influence of the multiple scattering on the large angle $\theta = 50$ mrad.

The very important goal made with this orientation is the high value of the calculated linear polarization i.e., 78% at the first peak. This was first pointed out Überall (5).

He suggested a different orientation; for experimental convenience we used the previously mentioned one.

During these measurements the γ-ray beam intensity was 10^{10} eQ/m, but an upper limit of 5×10^{11} e Q/m can be reached.

We thank G. Diambrini for his constant interest and useful collaboration at this work.

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DISCUSSION

WINICK: What are your plans for measuring the polarization of the peaks you observe?

BOLOGNA: We plan to use a second crystal inside the pair spectrometer, as proposed by G. Barbiellini, G. Bologna, G. Diambrini and G. P. Murtas (Nuovo Cimento *28*, 435, 1963).

QUANTUM DEPOLARIZATION OF ELECTRONS IN A MAGNETIC FIELD

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In the movement of electrons and positrons in a magnetic field the emission of protons, as shown in ref. (1), leads to their polarization. Thoug the probability of the emission with the change of spin direction is very small compared with the general probability of emission, high-energy electrons and positrons may appear to be highly polarized after a long time rotation in the modern storage rings (see for ex. (3)). The polarization time is in a homogeneous field equal to

$$\tau_{pol} = \frac{5\sqrt{3}}{8}\alpha m\left(\frac{\varepsilon}{m}\right)^2\left(\frac{H}{H_0}\right)^3, \quad \alpha = \frac{1}{137} \qquad [1]$$

here $H_0 \cdot m^2/e = 4{,}4 \cdot 10^{13}$ e. In a storage ring with the energy of $E = 6$ BeV and the field being $H = 8 \cdot 10^3$ e, $\tau_{pol} = 190$ sec.

In order to keep the arising particle polarization in the storage ring, it is necessary to select the particle energy ε so that there would arise no depolarizing resonances in the system due to the radial and azymuthal constituents of the magnetic field on a particle trajectory. Precisely, the resonance condition.

$$G\frac{\varepsilon}{m} = k + lQ_z + mQ_R + nQ_x, \quad k, l, m, n \text{ - integers} \qquad [2]$$

is to be fulfilled for as high values l, m, n as possible. Here $G = g - 2 = \alpha/2\pi$ stands for the anomalous part of g – factor of the electron, $Q_{z,k,x}$ for the oscillation number on the orbit for vertical, z, radial k and phase x-oscillations. The summands lQ_z and mQ_R arise due to the presence of terms z^l and r^m ($r = R - R_0$, R_0 is an equilibrial radius) in the fields H_R, H_x, and the term nQ_x takes into account the synchrotron energy oscillations and the corrections to the frequencies connected with these oscillations. For a correct choice of the energy ε one needs to have a detailed analysis of a particular storage ring taking into account the specific magnetic field nonlinearity, etc.

We admit that the energy ε may be chosen so that the depolarization effects due to the action of resonances [2] are non-essential. It turns to be that at an appropriately high energy there arises one more possibility of depolarization due to the quantum character of photon emission. This is also realized in the presence of perturbing fields H_R and H_X, only, but the fulfillment of the resonance condition [2] is not necessary for the moment. The effects is due to the fact that the energy jumps due to the quantum character of photon emission on being expanded in Fourier integral contain, in particular, harmonics giving the resonance [2]. Hence, the essence of the phenomenon lies in resonance too with that unpleasant peculiarity that it cannot be avoided by means of the choice of ε.

We would like at the same time to emphasize here that we failed in finding some other classical or quantum effects leading to beam depolarization, so that on elimination of the most dangerous perturbation field harmonics one can sharply decrease depolarization effects. This may be also true for the effect to be considered below.

