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CONTRIBUTION TO THE THEORY OF BEAM HEATING OF A PLASMA IN AN OPEN TRAP

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ABSTRACT. The author investigates the mechanism whereby an electron beam interacts with a cold plasma in an open trap to produce fast electrons, considering in particular the case where the electron plasma frequency, calculated from the cold plasma density, is considerably greater than the electron cyclotron frequency. It is assumed that the electron beam is the source of the electrons which are involved in the acceleration process. It is also assumed that the beam is the source of Langmuir oscillations with an anisotropic spectrum and that the electrons "ejected" from the beam, after interacting with the Langmuir oscillations, diffuse radially and at the same time accelerate. With these assumptions, it is possible to solve the kinetic equation with quasi-linear collision integral and to find the distribution function of the fast electrons at each point in the trap. It is also possible to calculate the "temperature" of the fast electrons and their total energy, and to determine how these quantities depend on the magnetic field strength and on the radius of the diaphragm limiting the diameter of the plasma. The results are in good agreement with experimental data.

1. INTRODUCTION

In 1961, Kharchenko, Fainberg and co-workers [1] found that, when an electron beam passes through a cold plasma, accelerated electrons with an energy greater than the energy of the beam appear in the plasma. Subsequently, various authors [2 - 9] conducted a series of experiments on the beam heating of a plasma in open (mirror) traps. In these experiments it was shown that the interaction of a beam with a cold plasma led to the formation of a certain quantity of hot plasma whose temperature exceeded the energy of the beam electrons by a factor of 5 - 15, and the hot plasma thus produced filled the entire trap (although the diameter of the beam was considerably less than that of the trap). The purpose of the present study is to investigate the mechanism whereby the hot plasma is formed.

The characteristic feature of this problem is that its solution depends on a large number of parameters whose relative importance is difficult to estimate a priori. We shall therefore have to refer frequently to experimental data - in particular the results of [5], in which certain relationships that are extremely important for understanding the heating mechanism were first established.

In [5], the plasma was produced in a mirror trap with distance between mirrors L = 80 cm. The diameter of the plasma could be limited by means of a set of diaphragms (maximum internal diaphragm radius R_{max} = 12 cm). The magnetic field H at the centre of the trap could be varied between 500 Oe and 1800 Oe; the mirror ratio was 5.5; the density of the cold plasma n_c was about 10^{12} cm^{-3} and did not vary significantly during the heating process. The temperature of the cold plasma probably did not exceed a few tens of electron-volts. The beam parameters were as follows: radius $r_b \sim 1$ cm, energy $\sim$ 25 keV, density $n_b \sim 10^{10}$ cm^{-3}, maximum duration $t_b \sim 250\ \mu$s.

It was established experimentally that the temperature, density and diameter of the hot plasma increased over a period of several tens of microseconds after the beam had been switched on. A steady state was then established in which the density of the hot plasma n_h was approximately 10^{10} cm^{-3} and its temperature T_h lay in the range 40 - 200 keV (depending on the strength of the magnetic field and on the diaphragm radius). In the steady state, the radius of the space occupied by the hot plasma was considerably greater than the radius of the beam. The steady state persisted until the beam was switched off, after which the hot plasma decayed over a period of the order of several hundreds of milliseconds. In this paper we shall concentrate on investigating the steady state; the process of plasma decay after the beam has been switched off will not be considered.

A complete theory of beam heating should undoubtedly include a description of the processes occurring both in the beam and in the space surrounding it. At present, however, such a theory cannot be constructed, mainly due to lack of experimental data on the processes occurring inside the beam. We shall therefore attempt to solve the narrower problem and confine ourselves to investigating the acceleration of electrons outside the beam. The beam itself will be considered simply as a source of oscillations with a known spectrum. In addition, we shall assume that the beam is also a source of fast electrons[1] (with an energy of the order of the energy of the beam electrons) which, having left the beam, are drawn into the acceleration process and ultimately acquire energy significantly greater than that of the beam electrons. The properties of the beam as a source of waves and particles are obviously determined by the processes occurring within it. Unfortunately, only crude qualitative assumptions

[1] The need for this assumption will be explained in section 2.

can be made regarding these processes. However, the final results are not very sensitive to these assumptions, so that we feel that our approach to the problem is more or less justified.

2. HEATING MODEL

There can be no doubt that the hot plasma component in beam heating experiments is produced by the interaction of electrons with oscillations whose source is the electron beam. We shall try to ascertain which type of oscillation is responsible for the heating effect.

It has been established experimentally [5] that the hot electrons acquire most of their final energy not in the vicinity of the beam, but at considerable (compared with the beam radius) distances from it. We can therefore disregard from the outset those specifically "beam" types of oscillation for whose existence charged particle fluxes are necessary (see, for example, Ref.[11]), since such oscillations would produce heating only in the vicinity of the beam.

The density of the cold plasma n_c treated in Ref.[5] was significantly higher than that of the hot plasma n_h. Consequently, the dispersion properties of the oscillations were determined by the cold plasma. Moreover, in these experiments the electron plasma frequency ω_q was substantially (3 - 10 times) greater than the electron cyclotron frequency ω_H inside the trap. It follows from the linear theory of beam instability that, under such conditions, of all the oscillation types capable of propagating outside the beam, electron Langmuir oscillations are excited most rapidly (see Ref.[12]) It may therefore be assumed that heating of the electrons is caused by their interaction with Langmuir oscillations. The above reasoning is in agreement with the results of experiments [6, 10] in which the h.f. oscillations of a plasma during beam heating were recorded and it was found that when $\omega_p \gg \omega_H$ these oscillations are concentrated close to the electron plasma frequency.

As the characteristic growth rate of the Langmuir oscillations γ_b we take ω_p (n_b/n_c), since the condition $\gamma_b L/v_b \gg 1$ (where v_b is the velocity of the beam as it enters the plasma) was satisfied in Ref.[5]; i.e. there is significant beam velocity "smearing" during the passage of the beam through the trap. It follows from the quasi-linear theory of beam instability that, as a result of this smearing (i.e. the formation of a plateau on the beam electron distribution function), the characteristic phase velocity v_{ph} of the Langmuir oscillations will be two to three times less than v_b, and their characteristic wave vector $\vec{k}$ will be of the order[2] $(2-3)\ \omega_p/v_b$.

