THE NATURE OF THE NEUGEBAUER-MARTZ-LEIGHTON OBJECTS

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Starting with observed facts about the Neugebauer-Martz-Leighton objects it is shown that they are consistent with a model for pre-main-sequence evolution, but these facts do not imply the correctness of Hoyle's stellar formation model.

In a recent infrared survey, Neugebauer et al. [1] discovered a number of very red objects. They found about ten such objects lying in the range $0^m < K < 2^m$, using the photometric system advocated by Low and Johnson [2]. These objects were studied by Johnson et al. [3,4], the first paper suggesting a temperature of about 700°K for one of the objects, while the second obtained a temperature in excess of 1200°K for it. The second paper points out that the method used in the first paper has a tendency to underestimate the temperature and suggests that 1000°K may be a reasonable value. We shall adopt this value. Ref. 4 also gives the visual magnitude of these objects fainter than $V = 16^m$ compared with the infrared magnitude of $0^m < K < 2^m$.

Penstone [5] gives the likely distance of these objects as 50 pc and their radius as $10^{13.5}$ cm. He also shows that existence time for one quasi-static model and for the free fall model is very short, but that the observations are consistent with the model for stellar formation proposed by Hoyle [6] provided the mass is small.

We show that the above observational facts are not in conflict with the stellar contraction model given by Huang [7] and McCrea and Williams [8]. Close to the main sequence this model has been superceded by Hayashi's model [9] but it is possible that in the very early stages of contraction a convective zone will not form and so the model may once again apply.

If we apply the bolometric correction appropriate to a temperature of 1000°K to a visible magnitude fainter than 16^m, the bolometric magnitude turns out to be in the same range as the infrared magnitude. It thus seems justifiable to take the effective apparent magnitude in this range. As the distance is 50 pc, this corresponds to a luminosity of about 2×10^{36} erg/sec.

This value is in very good agreement with the value obtained for the luminosity from the black body expression $L = 4\pi\sigma R^2 T_e^4$ when the above values for R and T_e are substituted. Thus the numerical values taken for the object are consistent with each other. McCrea and Williams [8] give the following equations which the model has to satisfy

$$\log \mathcal{L} = 18.2 \log \mathcal{M} + 3.76 \log \tau - 2.55$$

$$\log \mathcal{G} = 12.6 \log \mathcal{M} + 3.32 \log \tau - 2.25 \,.$$

Here all quantities are in solar units (time in Helmholtz-Kelvin units).

From these we obtain the values of $8\ M_\odot$ for the mass and 2×10^4 yrs ($\tau = 10^{-3}$) for the age of the object.

Hence a star contracting according to this model will be consistent with the observations if its mass is $8M_\odot$. Stars with masses much less than this would be correspondingly fainter and so could not be detected by the present survey.

We now consider Hoyle's model and show that the relation between it and the observations is by no means conclusive. Hoyle gives a relation between mass and radius as $M^3 = 16 \times 10^{104}/GR$. For the observed value for the radius the mass of the object is thus 0.45 $M_\odot$. (This value is more than double what Penstone obtained.) Setting the luminosity equal to the rate of change of gravitational energy as done by Penstone, we obtain $t = 3GM^2/2LR \simeq 10^9$ sec. In this paper, Hoyle uses the formula $tR^3H^2 = 4 \times 10^{51}$, to define the magnetic field. For consistency with the above values a magnetic field of over 10 gauss is required. This value is higher than envisaged by Hoyle and could well be too high for any real situation. In this model, the required mass is under $\frac{1}{2}M$; clearly more massive stars are also expected to be formed. If we consider $M = M_\odot$ say, then from the above equations we now find that the radius is reduced by a factor of 8. Stars of mass similar to the sun would thus have angular diameters one order of magnitude smaller than those observed and even more massive stars smaller still. Stars less massive would appear brighter and some explanation as to why these have not been found is required.

These recent observations cannot thus be taken as proof of the correctness of the Hoyle model for star formation. On the contrary, unless some explanation is found as to why stars either more or less massive than $\frac{1}{2}M_\odot$ have not been observed, or a reason for the existence of the high magnetic field, these observations actually weaken Hoyle's model. We have shown that another model is consistent with all the observation, though at this stage we would not suggest that it is the only one.

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RADIATIONAL POLARIZATION OF ELECTRONS IN INHOMOGENEOUS MAGNETIC FIELD

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The process of polarization of electrons due to radiation in an inhomogeneous magnetic field has been considered by means of operator formulation of a quasiclassical approximation. A general expression for the probability of a spin-flip radiational transition in an arbitrary magnetic field has been obtained.