The treatment of quantum depolarization can be carried out in the approximation when electron trajectory is regarded to be classical while the particle energy undergoes jumps in the each event of photon emission so that

$$\frac{\overline{d(\Delta\varepsilon)^2}}{dt} = \frac{55\sqrt{3}}{48}\frac{1}{R}\left(\frac{\varepsilon}{m}\right)^3 \overline{W}, \quad \overline{W} = \frac{2}{3}\left(\frac{\varepsilon}{m}\right)^4 \frac{r_o m}{R^2} \qquad [3]$$

here $\overline{W}$ stands for the classical power of photon emission by electron and r_o for a classical electron radius.

Equations for the 4-vector electron spin S^i, taking into account the perturbing fields H_R and H_X, are of the form [2]

$$\frac{dS^1}{d\tau} = G\frac{eH}{m}\left(\frac{\varepsilon}{m}\right)^2 S^2 - \left(1 + G\frac{\varepsilon^2}{m^2}\right)\frac{eH_R}{m}S^3 \qquad [4]$$

$$\frac{dS^2}{d\tau} = -G\frac{eH}{m}S^1 + (1+G)\frac{eH_x}{m}S^3 \qquad [5]$$

$$\frac{dS^3}{d\tau} = (1+G)\frac{eH_R}{m}S^1 + \left[G\frac{eH}{m}\frac{p}{m}\frac{dz}{d\tau} - (1+G)\frac{eH_x}{m}\right]S^2 \qquad [6]$$

$$\frac{dS^4}{d\tau} = G\frac{eH}{m}\frac{\varepsilon p}{m^2}S^2 - G\frac{eH_R}{m}\frac{\varepsilon p}{m^2}S^3 \qquad [7]$$

where $\tau = tm/\varepsilon$, H is an equilibrial field. One can easily see that both spatial and time variations of the field H do not lead to depolarization which is, of course, evident from the outset. Terms with H_R and H_X can give the resonant turn round of the spin, but we assume that the condition [2] is not fulfilled. Since the effect under consideration depends just indirectly on the type of beam focusing in the storage ring, we consider, for simplicity, an azymuthal symmetric weakly focusing field in which there act the k-th perturbation harmonics $H_R = h\cos[k\,eH/m\cdot(\tau-\tau_o)]$ (for $z = 0$), so that the field on a real orbit perturbed over a vertical $z = z_H$ can be written as

$$H_R = -\frac{k^2 h}{n-k^2}\cos\left[k\frac{eH}{m}(\tau-\tau_o)\right], \quad z_k = \frac{R}{H}\frac{h}{n-k^2} \qquad [8]$$

$$z_H = z_K\cos\left[k\frac{eH}{m}(\tau-\tau_o)\right] \qquad [9]$$

Since the r-oscillation in the linear in h approximation gives no contribution to the effect, we shall consider r to be equal to zero. The calculation of the effects due to the free z-oscillations in a non-homogeneous field as well as the field H_x can be carried out in the similar manner, and the corresponding results are given in the concluding part of the paper.

We write down the solutions of the equations [4]-[7] in the first order approximation in h in the rest particle system. In order to perform a transformation to this system, one is to perform the Lorentz transformation and the usual turn round to a small angle

$$\vartheta_z = \frac{m}{p}\frac{dz}{d\tau} = \frac{h}{H}\frac{k}{n-k^2}\sin\left[k\frac{eH}{m}(\tau-\tau_o)\right] \qquad [10]$$

In this system (which is denoted by the script "c") $S_c^4 = 0$

$$(S_c^1)^2 + (S_c^2)^2 \equiv S_\varrho^2 = \sigma_\varrho^2 + \sigma_\varrho\sigma_z\frac{h}{H}\frac{Gk}{n-k^2}F \qquad [11]$$

$$(S_c^3)^2 \equiv S_z^2 = \sigma_z^2 - \sigma_\varrho\sigma_z\frac{h}{H}\frac{Gk}{n-k^2}F \qquad [12]$$

$$F = \frac{1+k(\varepsilon/m)}{k-G(\varepsilon/m)}\sin\left\{\frac{eH}{m}\left[\left(k-G\frac{\varepsilon}{m}\right)\tau - k\tau_o\right]\right\} - \frac{1-k(\varepsilon/m)}{k+G(\varepsilon/m)}\sin\left\{\frac{eH}{m}\left[\left(k+G\frac{\varepsilon}{m}\right)\tau - k\tau_o\right]\right\} \qquad [13]$$