The Langmuir oscillations arising in the vicinity of the beam propagate from the axis towards the periphery of the trap and fill the entire space occupied by the cold plasma. When $\omega_p \gg \omega_H$, the interaction of the electrons with these oscillations is determined essentially by the Cherenkov effect, the interaction between an electron with velocity $\vec{v}$ and a Langmuir oscillation with wave vector $\vec{k}$ occurring only if the relation

$$\omega_p - \vec{k} \cdot \vec{v} = 0$$

is satisfied, or alternatively

$$v_{ph} = v \cos\psi \tag{1}$$

where ψ is the angle between the vectors $\vec{k}$ and $\vec{v}$, and $v_{ph} = \omega_p/k$ is the phase velocity. It is clear that the electrons of the cold plasma cannot interact with the oscillations excited by the beam because the velocity of these electrons is substantially less than v_{ph} and condition (1) cannot be satisfied in their case. Consequently, the beam cannot cause heating of the cold plasma electrons. We therefore have to assume that there is some mechanism which supplies to the region outside the beam electrons whose velocity exceeds v_{ph}, since only such electrons can interact with the oscillations and become accelerated.

It is reasonable to assume that, as the beam becomes unstable, some of the electrons are "ejected" from the beam[3] The velocities of the ejected electrons exceed v_{ph}, so that they are able to interact with the oscillations. We shall therefore assume that the electron beam is the source of the fast electrons which are drawn into the acceleration process. At present this assumption can scarcely be substantiated rigorously but, as will be seen subsequently, it enables one to understand many experimental results.

Since the electrons acquire during acceleration velocities substantially greater than the velocity of the beam (and consequently v_{ph}), we shall consider in greater detail the Cherenkov interaction when $v \gg v_{ph}$. It follows from relation (1) that $\psi \simeq \pi/2$ in such a case; i.e. each electron interacts only with those oscillations whose propagation is almost perpendicular to its velocity vector. We know [13] that this interaction reduces to two effects: elastic scattering of the electrons and electron acceleration, the characteristic acceleration time being greater than the elastic scattering time by a factor v^2/v_{ph}^2.

Elastic scattering causes diffusion of the electrons and their escape through the mirrors. Due to scattering into the loss cone, fast electrons can be thought to be generally unable to move away from the beam to distances exceeding their Larmor radius r_H: an electron is scattered into the loss cone within a time roughly equal to that of one elastic collision event, but within this time it cannot move along the radius of the trap through a

[2] It should be noted that the inequality $k r_b \gg 1$ was satisfied in [5]; i.e. in estimating the growth rate it is indeed possible to use the infinite-beam approximation, as we have done.

[3] Only 1-3% of the particles need be ejected from the beam in order to achieve the experimentally observed rate at which the trap is filled with hot plasma.

distance much greater than r_H. However, the situation changes considerably if the oscillations excited in the system are primarily those for which the angle θ' between the vector $\vec{k}$ and the trap axis is not too large. For the sake of simplicity, let us consider the case where there are no oscillations with θ' exceeding some critical value θ_0 ; in other words, let us assume that the spectral density of the oscillation energy $W_{\vec{k}}$ is zero when $\theta' > \theta_0$ (naturally, $\theta_0 < \pi/2$). As stated above, the condition for the interaction between fast electrons and oscillation has the form $\psi \simeq \pi/2$. This means that, with our assumption regarding the spectral function $W_{\vec{k}}$, only those electrons interact with oscillations (i.e. are scattered and accelerated) for which the angle θ between the velocity vector and the trap axis is sufficiently large (see Fig. 1).

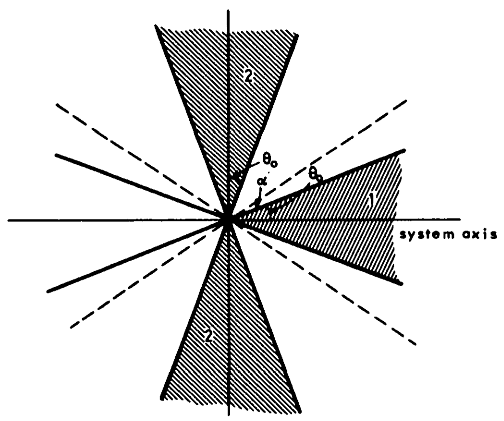

FIG. 1. Interaction of fast electrons with Langmuir oscillations having an anisotropic spectrum;
1 – region of angles in wave vector space in which Langmuir oscillations are excited;
2 – region of angles in velocity space in which there are electrons which interact with Langmuir oscillations (α-angle at the apex of the loss cone).

In the scattering process θ can vary only within the limits of region 2 in Fig. 1, from which it can be seen that scattering does not lead to an escape of electrons from the trap when the mirror ratio is sufficiently large. The corresponding limitation on the angle α at the apex of the loss cone has the form

$$\alpha < \frac{\pi}{2} - \theta_0 \tag{2}$$

If the inequality (2) is not satisfied, the electrons are scattered rapidly into the loss cone and there is no heating. This reasoning is in agreement with the results of experiments described in Refs [4-6], which revealed a threshold dependence of the heating effect on the mirror ratio η: when η was less than some critical value η_0 no heating took place. Expression (2) enables us to relate the value of η_0 to θ_0 :

$$\eta_0 = \frac{1}{\cos^2\theta_0} \tag{3}$$

It follows that the assumption regarding the anisotropy of the spectral function $W_{\vec{k}}$ is absolutely essential for explaining the heating of electrons through Cherenkov interaction with Langmuir oscillations. A reason for this anisotropy may be the dependence of the growth rate of the Langmuir oscillation on the angle θ' between the wave vector and the direction of the beam[4]. On the other hand, the anisotropy of the oscillation spectrum may in a certain sense be considered an experimental fact, since the occurrence of heating itself indicates the existence of such anisotropy. We shall therefore assume that the function $W_{\vec{k}}$ is anisotropic.

It should be noted that the degree of anisotropy which is really necessary to explain the experimental results is not very great; substituting into relation (3) the value of the critical mirror ratio $\eta_0 \simeq 4$ measured in Ref. [4], we find that $\theta_0 \simeq 60°$; i.e. oscillations should fail to occur only in the region of angles of $\sim 30°$. In reality, there is probably also some finite (although low) level of oscillations in the region $\theta' > \theta_0$. On interacting with these oscillations the electrons are scattered into the loss cone, but the process takes a long time.

Let us now formulate the main assumptions underlying the heating model considered here. It is assumed that the beam is the source of the electrons involved in the acceleration process, the initial energy of these electrons being of the same order of magnitude as the energy of the beam electrons. It is also assumed that the beam is a source of Langmuir oscillations with an anisotropic spectrum and that, when the electrons ejected from the beam interact with these oscillations, they diffuse radially and at the same time accelerate. Since, with the proposed model, filling of the trap by a hot plasma is the result of the diffusion of fast electrons away from the axis towards the periphery of the trap, we shall speak of a "diffusion" model of plasma heating.

The cold plasma plays an essentially passive role in the diffusion model; it is simply the medium in which the Langmuir oscillations propagate. For the sake of simplicity, we shall assume that within the magnetic surface which passes through the edge of the diaphragm the cold plasma is homogeneous, while beyond this surface its density falls rapidly to zero (over a distance of the order of the Larmor radius of the cold electrons).