Electrons and positrons when moving in a magnetic field can be polarized due to radiation. The polarization arises due to the fact that the probability of radiational spin-flip transition depends upon the orientation of the initial spin. The existence of the effect of radiational polarization in a homogeneous magnetic field was first pointed out by Sokolov, Ternov and co-workers [1,2]. Radiational polarization was also considered in ref. 3 where the authors formulated an approach which essentially takes into account the quasiclassical character of motion of high energy electrons in a magnetic field which allows one, in principle, to consider the radiational polarization in an inhomogeneous magnetic field.

In the present paper an operator formulation of the quasiclassical approximation * has been used in the developent of the approach of ref. 3 which turned out to be adequate for our problem and allows one to find the probability of spin-flip radiational transition in an arbitrary electromagnetic field.

It should be noted that the characteristic time of radiational polarization is of the same order as the time of operation of colliding beam accelerators, therefore the problem of radiational polarization in an inhomogeneous magnetic field is of great importance.

The motion of high energy electrons in a magnetic field may be regarded quasiclassical if the energy of radiated photons is much less than the electron energy

$$\hbar\omega \ll E\,, \qquad \omega \sim \omega_0 \gamma^3\,, \tag{1}$$

where

$$\gamma = \frac{E}{mc^2}\,, \qquad \omega_0 = \frac{v_t}{R}\,, \qquad R = \frac{cP_t}{eH}\,. \tag{2}$$

* Similar methods were applied by Schwinger [4] for finding the quantized corrections for the intensity of electron radiation in a magnetic field.

R is the instantaneous radius of curvature, H the magnetic field. In this case one can describe electron motion by means of classical characteristics. Since in all the existing machines the value $\hbar\omega/E$ is extremely small, we shall restrict ourselves to consideration of this case.

The expression for the probability of radiational transition will be written down in the form *

$$dw = \frac{\alpha}{(2\pi)^2}\frac{d^3k}{\omega} \times$$

$$\times \langle i \,|\int dt_1 \int dt_2 \exp\{+i\omega(t_1-t_2)\} M(t_1)M^*(t_2)\,|i\rangle \quad (3)$$

$$M(t) = U^+(\xi_f)\,(\alpha e)\exp\{-ikr\}\,U(\xi_i) \quad (4)$$

$U(\xi_f)$, $U(\xi_i)$ are the solutions of the Dirac equation in an arbitrary electromagnetic field in operator form; ξ_i, ξ_f are the initial and finite spin states.

For the description of the spin states we shall make use of two component spinors φ_i, φ_f. Since we are interested in the spin-flip transition, it will be convenient to express

$$\varphi_f = \varphi_i \exp(\tfrac{1}{2}\pi i\sigma a) = i(\sigma a)\,\varphi_i ,$$

where a is the unit vector, orthogonal to the direction of the axis of the quantization of the spin ξ.

By fulfilling the required commutations, one can easily obtain the following expression for the matrix element of the spin-flip radiational transition

$$M(t) = \{\hbar/2(E+m)\}\exp(-ikr)\,(b[qe]) , \quad (6)$$

where

$$b = a + i[\xi a] , \qquad q = P\omega/(E+m) - k . \quad (7)$$

It is evident that the expression (6) for the matrix element is proportional to $\hbar$, therefore one can ignore the non-commutation of the operators involved since its taking into account gives corrections of highest order with respect to $\hbar$ which are of no interest to us.

In the integral (3) the main contribution is given by the region $\omega_0\tau \equiv \omega_0(t_1-t_2) \sim 1/\gamma$, therefore we shall expand all the incoming values in powers of $\omega_0\tau$, which corresponds to expansion in $1/\gamma$ and leave only the expansion terms of highest order of magnitude. Moreover, as it is usually accepted in considering the classical radiation, we shall ignore the terms

$$|\dot{H}|\tau/|H| \ll 1 , \quad (8)$$

where $|\dot{H}|$ characterizes the change of the magnetic field on the trajectors. If the field will be described by the index of inhomogenity n, then the condition (8) has the form $n/\gamma \ll 1$.

We are interested in the total probability of the spin flip radiational transition, therefore we shall sum over photon polarization and integrate over its momentum. The latter turns to be convenient to perform up to the integration over τ with the aid of the relation

$$\exp\{-i(ky)\}f(k_\mu)\,d^3k/\omega = -f(i\partial_\mu)\,4\pi/(y^2 - i\epsilon) ;$$
$$y^2 = y_0^2 - \boldsymbol{y}^2 . \quad (9)$$

After this procedure, the integration over τ is reduced to taking simple contour integrals. As a result we obtain the following formula for the total probability of the spin flip radiational transition per unit time

$$\frac{dw^\zeta}{dt} \equiv W^\zeta = \frac{5\sqrt{3}}{16}\,\alpha\frac{\hbar^2}{m^2}\gamma^5|\dot{v}|^3\{1 - \tfrac{2}{9}(\zeta v)^2 - \frac{8\sqrt{3}}{15|\dot{v}|}(\zeta[\dot{v}v]) \quad (10)$$

In a homogeneous magnetic field the expression (10) has been transformed to the known probability of the spin-flip radiational transition for the cases of transverse polarization $(\xi v) = 0$ and longitudinal polarization $(\xi v) = 1$ for electrons $e < 0$ and positrons $e > 0$ [2]. In an inhomogeneous magnetic field it happens that for the longitudinal polarization the probability of the spin flip radiational transtition does not depend upon spin orientation, while for transverse polarization such a dependence generally takes place.

