working" and "non-working". Our consideration seems to be valid until phase stability regions of the working resonances are not overlapped according to the amplitude of betatron oscillations. This condition can be written down as

$$\sqrt{\frac{1}{q}\frac{d\nu}{da^2}B}\,\delta\nu \ll 1 \qquad [12]$$

where $\delta\nu$ is the distance to the nearest working resonance.

3) Let us consider now to what unpleasant effects may the presence of parasitic equilibrium orbits lead to. The main equilibrium orbit ($a = 0$) is left stable by the nonlinear resonances. They cannot hinder the exact matching of bunches since particle passes through resonances leaving the parasitic orbits aside. So, the harmful in-

fluence of the new orbits may be connected with the stochastic process, only.

It is due to the scattering from the residual gas and quantum fluctuations that particles get into the stability region of parasitic orbits and damp toward them. Then, however, particle will not have an infinitely long « life » on the orbit and can scatter and damp toward the main orbit. An equilibrium between the main and the parasitic orbit is established with time. Such an "instability" is likely to be developing within periods of time defined by the scattering and damping times. The estimation indicates that under some conditions the relative number of particles on parasitic orbits can be very large.

Besides, the appearance of strong modulation of the particle oscillation amplitude affected by the colliding bunch near resonances can decrease essentially the effective chamber aperture, decreasing thus the lifetime of particles in the storage ring.

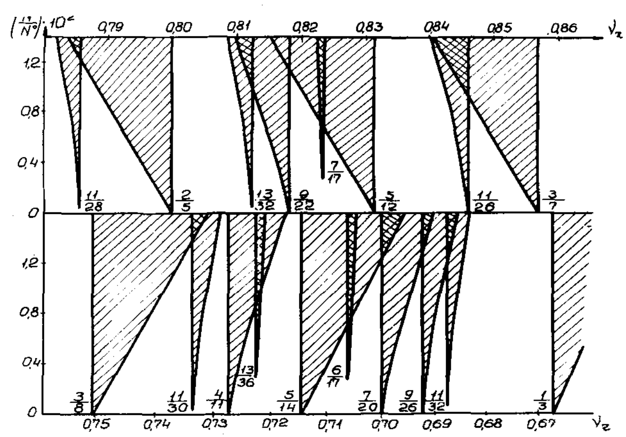

Fig. 3 - Zones of resonances action for the "colliding bunch with the density distribution to be

$$\sigma(z, r, \theta) = (\pi\, r_0^2)^{-1}\left(1 + \frac{z^2 + r^2}{r_0^2}\right)^2 \left(\sqrt{\pi}\, R\, \theta_0\right)^{-1} \exp^{-1}\left(\frac{\theta^2}{\theta_0^2}\right);$$

$(E = 200\,\text{MeV};\quad R = 150\ \text{sm};\quad \theta_0 = 0{,}03\ \text{rad}).$

4) For two-dimensional oscillations the perturbation from the colliding bunch leads to the nonlinear coupling between z and r-oscillations. The consideration of each coupling resonance seems to lead to effects similar to one-dimensional resonance. It may be shown that synchrobetatron resonances provide just the same effect.

5) As it was mentioned above, the consideration of individual resonance of one-dimensional oscillations is valid, if

$$\left(\sqrt{\frac{1}{q} B \frac{d\nu}{da^2}} \,/\, \delta\nu\right) \ll 1$$

In the inverse limiting case

$$\left(\sqrt{\frac{1}{q} B \frac{d\nu}{da^2}} \,/\, \delta\nu\right) \gg 1$$

and in accordance with the main result (5) the motion of the system [1], [2] loses its stability and becomes stochastic (the difference $\delta\nu$ is taken, naturally, between resonances for which the B values are of the same order). This criterion may, apparently, be generalized for the two-dimensional oscillations in the following manner: motion becomes stachastic if the total phase volume with phase stability for resonances acting in this area of the phase space exceeds greatly the volume of this area. The description of the motion in the stochastic region can only be statistical. Synchrotron oscillations of the particle perturbed by the colliding bunch substantially facilitate the arise of stochasticity.

6) The consideration of the influence of beam-beam interaction of synchrotron oscillations (neglecting betatron oscillations) lead to the conclusion that for electron-electron bunches the equilibrium phase becomes instable at sufficiently high currents. So, two new stable equilibrium phases arise and are located to the right and to the left of the old one with the distance $\sim 1/R$ (l is the bunch's length).

Thus, if orbits are intersected at the angle β such that $\beta l \gg b$ (b is the largest transverse size of bunch), than the unperturbed equilibrium phase becomes instable, when

$$V_{0\,KV} < 10^{-9}\frac{R}{l^2} \qquad (V_0 \text{ — r. f. amplitude}) \qquad [13]$$

For electron-positron bunches the linear effect is of no danger.

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(4) Here and in what follows we write don formulas for the even q; for the old q one must replace q → 2q, p → 2p.
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