3. SPECTRAL DENSITY OF THE OSCILLATION ENERGY

As stated above, we assume that the beam excites primarily oscillations in which the angle θ' between the direction of the wave vector and the axis of the system does not exceed some critical value θ_0. These oscillations propagate from the vicinity of the beam with a group velocity v_g which is determined by the dependence of their frequency on the wave vector. It can easily be seen that,

[4] The growth rate is known to decrease as θ' increases [14]. The wave vectors of the oscillations emitted by the beam are therefore oriented primarily along the beam axis.

under the conditions of the experiment described in Ref.[5], the dispersion relation for the Langmuir oscillations should be written in the form

$$\omega = \omega_p \left(1 + \frac{1}{2} \frac{k_\perp^2}{k_\perp^2 + k_\parallel^2} \frac{\omega_H^2}{\omega_p^2}\right) \tag{4}$$

where $k_\parallel$ and $k_\perp$ are wave vector components parallel and perpendicular to the magnetic field. The frequency correction for thermal motion of the electrons is only slight. We find from formula (4) that $v_{g\perp} \sim (k_\parallel^2 k_\perp / k^4)(\omega_H^2/\omega_p) \sim (\omega_H^2/\omega_p^2) v_{ph}$. On the basis of this result it can be shown that the oscillations reach the outer boundary of the plasma with virtually no damping (i.e. only an insignificant part of the oscillation energy is expended on heating the fast electrons). The time τ taken by the oscillation to propagate from the trap axis to the outer boundary of the plasma is $R/v_{g\perp} \sim (R/v_{ph})(\omega_p^2/\omega_H^2)$ (where R is the radius of the diaphragm), while the rate of damping of the oscillations, due to their Cherenkov interaction with the fast electrons, may be estimated as $\gamma_h \sim (n_h/n_c)\omega_p (v_{ph}/v_h)^3$ (where v_h is the characteristic thermal velocity of the fast electrons). The product $\tau\gamma_h$, which characterizes the damping of the oscillations, is equal to $(n_h/n_c)(\omega_p R/v_{ph})(\omega_p^2/\omega_H^2)(v_{ph}/v_h)^3$. It can be seen from the experimental results that $n_h/n_c \sim 10^{-2}$, $\omega_p R/v_{ph} \lesssim 30$, $\omega_p^2/\omega_H^2 \lesssim 30$, and $(v_{ph}/v_h)^3 \lesssim 10^{-2}$. At all events, therefore, $\tau\gamma_h$ does not exceed 10^{-1}; i.e. the damping of the oscillations as they propagate from the beam to the plasma boundary is not substantial.

On the other hand, at the boundary of the cold plasma the oscillations should be almost completely absorbed due to Cherenkov interaction with the cold electrons, since in this region the wave vector of the oscillations propagating from the plasma increases and becomes comparable with the Debye wave vector (owing to a decrease in the density of the cold plasma in the transition layer).

On the basis of this reasoning we shall assume that the oscillations propagate freely from the beam to the radial boundary of the cold plasma, where they are completely absorbed by the cold electrons. Consequently there are no oscillations reflected from the radial boundary of the plasma.

With regard to the boundary conditions at the ends of the trap, they are not important when $L \gg R$. Nevertheless, a decrease in the density of the cold plasma near the mirrors leads to absorption of the oscillations propagating from within the trap.

To find the dependence of the spectral function $W_{\vec{k}}$ on the co-ordinates, we use the equation

$$v_{g\vec{k}} \frac{\partial W_{\vec{k}}}{\partial \vec{r}} = 0 \tag{5}$$

which describes the free propagation (without damping) of oscillations in a homogeneous medium (see Ref.[15]). Investigation of this equation is facilitated by introducing the spherical system of co-ordinates (k, θ', φ') in wave vector space. The polar axis of this system is directed along the vector $\vec{H}$, which in its turn is parallel to the axis of the trap, while the angle φ' is read off from the direction of the radius drawn from the axis of the system to the point of observation (Fig. 2).

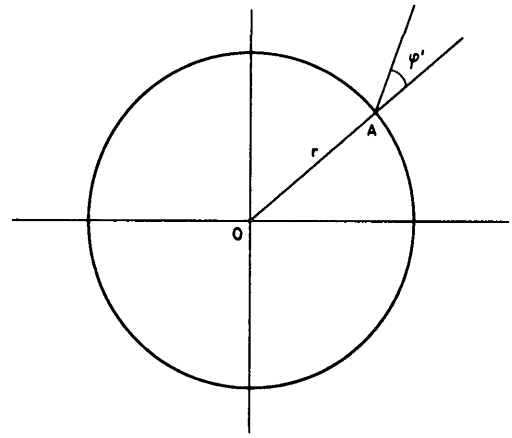

FIG.2. Determination of the azimuthal angle φ' in wave vector space; O - axis of system; A - point of observation, r - radius drawn from axis of system to point of observation.

The region of variation of the angle φ' is selected in the following manner:

$$-\pi \le \varphi' \le \pi$$

In the new variables, Eq.(5) is written in the form

$$v_{g\perp}\left(\cos\varphi' \frac{\partial W_{\vec{k}}}{\partial r} - \frac{\sin\varphi'}{r} \frac{\partial W_{\vec{k}}}{\partial \varphi'}\right) + v_{g\parallel} \frac{\partial W_{\vec{k}}}{\partial z} = 0 \tag{6}$$

where $W_{\vec{k}} \equiv W(z, r, k, \theta', \varphi')$. This equation has to be solved with the boundary condition $W_{\vec{k}}|_{r=r_b} = W^{(0)}(z, k, \theta', \varphi')$, where the spectral density of the oscillation energy at the boundary of the beam is denoted by $W^{(0)}(z, k, \theta', \varphi')$. The function $W^{(0)}$ has the following properties:

$$W^{(0)}(z, k, \theta', \varphi') = W^{(0)}(z, k, \theta', -\varphi') \tag{7}$$

$$W^{(0)}(z, k, \theta', \varphi') = 0 \text{ for } \varphi' < -\frac{\pi}{2} \text{ and } \varphi' > \frac{\pi}{2} \tag{8}$$

$$W^{(0)}(z, k, \theta', \varphi') = 0 \text{ for } \theta' > \theta_0 \tag{9}$$

The first of these properties results from the symmetry of the problem. The second relates to the fact that the oscillations at the beam boundary are all leaving the beam (there is no reflection from the outer boundary of the plasma). The meaning of Eq.(9) was discussed in section 2.

