VLEPP: TRANSVERSE BEAM DYNAMICS

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The transverse dynamics of an intense relativistic beam in a linac is mainly determined by radiation fields. Upon bunch deflection from the accelerating structure axis, the asymmetrical fields are emitted. These fields lead to the action of the transverse force on the particle in the bunch. At small deflections, the most substantial are the fields with one variation over the azimuth, whose amplitude is proportional to the bunch deflection. The transverse force, averaged over the structure period, may be represented in the form of an effective gradient:

$$F(z_0) = eG(z_0)\cdot\delta ,$$
$$G(z_0) = \frac{1}{a}\int_{-a/2}^{a/2}\left[E_z(0,z,t - H_\varphi(0,z,t)\right]dz ;$$
$$ct = z - z_0 ,$$

where δ is the emitting bunch deflection from the axis, a is the structure period, and z_0 is a longitudinal coordinate in the bunch.

The magnitude of gradient is determined by the parameters of the bunch and by the structure geometry. As the calculations show, the gradient is proportional, in a first approximation, to the number of diaphragms per unit length and is inversely proportional to the squared radius of the hole in a diaphragm. For the -structure the distribution of the gradient over the bunch, which has been obtained by numerical methods [1], is illustrated in Fig. 1. This figure also illustrates the charge distribution over the bunch.

The specific feature of the action of transverse forces is that the field, which is emitted by a part of the bunch, influences only behind the flying part, i.e. the magnitude of the force, which has influence on the 'tail' of the bunch, is determined by deflection of the 'head'. This results in the development of the transverse instability similar to the instability in a conventional linac. Focusing by means of quadrupole lenses changes the nature of development of the instability. In this case, the 'head' acts upon the 'tail' periodically at the frequency of free oscillations. Thie leads the resonant build-up of 'tail' oscillations.

The resonant mechanism of developing the instability predetermines the method of suppressing the instability by shifting the frequencies of transverse oscillations. The latter can be made by introducing the energy gradient of particles along the bunch. The equation, which describes the transverse motion of particles in the bunch, is similar to that for the sequence of bunches in a conventional linac[2]:

$$\frac{\partial}{\partial\tau}\left(\gamma\frac{\partial U}{\partial\tau}\right) + \gamma\nu^2 U = \frac{G}{mc^2}\int_{-\infty}^{x} U f(x-\xi)\,\rho(\xi)\,d\xi \quad (1)$$

where $\tau = ct$, γ is the current relativistic gactor, $\rho(\xi)$ is the charge distribution over the bunch; the function $f(x-\xi)$ is dependent on the structure of radiation fields, but one can put $f(x-\xi) = x-\xi$ in a first approximation.

This equation can be solved numerically. However, some regularities can be found on the basis of the solutions of a simpler equation. Let us assume that the acceleration is absent, the charge distribution is uniform on the interval [0; 1], and the function $f(x-\xi) = 1$. Then,

$$\frac{\partial^2 U(x,\tau)}{\partial\tau^2} + \nu^2(x)\,U(x,\tau) = \frac{G}{\mathcal{E}(x)}\int_0^x U\,d\xi .$$

Let us introduce a linear energy distribution over the bunch:

$$\mathcal{E} = \mathcal{E}_0\left(1 - \frac{\Delta\nu}{\nu_0}x\right) \quad ; \quad \nu = \nu_0\left(1 + \frac{\Delta\nu}{\nu_0}x\right)$$

where $\mathcal{E}_0$ and ν_0 are the energy and frequency of the 'head' respectively and $\frac{\Delta\nu}{\nu_0}$ is the full shift of frequencies.

The initial condition is the beam shift as a whole. Using the Laplace transform in time

$$V(x,p) = \int_0^\infty U(x,\tau)\,e^{-p\tau}d\tau ,$$

we obtain

$$V = \frac{\nu_0\mathcal{E}_0}{\mathcal{E}}\;\frac{1}{\nu^2+p^2}\left(\frac{\nu^2+p^2}{\nu_0^2+p^2}\right)^{\eta} ,$$

where

$$\eta = \frac{G}{2\mathcal{E}_0\nu_0^2\left(\frac{\Delta\nu}{\nu}\right)}$$

The inverse Laplace transform is easy to derive for integer values of η :

1. Free oscillations with eigen frequencies (there are no transverse forces):

$$\eta = 0 ;\; U(x,\tau) = U_0\frac{\mathcal{E}_0}{\mathcal{E}(x)}\cos[\nu(x)\tau] .$$

2. Bunch oscillation as a whole:

$$\eta = 1 ,\; U(x,\tau) = U_0\frac{\mathcal{E}_0}{\mathcal{E}(x)}\cos\nu_0\tau .$$