where σ_z, σ_ϱ are constant with the constant particle energy, and

$$\sigma_\varrho^2 + \sigma_z^2 = 1 \qquad [14]$$

Values S_z and S_ϱ have the meaning of the instantaneous spin projections on the axis z and on its perpendicular plane in the rest electron system; σ_z and σ_ϱ have the meaning of the means near which there occur low oscillations of the projections S_z and S_ϱ.

Due to the fact that the relative probability of the spin turn round at the moment of photon emission is negligibly small compared with that of emission without the spin turn round, the values of the true spin projections. S_z and S_ϱ do not change at the moemnt of photon, emission. However, at the moment of photon emission there occurs a jump-like change of F in the formulas [11], [12] proportional to the jump $\Delta\varepsilon$. Since S_z and S_ϱ are unchanged this leads to the jump-like change of the mean values σ_z and σ_ϱ. The compensation of the energy loss due to photon emission by the accelerating system of the storage ring makes the value F (more precise, the values of amplitudes in [13] come back to the initial one, but the set of such random jumps leads to the stochastic swing of σ_z and σ_ϱ and, consequently, to the swing of S_z and S_ϱ.

For the appropriately high energy $G(\varepsilon/m) \gg 1$, at which the effect under consideration is of interest, the main contribution to the process of swing is given by the jums of the denominator in the first term [13]. In this case a main part is played by the harmonics k for which $G(\varepsilon/m) \sim k$. This gives the following change of the angle ϑ between the spin direction and the axis z:

$$\frac{1}{\tau_{dep}} \simeq \frac{d\bar{\vartheta}^2}{dt} \simeq \frac{1}{8}\sum_k \left(\frac{z_k}{R}\right)^2 \left|\frac{kG\frac{\varepsilon}{m}}{k-G\frac{\varepsilon}{m}}\right|^4 \frac{1}{\varepsilon^2}\frac{\overline{d(\Delta\varepsilon)^2}}{dt} =$$

$$= \sum_k \frac{55}{192\sqrt{3}} \left|\frac{kG\frac{\varepsilon}{m}}{k-G\frac{\varepsilon}{m}}\right|^4 \cdot \frac{r_e}{mR^3}\left(\frac{z_k}{R}\right)^2\left(\frac{\varepsilon}{m}\right)^5 \qquad [15]$$

here z_k is the amplitude of the induced z-oscillations excited by the perturbation.

From [15] it is seen that the effect most strongly depends on the particle energy and highly depends on the number of the nearest resonant harmonics k, as well as on the distance to the resonance $k - G(\varepsilon/m)$ and on the quantity z_k. We evaluate the effect at reasonable values of the parameters: $E = 6$ BeV, $H = 8 \cdot 10^3$ e, $R = 3 \cdot 10^3$ cm$|(k - G\frac{\varepsilon}{m})| \simeq \frac{1}{2}$, for $k \sim 14,15$, $z_k = 0{,}1$ cm. In doing this the characteristic depolarization time (the change of ϑ angle by one) is equal to $\tau_{dep} = 25$ sec. This is by one order less than τ_{pol}. Thus, in this case, the beam is not obviously polarized.

This means that one is to take special steps for the keeping beam polarization. However, we would like once more emphasize a striking stability of polarization.