The characteristic scales of the variation in the function $W_{\vec{k}}$ along the co-ordinates z and r are equal to L and R, respectively, in order of magnitude. Since $L \gg R$, we can neglect the last term in Eq. (6):

$$\cos\varphi' \frac{\partial W_{\vec{k}}}{\partial r} - \frac{\sin\varphi'}{r}\frac{\partial W_{\vec{k}}}{\partial \varphi'} = 0 \qquad (10)$$

As stated in section 2, the fast electrons acquire most of their final energy in the region $r \gg r_b$. For investigating the heating problem, it is therefore sufficient to know the properties of the function $W_{\vec{k}}$ when $r \gg r_b$. At such distances from the beam, the solution of Eq. (10) can be written in the form

$$W_{\vec{k}} = W^{(0)}\left(z, k, \theta', \arcsin\frac{r}{r_b}\varphi'\right) \qquad (11)$$

Taking into account expression (8), we find that in the region $r \gg r_b$ the function $W_{\vec{k}}$ is non-zero only when $|\varphi'| \lesssim r_b/r \ll 1$; i.e. the wave vectors of the oscillations form a small angle with the plane passing through the axis of the system and the point of observation.

We shall now show that the function $W_{\vec{k}}^{(0)}$ is independent of the magnetic field. In fact, in the vicinity of the beam (i.e. when $r \lesssim r_b$) the form of the function $W_{\vec{k}}$ is determined by the balance of three processes: excitation of oscillations due to instability of the beam; non-linear interaction of the oscillations; escape of oscillations from the vicinity of the beam due to the non-zero group velocity. Accordingly, the equation for determining $W_{\vec{k}}$ in the vicinity of the beam can be written in the following schematic form:

$$\gamma_b W_{\vec{k}} + \hat{A}(W_{\vec{k}}) - \frac{v_{g\perp}}{r_b} W_{\vec{k}} = 0 \qquad (12)$$

The first term describes the build-up of oscillations by the beam, the second describes the non-linear interaction of the oscillations, and the third the escape of oscillations from the vicinity of the beam. It can easily be seen that $\gamma_b r_b \gg v_{g\perp}$ under the conditions of the experiment described in Ref. [5]; i.e. the last term in Eq. (12) can be neglected. Consequently, the function $W_{\vec{k}}$ is determined by the balance of two terms: $\gamma_b W_{\vec{k}}$ and $\hat{A}(W_{\vec{k}})$. However, if $\omega_p \gg \omega_H$, the instability growth rate and the speed of development of the non-linear interactions are independent of the magnetic field. The function $W_{\vec{k}}$, and consequently $W_{\vec{k}}^{(0)} \equiv W_{\vec{k}}|_{r=r_b}$, is therefore also independent of the magnetic field.

4. KINETIC EQUATION FOR FAST ELECTRONS

To give a quantitative description of the Cherenkov interaction of fast electrons with Langmuir oscillations, we shall use a kinetic equation with a quasi-linear collision integral:

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} - \frac{e}{mc}[\vec{v}\times\vec{H}]\frac{\partial f}{\partial \vec{v}} = \mathrm{Stf} \qquad (13)$$

Here f is the fast electron distribution function, $\vec{H}$ the external magnetic field and Stf the quasi-linear collision integral, which has the form

$$\mathrm{Stf} = \frac{\partial}{\partial v_\alpha} D_{\alpha\beta}\frac{\partial f}{\partial v_\beta} \qquad (14)$$

the tensor $D_{\alpha\beta}$ being expressed in terms of the spectral density of the oscillation energy $W_{\vec{k}}$ (see Ref. [14]):

$$D_{\alpha\beta} = \frac{4\pi^2 e^2}{m^2}\int d^3\vec{k}\,\frac{k_\alpha k_\beta}{k^2} W_{\vec{k}}\,\delta(\omega_p - \vec{k}\cdot\vec{v}) \qquad (15)$$

As indicated in section 3, the spectral density of the oscillation energy may depend on the z co-ordinate. Strictly speaking, therefore, the distribution function f may also depend on z. However, if the characteristic frequency of the electron-oscillation collisions ν is small compared with the reciprocal transit time of an electron between the mirrors (as will be seen below, it is this case which applies in experiments of the type described in Ref. [5]), f is virtually independent of z: changes in f along a field line in the space between the mirrors are equal to $(\nu L/v) f \ll f$ in order of magnitude. Bearing this in mind, we average Eq. (13) over z, obtaining an equation which has the same form as Eq. (13) but in which $W_{\vec{k}}$ is replaced by $\overline{W}_{\vec{k}} = (1/L)\int_0^L W_{\vec{k}}\,dz$ and the term $v_z\ (\partial f/\partial z)$ vanishes.

As in section 3, we introduce the spherical system of co-ordinates (v, θ, φ) in velocity space. In this system, the kinetic equation for f is written as follows:

$$\frac{\partial f}{\partial t} + v\sin\theta\left(\cos\varphi\frac{\partial f}{\partial r} - \frac{\sin\varphi}{r}\frac{\partial f}{\partial \varphi}\right) + \omega_H\frac{\partial f}{\partial \varphi} = \mathrm{Stf} \qquad (16)$$

the collision integral having the form

$$\mathrm{Stf} = \frac{1}{v^2}\frac{\partial}{\partial v} v^2\left(D_{vv}\frac{\partial f}{\partial v} + \frac{D_{v\theta}}{v}\frac{\partial f}{\partial \theta} + \frac{D_{v\varphi}}{v\sin\theta}\frac{\partial f}{\partial \varphi}\right)$$
$$+ \frac{1}{v\sin\theta}\frac{\partial}{\partial \theta}\sin\theta\left(D_{v\theta}\frac{\partial f}{\partial v} + \frac{D_{\theta\theta}}{v}\frac{\partial f}{\partial \theta} + \frac{D_{\theta\varphi}}{v\sin\theta}\frac{\partial f}{\partial \varphi}\right)$$
$$+ \frac{1}{v\sin\theta}\frac{\partial}{\partial \varphi}\left(D_{v\varphi}\frac{\partial f}{\partial v} + \frac{D_{\theta\varphi}}{v}\frac{\partial f}{\partial \theta} + \frac{D_{\varphi\varphi}}{v\sin\theta}\frac{\partial f}{\partial \varphi}\right) \qquad (17)$$

As before, the tensor $D_{\alpha\beta}$ is determined by relation (15), with $\overline{W}_{\vec{k}}$ instead of $W_{\vec{k}}$ and with projections of $\vec{k}$ onto the orthogonal unit vectors of the spherical system (v, θ, φ) instead of the Cartesian system:

$$k_v = k\,[\sin\theta\,\sin\theta'\cos(\varphi-\varphi') + \cos\theta\,\cos\theta']$$

$$k_\theta = k\,[\cos\theta\,\sin\theta'\cos(\varphi-\varphi') - \sin\theta\,\cos\theta']$$

$$k_\varphi = -k\sin\theta'\sin(\varphi-\varphi')$$

Those terms in collision integral (17) which contain the components $D_{\theta\theta}$, $D_{\theta\varphi}$ and $D_{\varphi\varphi}$ of the tensor $D_{\alpha\beta}$ correspond to elastic scattering of the electrons on the oscillations, while the remaining terms take into account the exchange of energy between oscillations and electrons leading to heating of the electrons.

