QUANTUM EFFECTS IN MAGNETIC BREMSSTRAHLUNG

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Quantum effects in magnetic bremsstrahlung have been considered by means of an operator method.

In this paper an operator method for consideration of quantum effects when ultrarelavistic (UR) particles radiate in an external field is given. It is essential, that for this method only knowledge of the Heisenberg equations of motion of the particle is sufficient, while usually solutions of wave equations are used. Here for definiteness we consider an inhomogeneous magnetic field.

The method is based on fact that quantum effects in the motion of UR particles in a magnetic field can be of two types. The first type is connected with the quantum character of particle motion. The non-commutativity (NC) of particle variables in this case is to be $\hbar\omega_0/E$ (where $\omega_0 = v_t/R$, R - is instantaneous radius of the curvature, E-particle energy, we put $\hbar = c = 1$). Hence with increasing energy the particle motion in a magnetic field becomes more and more "classical". The second type is connected with particle recoil in radiation and is of the order $\hbar\omega/E$ (ω-radiated photon frequency).

The quantum effects in magnetic bremsstrahlung may be conveniently characterized by the parameter

$$\chi = |\dot{v}|\frac{\gamma^2}{m} = \frac{e\sqrt{|(F_{\mu\nu}p^{\nu})^2|}}{m^3}\,, \tag{1}$$

here $\gamma = E/m$.

For $\chi \ll 1$ recoil (that is quantum effects magnitude) is small, then $\omega \approx \omega_0\gamma^3$. In the essentially quantum region $\chi \gtrsim 1$, $\omega \approx E$. Hence it follows that for UR particles at any χ quantum effects of the 1st type are negligible as compared with radiation effects. Therefore we shall neglect NC of operators of the dynamic particle variables taking into account only their NC with the field of the radiated photon. Moreover, we shall regularly expand all the values in powers γ^{-1} keeping only highest order terms.

The expression for the radiation intensity $\mathrm{d}I$ in lowest order of perturbation theory in the radiation field has the form:

$$\mathrm{d}I = \frac{e^2}{4\pi}\langle \mathrm{i}|\int \mathrm{d}t_1 \int \mathrm{d}t_2\, \mathrm{e}^{-\mathrm{i}\omega(t_2-t_1)}\, M^*(t_2)\, M(t_1)|\mathrm{i}\rangle \frac{\mathrm{d}^3k}{(2\pi)^2} \tag{2}$$

where

$$eM(t) = \psi^{+}_{s'}(\mathcal{P}(t))\,\{(ej),\ \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\,\psi_s(\mathcal{P}(t)). \tag{3}$$

Here $j_\mu(t)$, $\boldsymbol{r}(t)$ are current and particle coordinate operators, respectively, e_μ - photon polarization vector, the braces { , } indicate a symmetrized operator product $\psi_s(\mathcal{P}(t))$ wave function of particle in magnetic field in operator form (this means that in the free wave function instead of $E, \boldsymbol{p}$ one must substitute operators of energy $\mathcal{H} = \sqrt{\mathcal{P}^2 + m^2}$ and momentum $\mathcal{P}(t)$ in a given field), the indices s, s' are related to spin characteristics of particles, $|\mathrm{i}\rangle$ - initial state.

In the expression for $M(t)$ one should take into account only photon field ($\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$) commutators with momentum $\mathcal{P}$. Then $\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_1)}$ can be taken out to the left in $M(t_1)$ and $\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_2)}$ to the right in $M^*(t_2)$. The main point of the method is disentangling of the combination $\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_2)}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_1)}$. The problem reduces to an integral equation on solving which with our accuracy one obtains:

$$\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_2)}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(t_1)} = \mathrm{e}^{\mathrm{i}\omega(t_2-t_1)}\exp\left\{\mathrm{i}\frac{\mathcal{H}}{\mathcal{H}-\omega}[-\omega(t_2-t_1) + \boldsymbol{k}\cdot(\boldsymbol{r}(t_2)-\boldsymbol{r}(t_1))]\right\}. \tag{4}$$

After disentangling all the operators in (2) which are in the brackets of the initial state can be replaced by their classical values.

The subsequent calculations were performed like those for the classical description of magnetic bremsstrahlung. By carrying out integration in (2) over relative time $t_2 - t_1$ and azimuthal angle of photon emission we obtain spectral and angular distributions of radiated photon intensity per unit time

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{e^2}{4\pi}\frac{1}{3\pi^2}\frac{E^3}{|\dot{v}|}\frac{\alpha^2}{(1+\alpha)^4}\mu\{\mu(1+\alpha+\delta\mathrm{d}^2)\,[K^2_{\frac{1}{3}}(\eta) + K^2_{\frac{2}{3}}(\eta)] - \frac{1+\alpha}{\gamma^2}K^2_{\frac{1}{3}}(\eta)\}\,\mathrm{d}\alpha\,\mathrm{d}\sin\theta\,, \tag{5}$$

where

$$\alpha = \omega/(E-\omega), \qquad \mu = 1 - v^2\cos^2\theta, \qquad \eta = \frac{\alpha E}{3|\dot{v}|}\mu^{\frac{3}{2}} = \frac{\alpha}{3\chi}(\gamma^2\mu)^{\frac{3}{2}}\,,$$

θ-angle between the photon momentum and orbital plane, $\delta = 0$ for scalar and $\delta = \frac{1}{2}$ for spinor particles. This expression depends on kinematic particle characteristics $\boldsymbol{v}(t)$, $\dot{\boldsymbol{v}}(t)$ in the given field, while in a homogeneous field it reduces to a known expression [1]. Integration of (5) is performed in a well-known way, it results can be represented as a series in χ for $\chi \ll 1$ and series in the inverse powers of χ for $\chi \gg 1$. Evidently all the expressions contain characteristics of magnetic field inhomogeneity only in χ. This question caused recently discussion (for the 1st expansion term for $\chi \ll 1$, see [1]).

It is of interest to note, that the operation of disentangling does not touch upon spin of particles, therefore this method is suitable for particles with arbitrary spin.

The complete results and details of calculation will be published in "JETP".

Reference

1. Synchrotron radiation, eds. A. Sokolov and I. Ternov, (Moscow, 1966).

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ANALYTICITY PROPERTIES OF A LATTICE GAS

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Analyticity properties are given for a system of classical particles on a lattice interacting through many-body potentials.

In this note we give certain analyticity properties of the pressure and correlation functions of a system of classical particles on a lattice interacting through many-body potentials. Our methods generalize those of Dobrushin et al. [1] for two-body potentials.

Assume there can be either 0 or 1 particle at each point of a ν dimensional lattice Z^ν; a finite subset $X \subset Z^\nu$ specifies a finite configuration. Suppose the particles interact through symmetric translationally invariant many-body potentials $\phi^{(k)}(x_1, \ldots, x_k)$ and consider these as a function ϕ on the finite subsets $X \subset Z^\nu$ defined by $\phi(X) = \phi^{(k)}(x_1, \ldots, x_k)$ if $X = \{x_1, \ldots, x_k\}$. We consider only interactions involving a finite number of particles such that