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The result can be written as

$$d\sigma_{soft} = d\sigma_o \frac{\alpha}{\pi} \ln \frac{E}{\Delta E} \left[1 + Z^2 - 2\ln \frac{4E^2}{m^2} - Z^2 \frac{2E^2 - M^2}{EP_A} \ln \frac{(E+P_A)^2}{M^2} + 4Z \ln \frac{(p_+ \cdot p_A)}{(p_+ \cdot p_B)}\right]$$
$$- d\sigma_o \left[\frac{\alpha}{\pi} K(+,+) - K(+,-) + Z^2 K(A,A) - Z^2 K(A,B) + 2ZK(+,A) - 2ZK(+,B)\right] \quad (12)$$

where [1] K's are the infrared terms and always cancel out completely against similar terms in the virtual radiative corrections.

3. If we ignore the radiative corrections, all the processes of the type $e^+ + e^- \to A + B$ must be symmetric with respect to $90°$ in the c.m. ($e^+ + e^- \to e^+ + e^-$ is the only exception). From the term linear in Z in Eq. (12) we notice that if the final particles are charged, then there will be more positively charged final particles going in the direction of P_+ than the negatively charged ones. This phenomenon is very similar to the difference between the e^+p and e^-p scatterings where e^+p in general has a larger cross section at a finite fixed angle than e^-p if higher order terms are included [1].

4. In reaching the above conclusion we have assumed that the two-photon exchange graphs do not contribute anything significant except for supplying infrared terms

$$d\sigma_o \frac{\alpha}{\pi} 2Z[K(+,A) - K(+,B)] \quad (13)$$

which cancel out with the similar terms in Eq. (12). This assumption has been verified in all the calculations [2,4,5] done by perturbation theory for ee, $e\mu$, and $e\pi$ scatterings. When the final particles A and B are strongly interacting, there may be some additional non-negligible contributions beside the infrared terms in the two-photon exchange diagrams.

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Double Bremsstrahlung in Colliding Beam Experiments

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1. Of considerable interest in the experiments with colliding beams is the process of double bremsstrahlung, i.e. the process in which collision between electrons or between electron and positron is accompanied by the emission of two photons. This process can be used as a monitor for the registration of beam collisions. The simplest scheme seems to be such in which quanta emitted in opposite directions are registered. With the availability of such a scheme the process of the double bremsstrahlung competes with the process of the two-quanta annihilation (the case of electron-positron collisions). In case of high-energy electrons and within a sufficiently wide range of frequencies of photons under registration the cross-section of this process can exceed that of the two-quanta annihilation. This is connected with the fact that the cross-section of the double bremsstrahlung in contrast to that of two-quanta annihilation does not decrease with the increase of colliding particle energy.

The use of double bremsstrahlung as a monitor requires the knowledge of theoretical formulas for the cross-section of this process with a sufficient degree of accuracy. Taking into account that photon emission occurs mainly into the angle of $1/\gamma$ order, and that this is less than the angular dimension of counters at large energy, it becomes evident that the cross-section integrated over the angles of photon emission is of interest. Since electrons are not registered in their state, it is also necessary to carry out the integration over their final states. The quantity thus obtained will be a differential cross-section over both quanta frequencies of the double bremsstrahlung – $d\sigma_{\omega_1\omega_2}$ where ω_1 is the frequency of photon emitted in the direction 1, and ω_2 is the frequency of photon emitted in the direction 2, opposite to the direction 1.

The calculation of this cross-section for the case of classical quanta bremsstrahlung ($\omega_{1,2} \ll \varepsilon$) is performed in ref. [1], and that for the case when only one of the emitted photons is soft while the second has an arbitrary energy obtained in ref. [2]. The paper reports the results of the calculation of the cross-section of the double bremsstrahlung of photons with the arbitrary energy. It is assumed that the electron energy is high so that the expansion in the inverse powers γ can be performed.

2. The above formulation of the question naturally determines the choice of diagrams. In fact, since the small scattering angles are of importance, and each of electrons emits into a narrow cone with the angle of the order of γ^{-1}, the main contribution to the cross-section is given by the diagrams corresponding to the emission of quanta by the different particles. Since the interference is non-essential, one is to consider four diagrams presented in the Fig. 1. The results obtained will be equally valid for electron-electron and electron-positron scattering.