The argument of the δ-function in formula (15) may be written in the form $\omega_p - k_v v$, from which it follows that $k_v = \omega_p / v$. On the other hand, k_θ and k_φ are equal to ω_p / v_{ph} in order of magnitude. Accordingly,

$$D_{vv} \sim \frac{v_{ph}}{v} (D_{v\theta}, D_{v\varphi}) \sim \frac{v_{ph}^2}{v^2} (D_{\theta\theta}, D_{\theta\varphi}, D_{\varphi\varphi}) \tag{18}$$

i.e. for hot electrons $D_{vv} \ll D_{v\theta}$, $D_{v\varphi} \ll D_{\theta\theta}$, $D_{\theta\varphi}$, $D_{\varphi\varphi}$, and elastic collisions predominate over inelastic ones.

Interacting with the oscillations, the electrons are scattered with a given angular distribution and diffuse across the magnetic field. In accordance with expression (18), in investigating this process one can confine oneself to elastic collisions:

$$\frac{\partial f}{\partial t} + v \sin\theta \left(\cos\varphi \frac{\partial f}{\partial r} - \frac{\sin\varphi}{r} \frac{\partial f}{\partial \varphi} \right) + \omega_H \frac{\partial f}{\partial \varphi}$$
$$= \frac{1}{v^2} \left[\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \sin\theta \left(D_{\theta\theta} \frac{\partial f}{\partial\theta} + \frac{D_{\theta\varphi}}{\sin\theta} \frac{\partial f}{\partial\varphi} \right) + \frac{1}{\sin\theta} \frac{\partial}{\partial\varphi} \left(D_{\theta\varphi} \frac{\partial f}{\partial\theta} + \frac{D_{\varphi\varphi}}{\sin\theta} \frac{\partial f}{\partial\varphi} \right) \right] \equiv \mathrm{St}_{\theta\varphi} f \tag{19}$$

The right-hand side of Eq. (19) is of the same order of magnitude as νf, where ν is the effective electron-oscillation collision frequency. It will be shown in section 5 that $\nu \ll \omega_H$. In investigating Eq. (19) one can therefore make use of the smallness of ν/ω_H.

Let us first of all write f in the form

$$f = \langle f \rangle_{\theta\varphi} + \delta f$$

where $\langle \ldots \rangle_{\theta\varphi}$ denotes averaging over the angular variables:

$$\langle \ldots \rangle_{\theta\varphi} = \frac{1}{4\pi} \int_0^\pi \sin\theta \, d\theta \int_{-\pi}^{\pi} \ldots d\varphi$$

It follows from Eq. (19) that

$$\frac{\partial}{\partial t} \langle f \rangle_{\theta\varphi} = -\frac{v}{r} \frac{\partial}{\partial r} r \langle \sin\theta \cos\varphi \, \delta f \rangle_{\theta\varphi}$$

The quantity $v \langle \sin\theta \cos\varphi \, \delta f \rangle_{\theta\varphi}$ represents diffusion flux. The equation for the correction δf to the distribution function has the form

$$\frac{\partial \delta f}{\partial t} + v \sin\theta \left(\cos\varphi \frac{\partial}{\partial r} - \frac{\sin\varphi}{r} \frac{\partial}{\partial\varphi} \right) \delta f + v \sin\theta \cos\varphi \frac{\partial}{\partial r} \langle f \rangle_{\theta\varphi} + \omega_H \frac{\partial \delta f}{\partial \varphi} = \mathrm{St}_{\theta\varphi} \delta f \tag{20}$$

Neglecting the first two terms on the left-hand side of Eq. (20) as being small compared with the third (this is possible since $\delta f \sim r_H \partial \langle f \rangle_{\theta\varphi} / \partial r \ll \langle f \rangle_{\theta\varphi}$), we obtain

$$\frac{\partial \delta f}{\partial \varphi} - \frac{1}{\omega_H} \mathrm{St}_{\theta\varphi} \delta f = -\frac{v}{\omega_H} \sin\theta \cos\varphi \frac{\partial}{\partial r} \langle f \rangle_{\theta\varphi} \tag{21}$$

From this the function δf can be found by the method of successive approximations with respect to the parameter ν/ω_H : $\delta f = \delta f^{(0)} + (\nu/\omega_H) \delta f^{(1)} + \ldots$. However, it is better first to average Eq. (21) over the angular variables with the weight $\sin\varphi \sin\theta$; this gives the following result:

$$\langle \sin\theta \cos\varphi \, \delta f \rangle_{\theta\varphi} = \frac{1}{4\pi\omega_H} \int_{-\pi}^{\pi} d\varphi \int_0^\pi d\theta \sin\theta \times \left[(\cos\theta \sin\varphi D_{\theta\theta} + \cos\varphi D_{\theta\varphi}) \frac{\partial \delta f}{\partial\theta} + (\cos\theta \sin\varphi D_{\theta\varphi} + \cos\varphi D_{\varphi\varphi}) \frac{\partial \delta f}{\partial \varphi} \right]$$

Substituting the function

$$\delta f^{(0)} = -\frac{v}{\omega_H} \sin\theta \sin\varphi \frac{\partial}{\partial r} \langle f \rangle_{\theta\varphi}$$

on the right-hand side of this expression, we find the diffusion flow

$$-v \langle \sin\theta \sin\varphi \, \delta f \rangle_{\theta\varphi} = D(v, r) \frac{\partial}{\partial r} \langle f \rangle_{\theta\varphi}$$

where the diffusion coefficient is determined by the expression

$$D(v, r) = \frac{v^2}{4\pi\omega_H^2} \int_{-\pi}^{\pi} d\varphi \int_0^\pi [\cos^2\theta \sin^2\varphi D_{\theta\theta} + 2 \cos\theta \sin\varphi \cos\varphi D_{\theta\varphi} + \cos^2\varphi \, D_{\varphi\varphi}] \sin\theta d\theta$$

Thus, we have obtained a diffusion equation for the fast electrons:

$$\frac{\partial F}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} r D \frac{\partial F}{\partial r} \tag{22}$$

Here and subsequently F denotes $\langle f \rangle_{\theta\varphi}$.