The expression (10) contains values which depend upon time. We are naturally interested in the mean values over time. For the general analysis of radiational polarization under specific conditions, it is necessary to solve the classical equations of motion ofparticle and spin vector [5] to replace them in (10) and to carry out averaging over time. In the case of an axial-symmetric weak-focussing inhomogeneous magnetic field, the average expression over time for the probability of the spin flip radiational transition with the accuracy up to the correction terms $\sim a^2/\bar{R}^2$ (a is the amplitude of transverse oscillations, $\bar{R}$ the mean radius of the orbit curvature) has the same form as in a homogeneous magnetic field ($\bar{R}$ enters as a radius). The quantities $a^2/\bar{R}^2$ are extremely small (10^{-3} - 10^{-4}) for all modern machines.

In this way, the effect of radiational polarization, generally speaking, also takes place in an inhomogeneous magnetic field and, consequently, may be observed in the modern storage rings, if the effects of depolarizing factors were eliminated [6].

* For the further c=1.

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FIRST ORDER IMPACT PARAMETER TREATMENT OF THE EXCITATION OF HELIUM BY PROTONS

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A formula is presented by which analytical expressions can be derived for the cross section of excitation of n electron atoms by protons, neglecting the correlation of electrons. The procedure is applied to the excitation of helium from the ground state to $(1s, 3p)^1P$ and $(1s, 3d)^1D$.

An expression for the excitation cross section of hydrogen by protons in a first order impact parameter formulation [1] can, after suitable transformation, be generalized for atoms with n electrons, yielding

$$\sigma_{if} = \frac{8}{v^2}\int_{|\alpha|}^{\infty}\frac{|f(k)|^2 dk}{k^3}(\pi a_0^2) \quad (1)$$

in which $f(k) = \sum_j \langle\Psi_f|\exp(ik.r_j|\,\Psi_i\rangle$ with $k^2 = k_x^2 + k_y^2 + \alpha^2$ and $\alpha = \Delta E/v$.

Eq. (1) is used to determine analytical expressions for the cross sections for excitation of helium from the ground state to the final states $(1s, 3p)^1P$ and $(1s, 3d)^1D$.

3^1P *excitation*. For the excitation of the $(1s, 3p)^1P$ level the following wave functions are used (here the $\varphi_{nlm}(Z, r_j)$ represent hydrogenic wave functions for electron j with nuclear charge Z):

$$|(1s^2)^1S\rangle = \psi_0(r_1)\,\psi_0(r_2) , \quad (2)$$

with [2] $\psi_0(r) = (N_{1s}/\sqrt{\pi})\,(\exp{-\epsilon r} + \eta\exp{-2\epsilon r})$ and $|(1s, 3p)^1P\rangle = \frac{1}{2}\sqrt{2}\,[\varphi_{1s}(2, r_1)\,\psi_{3p}(r_2) + \varphi_{1s}(2, r_2)\,\psi_{3p}(r_1)]$ with [3]

$$\psi_{3pm}(r) = \frac{N_{3p}}{\sqrt{\pi}}\left(\frac{5A}{\mu} - r\right)\exp(-\mu r)\,r\,F_m(\theta, \Phi) . \quad (3)$$

One derives for the excitation of the 3^1P-level:

$$\sigma_{1s\text{-}3p} = \frac{2^{13}N_{1s}^2N_{3p}^2C^2}{v^2}\left[\frac{\alpha^2 p_a^2}{7(\alpha^2+a^2)^7} + \frac{\eta^2\alpha^2p_b^2}{7(\alpha^2+b^2)^7} + \frac{p_a^2}{42(\alpha^2+a^2)^6} + \frac{2p_aq_a}{7(\alpha^2+a^2)^7} + \frac{q_a^2T_{3px}^a}{a^{16}} + \right.$$
$$\left. + \eta^2\left\{\frac{p_b^2}{42(\alpha^2+b^2)^6} + \frac{2p_bq_b}{7(\alpha^2+b^2)^7}\frac{q_b^2T_{3px}}{b^{16}}\right\} + 2\eta\{p_ap_b\,M_{-14} + (p_aq_a + p_bq_a)\,M_{04} + q_aq_b\,M_{14}\}\right] \quad (4)$$