It turns out to be reasonable to separate the integration over the final states of each electron and over the angles of the photon emitted by it by introducing the additional δ-function. Thereby, the cross-section of the double bremsstrahlung will be[1]

$$d\sigma_{\omega_1\omega_2} = \frac{4\alpha^4}{(2\pi)^4\sqrt{(p_1p_2)^2-1}}\,\frac{d\omega_1}{\omega_1}\,\frac{d\omega_2}{\omega_2}\int\frac{d^4\Delta}{\Delta^4}\,K_{1\mu\nu}K_2^{\mu\nu} \qquad (1)$$

Quantity $K_{1\mu\nu}$ is proportional to the scattering cross-section of the arbitrary polarized photon on electron integrated over all the final states, except for frequency, with the square of mass of the initial photon being $-\Delta^2$. Tensors $K_{\mu\nu}$ depend on the reference system in which the quanta energy ω is fixed. In

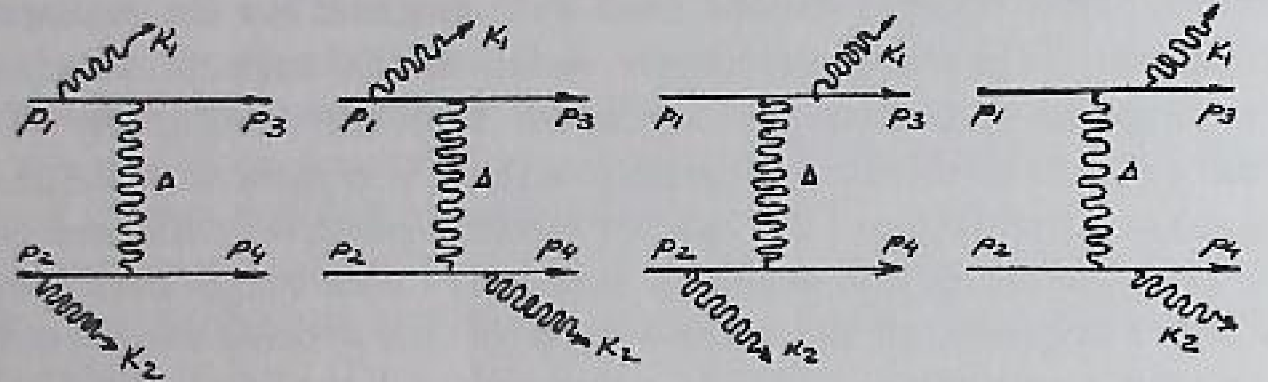

Fig. 1

order to write them down in a covariant form it is convenient to introduce 4-vector n_μ defined so that in the reference system being of interest to us (c.m.s.) its components are equal to $n_\mu = n_\mu\ (0,0,0,1)$. Then the tensor $K_{\mu\nu}$ can be expressed in terms of vectors p_μ, Δ_μ, n_μ with the coefficients by their quadratic combinations defined by the following four covariant integrals

$$J_1^{(i)} = g^{\mu\nu}K_{i\mu\nu},\quad J_2^{(i)} = p_i^\mu p_i^\nu K_{i\mu\nu},\quad J_3 = n^\mu n^\nu K_{i\mu\nu},$$
$$J_4^{(i)} = (n^\mu p_i^\nu + n^\nu p_i^\mu\ K_{i\mu\nu}) \qquad (i=1,2) \qquad (2)$$

3. In the subsequent calculations one makes use of the smallness of the parameter ε^{-2}. This approximation is valid as long as the dropped diagrams give the contribution of the above order.

In this approximation the only essential term in the product of tensors of the formula (1) proves to be the product $J_3^{(1)}\cdot J_3^{(2)}$. This is due to the fact that in the reduction of the matrix elements with the vector n_μ there appear terms of the form (np) and (nk) whose order is ε while the products of the form (pp) and (kp) are of the order of unity. Hence, the most essential are those summands which contain the maximum number of vectors n_μ, i.e. "the least covarient terms".

The quantity J_3 has a transparent physical meaning. It is the 00-component of the tensor $K_{\mu\nu}$, namely the component entering the cross-section of the electron bremsstrahlung in the Coulomb center.

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[1] Here and in what follows one uses the metric $(ab) = ab - a_0b_0$ and the system of unities $\hbar = c = m = 1$

The integration over Δ_0 and Δ_z (Δ_z is the component of the vector Δ parallel to the direction of the initial electron momentum) is carried out simply. On choosing the terms of the highest order over a small parameter ε^{-2}, the result of integration is written down as

$$d\sigma_{\omega_1\omega_2} = \frac{16\alpha^4}{\pi}\,\frac{d\omega_1}{\omega_1}\,\frac{d\omega_2}{\omega_2}\int\frac{d\Delta^2}{\Delta^4}\left\{\left(1-\frac{\omega_1}{\varepsilon}\right)\Phi\left(\frac{\Delta^2}{4}\right)+\right.$$
$$+\frac{\omega_1^2}{\varepsilon^2}\,\frac{\Delta}{2\sqrt{1+\frac{\Delta^2}{4}}}\ln\left(\frac{\Delta}{2}+\sqrt{1+\frac{\Delta^2}{4}}\right)\Bigg\}\left\{\left(1-\frac{\omega_2}{\varepsilon}\right)\Phi\left(\frac{\Delta^2}{4}\right)+\right. \qquad (3)$$
$$\left.+\frac{\omega_2^2}{\varepsilon^2}\,\frac{\Delta}{2\sqrt{1+\frac{\Delta^2}{4}}}\ln\left(\frac{\Delta}{2}+\sqrt{1+\frac{\Delta^2}{4}}\right)\right\}$$

$$\Phi(x) = \frac{1+2x^2}{x\sqrt{1+x^2}}\ln\left(x+\sqrt{1+x^2}\right)-1$$

At $\omega\to 0$ the braced expressions are equal to $\Phi\left(\frac{\Delta^2}{4}\right)$, i.e. the quantity proportional to the probability of emission of the classical photon integrated over the angles of photon emission in the Δ momentum transfer to electron. Therefore, these expressions may be regarded as a generalization of such a probability to the case of arbitrary energy photons. For small Δ^2 such a generalized probability is proportional to Δ^2 so that in the integral of the formula (3) the small Δ are non-essential, and the lower integration limit may be put equal to zero. This implies the inapplicability of Weizsäcker-Williams method for this problem.