The dependence of the diffusion coefficient on the spectral density of the oscillation energy can be found by means of formula (15):

$$D(v, r) = \frac{4\pi^2 e^2}{m^2 \omega_H^2} \frac{1}{v} \int_0^\infty k dk \int_0^\pi \sin^3\theta' d\theta' \int_{-\pi}^{\pi} \sin^2\varphi' \overline{W} \times (r, k, \theta', \varphi') d\varphi'$$

(we confine ourselves to the zero-order approximation with respect to the parameter v_{ph}/v). Taking formula (11) into account, it is possible to find the explicit dependence of D on r for $r \gg r_b$:

$$D(v, r) = \frac{4\pi^2 e^2}{m^2\omega_H^2} \frac{1}{v} \left(\frac{r_b}{r}\right)^3 \int_0^\infty k dk \int_0^{\theta_0} \sin^3\theta' d\theta' \times \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\varphi' \cos\varphi' \overline{W}^{(0)}(k, \theta', \varphi') d\varphi' \quad (23)$$

It can be seen that $D(v, r) \sim r^{-3} v^{-1}$.

We introduce

$$U = \int_0^\infty k^2 dk \int_0^{\theta_0} \sin\theta' d\theta' \int_{-\pi}^{\pi} \overline{W}(r, k, \theta', \varphi') d\varphi'$$

which represents the energy density of the Langmuir oscillations. For $r \gg r_b$

$$U = \frac{r_b}{r} U_0 \quad (24)$$

where U_0 is the oscillation energy density for $r = r_b$. It follows from formula (23) that

$$D = A \left(\frac{r_b}{r}\right)^2 r_H^2 \omega_p \frac{v_{ph}}{v} \frac{U}{mn_c v^2}, \quad r_H = \frac{v}{\omega_H}$$

where A is a numerical coefficient of the order of unity dependent on the detailed form of the function $W_{\vec{k}}$. In this expression it is possible to isolate the factor

$$\nu = \omega_p \frac{v_{ph}}{v} \frac{U}{mn_c v^2} \quad (25)$$

which signifies the effective electron-oscillation collision frequency [13]. Thus

$$D = A \left(\frac{r_b}{r}\right)^2 r_H^2 \nu \quad (26)$$

A particular feature of this expression is the factor $(r_b/r)^2$ by which the latter result differs from the usual estimate for the magnetized diffusion coefficient. The appearance of this factor is due to the close dependence of the function $W_{\vec{k}}$ on the angle φ': when $r \gg r_b$, the function $W_{\vec{k}}$ is non-zero only for $|\varphi'| \lesssim r_b/r$ (see section 3).

By averaging over the angular variables it is possible to consider also effects associated with inelastic collisions. We present only the final result, without going into the intermediate calculations (which are completely analogous to the previous ones). It appears that allowance for inelastic collisions leads to the appearance on the right-hand side of Eq. (22) of a new term having the form

$$\frac{1}{v^2} \frac{\partial}{\partial v} v^2 d(v, r) \frac{\partial F}{\partial v}$$

where

$$d(v, r) = \left\langle \langle D_{vv} \rangle_\varphi - \frac{\langle D_{v\theta} \rangle_\varphi^2}{\langle D_{\theta\theta} \rangle_\varphi} \right\rangle_\theta$$

is the "coefficient of diffusion" in velocity space. The angular brackets in this formula have the following meaning:

$$\langle \ldots \rangle_\varphi = \frac{1}{2\pi} \int_{-\pi}^{\pi} \ldots d\varphi, \qquad \langle \ldots \rangle_\theta = \frac{1}{2} \int_{\theta_0}^{\pi-\theta_0} \ldots \sin\theta d\theta$$

The connection between d and the spectral function $W_{\vec{k}}$ can be found by means of Eq. (15). The resulting expression is very cumbersome and will not be written out in full. It should merely be noted that the coefficient of diffusion d is proportional to $v^{-3} r^{-1}$ and can be estimated by means of the formula

$$d = B v_{ph}^2 \nu \quad (27)$$

where B is a numerical factor of the order of unity.

So far we have assumed that the function $W_{\vec{k}}$ is exactly zero when $\theta' > \theta_0$. However, in an actual experiment there is probably also a non-zero (albeit low) level of oscillations even in the region $\theta' > \theta_0$. Interacting with these oscillations, the electrons will be scattered into the loss cone and will escape from the trap. The corresponding losses can be taken into account by including on the right-hand side of Eq. (22) the term $\nu_s F$, where ν_s is the frequency of electron scattering into the loss cone ($\nu_s \ll \nu$). Since the phase velocity of the oscillations excited by the beam is small compared with the velocity of the fast electrons, and the energy density of these oscillations decreases along the radius in proportion to r^{-1}, we can write

$$\nu_s \sim r^{-1} v^{-3} \quad (28)$$

(it should be remembered that for $v \gg v_{ph}$ the electron-oscillation collision frequency is proportional to v^{-3}). Thus, the kinetic equation taking into account the diffusion and acceleration of the electrons and their scattering into the loss cone has the form

$$\frac{\partial F}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} r D \frac{\partial F}{\partial r} + \frac{1}{v^2} \frac{\partial}{\partial v} v^2 d \frac{\partial F}{\partial v} - \nu_s F \quad (29)$$

Let us now formulate the boundary conditions for Eq. (29). The actual form of the function F(v) for $r = r_b$ (we shall use the notation $F(v, r_b) \equiv F_0(v)$) is determined by the processes occurring in the beam itself. However, a line of reasoning exactly analogous to that employed at the end of section 3 demonstrates that the function $F_0(v)$ is independent of the magnetic field. Moreover, on the basis of experimental data it may be stated that the values of v at which the function F_0 is substantially non-zero are small compared with the final velocity acquired by the electrons in the course of acceleration. Subsequent results are therefore only slightly dependent on the actual shape of the function F_0. For the sake of simplicity, we assume $F_0 = (n_0/4\pi v_0^2)\,\delta(v - v_0)$, where n_0 and v_0 are fixed quantities independent of the magnetic field and determined by the beam parameters. Thus, we shall use the boundary condition

$$F(v, r_b) = F_0(v) = \frac{n_0}{4\pi v_0^2}\,\delta(v - v_0) \tag{30}$$

The quantity n_0 signifies the fast electron density near the beam, while v_0 represents the velocity of these fast electrons.