3. Build-up of oscillations at a frequency of the 'head':

$$\eta = 2 ;\; U(x,\tau) = U_0\frac{\mathcal{E}_0}{\mathcal{E}(x)}\left\{\cos\nu_0\tau + \frac{\nu^2-\nu_0^2}{2\nu_0^2}(\nu_0\tau)\sin\nu_0\tau\right\}$$

4. Build-up of oscillations at natural frequencies:

$$\eta = -1 ;\; U(x,\tau) = U_0\frac{\mathcal{E}_0}{\mathcal{E}(x)}\left\{\cos[\nu(x)\tau] - \frac{\nu^2-\nu_0^2}{2\nu^2}(\nu\tau)\sin[\nu(x)\tau]\right\}$$

Thus the instability can be suppressed at a particular magnitude and sign of the spread, i.e. when the condition

$$\frac{\Delta\nu}{\nu} \geq \frac{G}{2\mathcal{E}_0 \nu_0^2}$$

is satisfied. Of great interest is the solution at $\eta = 1/2$. Provided that $\frac{\Delta\nu}{\nu} \ll 1$ the approximate solution is as follows:

$$U(x,\tau) \approx U_0 \frac{\mathcal{E}_0}{\mathcal{E}} J_0\left(\frac{\nu-\nu_0}{2}\tau\right) \cdot \cos\nu\tau ,$$

where J_0 is the Bessel function. As is clear, the amplitude of oscillations is damped, and $U \sim \frac{1}{\sqrt{\tau}}$. Unfortunately, as the analysis shows, this interesting result holds only for the function $f(x-\xi) = 1$.

The results of the numerical integration of equation (1) exhibit the similar regularities of the transverse motion of the bunch. Fig. 2 gives the phase images of particles along the bunch at different spread in energies. In Fig. 3 one can see one of the results of numerical simulation for the 100 GeV section of the accelerator VLEPP. As is seen, to suppress the transverse instability, an initial energy spread of ±10% needs to be introduced. In the course of acceleration this spread can be decreased down to ±3% and can reach a minimally achievable one on the final section.

We would like to mention that the efficiency of the presented method of instability suppression is the higher the larger the number of periods of transverse oscillations, made by the bunch on the whole length of the accelerator.

References

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2. Burshtein E.L. and Voskresensky G.V. Linear electron accelerators with intense beams. Moscow, Atomizdat, 1970.

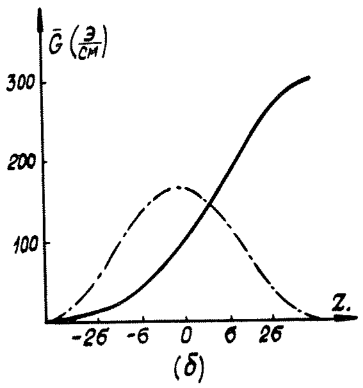

Fig.1. Distribution of the transverse force of radiation fields along the bunch.

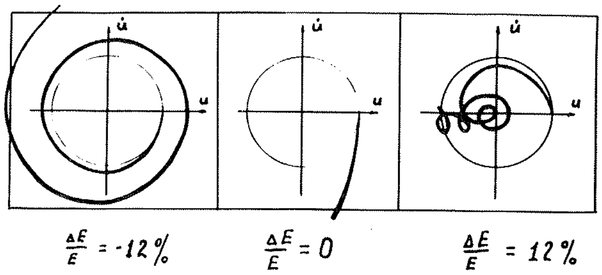

Fig.2. The phase picture of the particles in the bunch at different energy spreads.

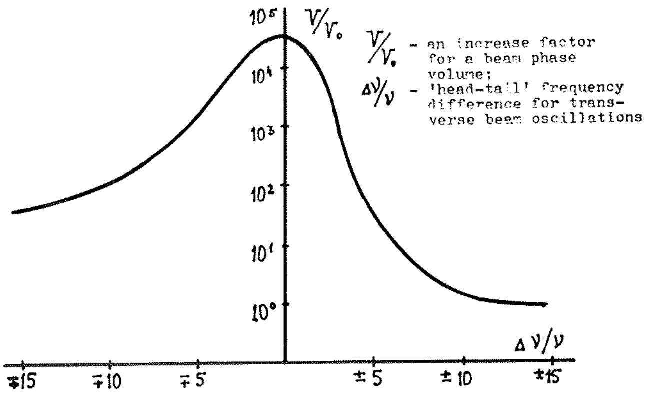

Fig.3. Relative phase volume at the exit of the accelerator's 100 GeV section versus the initial energy spread.