We give also formulas for S_ϱ and S_z due to the perturbation H_x and the presence of the free z-oscillations (these effects, generally speaking, yield a small contribution compared with the considered above):

$$S^R_{\varrho,z} = \sigma^R_{\varrho,z} \pm \sigma_\varrho\sigma_z\frac{h_{xk}}{H}\left\{\frac{1+G}{k+G\frac{\varepsilon}{m}}\sin\left[\frac{eH}{m}\left(k+G\frac{\varepsilon}{m}\right)\cdot \tau - k\tau_1\right] - \frac{1+G}{k-G\frac{\varepsilon}{m}}\sin\left[\frac{eH}{m}\left(k-G\frac{\varepsilon}{m}\right)\tau - k\tau_1\right]\right\} \qquad [16]$$

$$H_x = h_{xk}\sin\left[k\frac{eH}{m}(\tau-\tau_1)\right]$$

$$S^2_{\varrho,z} = \sigma^2_{\varrho,z} \pm \sigma_\varrho\sigma_z\frac{Z_{max}}{R}G\left\{\frac{1+\frac{\varepsilon}{m}\sqrt{n}}{G\frac{\varepsilon}{m}\frac{1}{\sqrt{n}}-1}\sin\left[\frac{eH}{m}\left(G\frac{\varepsilon}{m}-\sqrt{n}\right)\tau+\sqrt{n}\,\tau_1\right] + \frac{\frac{\varepsilon}{m}\sqrt{n}-1}{G\frac{\varepsilon}{m}\frac{1}{\sqrt{n}}+1}\cdot\sin\left[\frac{eH}{m}\left(G\frac{\varepsilon}{m}+\sqrt{n}\right)\tau-\sqrt{n}\,\tau_2\right]\right\} \qquad [17]$$

As one should expect, the consideration of the linear z-oscillation leads to the effect dependence on the difference

$$G(\varepsilon/m) - Q_z, \quad Q_z = \sqrt{n} \quad (\text{cf.}\,[2]\ \text{for}\ l = 1).$$

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NUMERICAL-ANALITICAL CALCULATION OF THE ACCELERATION OF THE POLARIZED ELECTRONS IN SYNCHROTRON

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In recent years a series af theoretical works (1, 2, 3, 5,) on the acceleration of the polarized protons in different types of accelerators have been carried out.

It is found out that the vertically polarized particles pass through a series of depolarizing resonances during the acceleration process.

The theoretical investigation of some effects which may be expected for the polarized electron beam of the Yerevan Synchrotron (EKU) has been carried out by the author in (4). In this paper (as well as in the papers of other authors) in order to estimate the effect of the passage through the resonance it has been used the Froissart and Stora's method (1) based on the application of the perturbation theory.

In this paper it is adopted for EKU another approach to the problem consisting in the use of a high speed electronic computer for the simultaneous calculation of the amplitudes of the oscillation harmonics and of the spin motion in various energy region.

The use of this method gives a certain confirmation to the analitical calculations concerning to the prediction of the places of the resonances (4) and the spin motion just at the resonance (6).

EQUATION OF MOTION

For the preliminary calculations in order to obviate some additional difficulties in the programming it has been considered the case when only z-motion is present.

$$\frac{du^i}{d\tau} = \begin{cases} 0 & i = 0, 1, 2 \\ \dfrac{eH_r}{mc} \cdot \dfrac{p}{mc} & i = 3 \end{cases}$$

(just it is the only important one for the vertically polarized particles). Here u^i is the velocity four-vector, p is the particle momentum, τ is the proper time, H_r is the radial component of the magnetic field. The relativistic invariant equations of the spin motion (7) take the form:

$$\frac{dS^1}{d\tau} = G \frac{eH_o}{mc} \gamma^2 S' - (1 + G\gamma^2) \frac{eH_r}{mc} S^3$$

$$\frac{dS^2}{d\tau} = -G \frac{eH_o}{mc} S^1 \qquad [1]$$

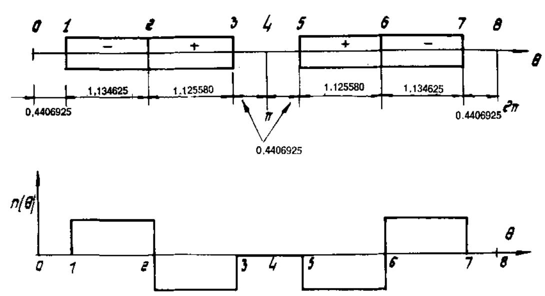

Fig. 1