For $v = 0$ the boundary condition can be found in the following manner. If the electron velocity is comparable with v_{ph}, then - as can easily be seen from formula (15) - in spite of the anisotropy of the oscillation spectrum, the components of the tensor $D_{\alpha\beta}$ are non-zero throughout the range of variations in $\theta: 0 \leqslant \theta \leqslant \pi$. Thus, if the electron velocity becomes comparable with v_{ph}, the electron is rapidly (in terms of the acceleration time) scattered into the loss cone and leaves the trap. Consequently, the function F vanishes when $v \sim v_{ph}$. However, since v_{ph} is small compared with the velocity of the fast electrons, this condition may be written in the form

$$F(0, r) = 0 \tag{31}$$

For $r = R$ and $v = \infty$ the boundary conditions are obvious:

$$F(v, R) = 0 \tag{32}$$

$$F(\infty, r) = 0 \tag{33}$$

5. INVESTIGATION OF THE KINETIC EQUATION AND DISCUSSION OF THE RESULTS

By solving Eq. (29) with boundary conditions (30) - (33) it is possible to determine the parameters of the hot plasma inside the trap at any moment of time. We shall confine ourselves to investigating the steady state, which is described by the equation

$$\frac{1}{v^2}\frac{\partial}{\partial v} v^2 d(v, r)\frac{\partial F}{\partial v} + \frac{1}{r}\frac{\partial}{\partial r} r D(v, r)\frac{\partial F}{\partial r} - \nu_s(v, r) F = 0 \tag{34}$$

Taking into account the relations $d(v, r) \sim v^{-3} r^{-1}$, $D(v, r) \sim v^{-1} r^{-3}$, $\nu_s \sim v^{-3} r^{-1}$, we can write

$$d(v, r) = d_0\left(\frac{v_0}{v}\right)^3 \frac{r_b}{r}$$

$$D(v, r) = D_0\left(\frac{v_0}{v}\right)\left(\frac{r_b}{r}\right)^3$$

$$\nu_s(v, r) = \nu_{s0}\left(\frac{v_0}{v}\right)^3\left(\frac{r_b}{r}\right)$$

where $d_0 \equiv d(v_0, r_b)$, $D_0 \equiv D(v_0, r_b)$, $\nu_{s0} \equiv \nu_s(v_0, r_b)$. We then introduce the new variables

$$\rho = \frac{r}{R}, \qquad \epsilon = \frac{v^2}{v_0^2}$$

which represent the dimensionless radius and dimensionless energy, respectively. In these variables, Eq. (34) and boundary conditions (30) - (33) assume the form

$$\frac{1}{\tau_a}\frac{\partial^2 F}{\partial \epsilon^2} + \frac{1}{\tau_D}\frac{\partial}{\partial \rho}\frac{1}{\rho^2}\frac{\partial F}{\partial \rho} - \frac{1}{\tau_s}\frac{F}{\epsilon} = 0 \tag{35}$$

$$F\left(\epsilon, \frac{r_b}{R}\right) = \frac{n_0}{4\pi v_0^3}\,\delta(\epsilon - 1), \qquad F(\rho, 0) = 0$$
$$F(1, \epsilon) = 0, \qquad F(\rho, \infty) = 0 \tag{36}$$

where $\tau_D = (R^2/D_0)(R/r_b)^3$, $\tau_a = (4v_0^2/d_0)(R/r_b)$, $\tau_s = \nu_{s0}^{-1}(R/r_b)$. The physical meaning of these constants is the following: τ_a represents the time within which the energy of an electron changes during acceleration by an amount of the order of unity; τ_D is the time of electron diffusion from the trap axis to the diaphragm; τ_s is the effective time of electron scattering into the loss cone.

The final energy acquired by the fast electrons during acceleration is determined by their lifetime in the trap. As was noted in section 2 (and as is reflected formally in Eq. (35)), there are two mechanisms whereby the fast electrons are lost: diffusion losses to the diaphragm (characteristic time τ_D) and scattering into the loss cone (characteristic time τ_s). The characteristic time τ_D is proportional to H^2, while τ_s is independent of H. When considering the heating effect in the region of weak magnetic fields, one can therefore neglect losses connected with electron scattering into the loss cone and omit the last term on the left-hand side of Eq. (35):

$$\frac{1}{\tau_a}\frac{\partial^2 F}{\partial \epsilon^2} + \frac{1}{\tau_D}\frac{\partial}{\partial \rho}\frac{1}{\rho^2}\frac{\partial F}{\partial \rho} = 0$$

The characteristic feature of the resulting equation is that it does not contain the oscillation energy density U_0, since the constants τ_a and τ_D are proportional to U_0. This equation can be solved by the method of separation of variables. The result has the following form:

$$F = \frac{n_0}{2\pi^2 v_0^3}\left(\frac{\rho R}{r_b}\right)^{3/2}\sqrt{\frac{\tau_a}{\tau_D}}\int_0^\infty dq \sin\left(q\sqrt{\frac{\tau_a}{\tau_D}}\right) \times \sin\left(q\epsilon\sqrt{\frac{\tau_a}{\tau_D}}\right)\frac{I_{\frac{3}{4}}(q\rho^2)\, I_{-\frac{3}{4}}(q) - I_{\frac{3}{4}}(q)\, I_{-\frac{3}{4}}(q\rho^2)}{I_{\frac{3}{4}}\left(q\frac{r_b^2}{R^2}\right) I_{-\frac{3}{4}}(q) - I_{\frac{3}{4}}(q)\, I_{-\frac{3}{4}}\left(q\frac{r_b^2}{R^2}\right)} \tag{37}$$

where the Bessel functions of an imaginary argument are designated by $I_{\frac{3}{4}}$ and $I_{-\frac{3}{4}}$.

The function F is not measured in actual experiments; only certain of its integral characteristics are determined. These will be designated by the letters Q and E:

$$Q = 8\pi^2 \int_{r_b}^{R} r\,dr \int_0^\infty \frac{mv^2}{2} F v^2 dv \tag{38}$$

$$E = \frac{\int_{r_b}^{R} r\,dr \int_0^\infty \frac{mv^2}{2} F v^2 dv}{\int_{r_b}^{R} r\,dr \int_0^\infty F v^2 dv} \tag{39}$$

Q represents the hot plasma energy per unit length of system, while E represents the mean energy of the hot electrons[5]. Calculation of Q and E by means of formulas (38) and (39) gives the following result:

$$Q = A'\pi R_0^2 \frac{mn_0 v_0^2}{2} \left(\frac{\tau_D}{\tau_a}\right)^{\frac{1}{4}} \tag{40}$$

$$E = B' \frac{mv_0^2}{2} \left(\frac{\tau_D}{\tau_a}\right)^{\frac{1}{2}} \tag{41}$$

where A' and B' are numerical constants of the order of unity expressed in the form of complex Bessel function integrals[6]. These formulas can be represented in the following form:

$$Q \sim mv_0^2 n_0 R^2 \sqrt{\frac{\omega_H R^2}{v_0 r_b}} \tag{42}$$

$$E \sim mv_{ph} R \omega_H \frac{R}{r_b} \tag{43}$$

It follows that $Q \sim R^3 H^{\frac{1}{2}}$ and $E \sim R^2 H$. This form of the dependence of Q and E on R and H is in good agreement with the results reported in Ref. [5] for the region of weak fields. With regard to the absolute values of Q and E, they depend on the constants v_0, n_0 and v_{ph}, which at present cannot be determined theoretically. Agreement with experiment is obtained if one assumes

$$v_0 \sim v_b$$
$$n_0 \sim n_b \tag{44}$$
$$v_{ph} \sim (\tfrac{1}{2} \div \tfrac{1}{3}) v_b$$

Expressions (44) are completely rational from the viewpoint of the theory of quasi-linear beam relaxation in a plasma [14].