The upper integration limit over Δ^2 is proportional to ε^2 and in view of convergence of the integral may be put equal to infinity. On integration one obtains the following resulting formula for the cross-section of the double bremsstrahlung

$$d\sigma_{\omega_1\omega_2} = \frac{8r_0^2\alpha^2}{\pi}\left\{\left(1-\frac{\omega_1}{\varepsilon}\right)\left(1-\frac{\omega_2}{\varepsilon}\right)\left[\frac{5}{4}+\frac{7}{8}\zeta(3)\right]+\left[\left(1-\frac{\omega_1}{\varepsilon}\right)\frac{\omega_2^2}{\varepsilon^2}+\right.\right.$$
$$\left.\left.+\left(1-\frac{\omega_2}{\varepsilon}\right)\frac{\omega_1^2}{\varepsilon^2}\right]\left[\frac{1}{2}+\frac{7}{8}\zeta(3)\right]+\frac{\omega_1^2\omega_2^2}{\varepsilon^4}\,\frac{7}{8}\zeta(3)\right\}\frac{d\omega_1}{\omega_1}\,\frac{d\omega_2}{\omega_2} \qquad (4)$$

$$\frac{7}{8}\zeta(3) = 1{,}052.$$

At $\omega_{1,2} \ll \varepsilon$ this expression is reduced to the formula (22) of the ref. [1] in which the numerical coefficient is not correct and is to be 4 times decreased.

Formula (4) is invalid in the hardest part of the spectrum when $\varepsilon - \omega_{1,2}$ is of the order of unity. However, in view of the narrowness of the interval this region gives no final contribution to the integral cross-section.

The estimation of the dropped terms in the expression (4) for the quanta of the arbitrary energy turns out to be rather complex. However, this estimation is easily carried out for the soft quanta $\omega_{1,2} \ll \varepsilon$. Then the correction to the cross-section is of the order

$$d\sigma \simeq r_0^2 \alpha^2 \frac{\ln^3 \varepsilon}{\varepsilon^2} \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} \tag{5}$$

For electron energy 50 Mev $d\sigma$ is 2–3% and decreases rapidly with the increase of energy.

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VII. Experimental Techniques

Electron Showers

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This is an attempt to correlate five papers [1,2,3,4,5] submitted to the conference concerning electron showers in the light of other recent developments.

Shower theory in its modern form was initiated by R. R. Wilson [6] using graduate students to make Monte Carlo calculations. Lately the power of the computing machine has been brought to bear in a series of works by Crawford and Messel [7,8,9,10] and by Nagel and Schlier [11]. An early disagreement between the results of Wilson and of Crawford and Messel has been understood [7,12]. There is a general feeling that the showering process can be adequately simulated by the Monte Carlo technique.

Several experiments have been reported which directly check the results of the Monte Carlo calculations on the number and the variance of particles as a function of depth. H. Thom [12] has studied showers in a lead plate expansion cloud chamber while Heusch and Prescott [13] have used a lucite Cerenkov radiator embedded in lead. They all find reasonable agreement with the calculations.

Earlier in the conference Hofstadter [14] reported that the early classic work of Kantz and Hofstadter [15] on showers from 185 MeV electrons was being extended to higher energies. Murata [16] et al have been engaged in similar work in Japan using a technique where X-ray films are embedded in the absorber. By the use of different exposures and film type they have an intensity range of 10^4. They have made measurements of 200 MeV electrons in lead, and find rough agreement with Kanth and Hofstadter [15]. They differ in some of the details, for example, finding a slower decrease in the tail. Murata et al have also measured bremsstrahlung induced showers in lead at 200 MeV and in lead, copper, and aluminum at 720 MeV. The various showers in lead are much the same. The bremsstrahlung induced showers in copper and aluminum have a much smaller lateral spread than do the 185 MeV results of Kantz and Hofstadter. In this field we then can expect new results which will allow for counter design at higher energy. Bauer et al. [4] have measured the lateral spread in the beginning of a shower to see if it could be used to estimate the energy. They find little or no correlation between 1.15 GeV and 6.9 GeV. Gamma induced showers have a greater lateral spread than do electron induced showers.

Shower calculations should be extended to non-homogeneous system like the lead scintillator sandwich, and the MonteCarlo output be made to simulate the