In Ref. [5], the time of electron diffusion from the beam to the diaphragm was also determined and found to be proportional to H^2 - in agreement with the formula for τ_D presented above.

Using the measured value of τ_D and the relation

$$\frac{U_0}{mn_b v_b^2} \sim \frac{\omega_H^2 R^2}{v_b^2} \frac{1}{\omega_p \tau_D} \frac{v_0}{v_{ph}} \left(\frac{R}{r_b}\right)^3 \cdot \frac{n_c}{n_b} \tag{45}$$

it is possible to estimate the energy density of the oscillations near the beam. Substituting $\omega_H = 1.7 \times 10^{10}\ s^{-1}$, $\tau_D \sim 10^{-4}$ s, $R \sim 10$ cm, $v_b \sim 10^{10}$ cm/s, $r_b \sim 1$ cm, $\omega_p \sim 5 \times 10^{10} s^{-1}$(see Ref. [5]) into expression (45) and taking into account expressions (44), we obtain

$$\frac{U_0}{mn_b v_b^2} \sim 1$$

i.e. the energy density of the oscillations near the beam is comparable with the energy density of the beam electrons. This conclusion is in good agreement with current ideas regarding the mechanism of quasi-linear beam relaxation [14]. With increasing distance from the beam the energy density of the oscillations decreases in proportion to r^{-1} (this follows from expression (24)).

Knowing U_0, it is possible to estimate the electron-oscillation collision frequency in the vicinity of the beam with the help of formula (25):

$$\nu_0 \equiv \nu(v_0, r_b) \sim 3 \times 10^{-3}\, \omega_p \sim 10^8\ s^{-1}$$

(ν_0 should not be confused with ν_{s0}). The collision frequency for arbitrary values of r and v is found from the relation

$$\nu = \nu_0 \frac{r_b}{r} \left(\frac{v_0}{v}\right)^3$$

It can easily be seen that for fast electrons the collision frequency is low compared with the transit frequency v/L (use was made of this fact in section 4).

Let us now consider Eq. (35) in the region of strong magnetic fields when escape through the mirrors becomes the principal electron loss mechanism. A comparison of the second and third terms in Eq. (35) shows that this mechanism is comparable with the diffusion mechanism when

$$\tau_D \tau_a \sim \tau_s^2$$

or when

$$H \sim H_0 = \frac{mc}{e} \frac{r_b}{R} \frac{\nu_0}{\nu_{s0}} \frac{v_{ph}}{R}$$

In the region $H > H_0$, the lifetime of a fast electron in the trap is determined by τ_s, which is independent of the magnetic field. Accordingly, the mean energy of the fast electrons E is also independent of the magnetic field:

$$E \sim \frac{mv_0^2}{2} \frac{\tau_s}{\tau_a} \tag{46}$$

Expression (46) is obtained by comparing the first and third terms in Eq. (35). It follows from relations (41) and (46) that the function E(H) should have

[5] In Ref. [5], Q was called the energy content of the plasma and E the temperature of the fast electrons.

[6] In principle, it is possible to find the exact values of A' and B', but there is little point in doing so since τ_a and τ_D are themselves determined to within factors of the order of unity.

the form shown in Fig. 3a - in accordance with the results reported in Ref. [5]. If one uses the value of E measured at high values of H, it is possible to estimate the frequency of scattering into the loss cone ν_{s0} with the help of expression (46). It is found that

$$\nu_{s0} \sim 10^{-2}\, \nu_0$$

i.e. the oscillation level in the region $\theta' > \theta_0$ is approximately one hundred times lower than the oscillation level in the region $\theta' < \theta_0$.

On the basis of Eq. (35) it can also be shown that when $H \gg H_0$

$$Q \sim Q_0 \frac{H_0}{H} \qquad (47)$$

where $Q_0 \equiv Q(H_0)$. From a comparison of formulas (40) and (47) it can be seen that the function Q(H) has a maximum at $H \sim H_0$. The shape of the function Q(H) obtained by us (Fig. 3b) is in good agreement with experiment [5].

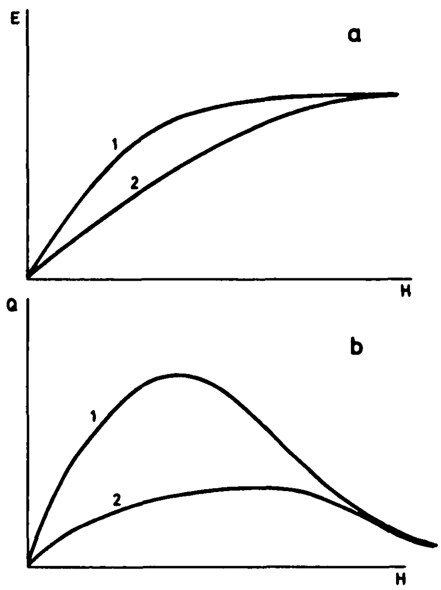

FIG. 3. Mean energy E and energy content Q of fast electrons as functions of the magnetic field. Curves 1 and 2 correspond to two different diaphragm radii ($R_1 > R_2$).

The decrease in Q at high values of H can be explained in the following manner. In the region $H > H_0$, the diffusion time exceeds the time taken for scattering into the loss cone. This means that the fast electrons enter the mirrors before they reach the diaphragm; i.e. the radius of the hot plasma R_h becomes less than the radius of the diaphragm R. An increase in the magnetic field leads to a decrease in R_h (since the diffusion coefficient D is proportional to H^{-2}). On the other hand, since Q is proportional to ER_h^2, a decrease in R_h (for E = const) causes a corresponding decrease in Q.

Let us summarize the main conclusions derived from the diffusion model:

1. The diffusion model satisfactorily explains the observed form of the functions Q(H, R) and E(H, R);

2. With reasonable assumptions regarding the properties of the beam as a source of waves and particles, the diffusion model gives correct absolute values for Q and E;

3. The diffusion model makes it possible to understand many qualitative aspects of experiments (e.g. the threshold dependence of the heating effect on the mirror ratio the dependence of the diffusion time on the magnetic field, and the decrease in plasma radius in the region of strong magnetic fields).

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