VLEPP: STOCHASTIC BEAM HEATING

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To achieve a high luminosity in the accelerator VLEPP (colliding linear electron-positron beams)[1], it is necessary that the bunches at the interaction point have a very small size in one of the transverse directions.

The major effect, which leads to a considerable increase of the phase volume, is the transverse instability of a single bunch in a linac[2]. The authors have previously shown that this instability can be suppressed if the frequencies of transverse oscillations in the bunch are shifted. This can be made by introducing the energy spread over the bunch. Note that the required relative spread is in definite relation to the frequency of transverse oscillations ν and to the energy $\mathcal{E}$. In the VLEPP accelerator the following dependence

$$\nu = \nu_0 \left(\frac{\mathcal{E}}{\mathcal{E}_0}\right)^{-\varkappa}$$

has been chosen. In this case, the spread changes as follows:

$$\frac{\Delta\mathcal{E}}{\mathcal{E}} = \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)_0 \cdot \left(\frac{\mathcal{E}}{\mathcal{E}_0}\right)^{2\varkappa-1}$$

where ν_0 and $\mathcal{E}_0$ are the initial frequency and energy.

It is worth noting that for the sufficiently large number of particles in the bunch to be accelerated, a sufficiently large energy spread is required (± 10 at the beginning of acceleration).

However, any random perturbations of a trajectory of the transverse bunch motion, which result in increasing the amplitude of transverse oscillations, give rise to an increase in the phase volime just because of the energy (frequency) spread[3]. Let us treat this process on the phase plane in the time representation (Fig. 1). At the initial instant, the bunch, which has the zero phase volume, acquires random transversal angle (strike). The particles in the bunch begin to oscillate at natural frequencies and gradually 'run' over phases. The bunch is extended over the perimeter of the ellipse on the phase plane and completely occupies it during the period $\Delta t = \frac{1}{\Delta\nu}$ (where is the full shift in frequency). It is the ellipse area which determines the acquired phase volume of the bunch. As the bunch travels in the accelerator, it is subjected to a great number of random strikes, and the increments in amplitudes of different particles of the bunch are different just because of the frequency spread. As a result, the originally point-like bunch 'spreads' on the phase plane, as is shown in Fig. 2. This Figure also presents the distribution in frequencies. If random strikes are independent, the distribution, normalized over the r.m.s. size σ ($\sigma^2 \sim N$, N is the number of strikes) remains the same.

Random strikes are caused by the errors of the focusing system (displacement of the centers of quadrupole lenses with respect to the accelerator axis, rotation and inclination of the lenses relative to the axis, deviation of the field gradient of the lenses) and of the accelerating system (displacement and inclination of the accelerating sections, deviations of the acquired energy from a nominal one)[4]. It is necessary to emphasize that, in addition to random perturbations, the regular perturbations are possible, for example, deformation of the accelerator axis because of the ground waves[4,5].

Let us consider in detail the influence of particular perturbations on an increase in phase volume of the bunch. For estimations we use the model of thin lenses, and the relation between the amplitudes of deviation A and of the angle x'_m is defined by means of the function $\beta = \beta(\nu)$: $A = \beta(\nu) \cdot x'_m$.

1. Random displacement of the centres of quadrupole lenses (vibrations)

These errors of the focusing structure is the most significant. Indeed, if the centre of a lens with the focal distance f is shifted with respect to the accelerator axis at a distance of Δx, the bunch, after its flight through this lens, acquires an additional angle

$$\Delta x' = \frac{\Delta x}{f}.$$

The maximum increase in the amplitude of oscillation will be equal to

$$\Delta A = \beta\,\frac{\Delta x}{f}\,, \quad \beta = 2f\sqrt{\frac{1+\ell/2f}{1-\ell/2f}}\,,$$

where ℓ is the distance between the lenses.

Thus, the increase in the amplitude of oscillation exceeds more than by a factor of two the displacement of the quadrupole lens. At the end of acceleration the amplitude will be determined by the sum of the increments in the amplitude of oscillations due to the displacements of all the lenses with the adiabatic damping taken into account. From the above considerations, one can obtain the estimate for the phase volume determining the r.m.s. size of the bunch:

$$\Phi_1 = \frac{(\Delta x)^2 L}{\ell_0^2} \cdot \frac{2+\sqrt{2}}{2(1-\varkappa)} \cdot \left(\frac{\mathcal{E}_0}{\mathcal{E}_f}\right)^{2\varkappa} \qquad (3)$$

where L is the length of an accelerator, ℓ_0 is the initial distance between the lenses, and $\mathcal{E}_f$ is the final energy of the accelerator. The number of lenses on the wavelength of transverse oscillations is assumed here to remain the same and to be equal to 8. However, if the action of one strong lens is replaced by that of a set of weaker lenses of the same sign, as acceleration continues, the smaller phase volume will be excited:

$$\Phi_2 = \frac{(\Delta x)^2 L}{\ell_0^2} \cdot (2+\sqrt{2}) \left(\frac{\mathcal{E}_0}{\mathcal{E}_f}\right)^{3\varkappa}$$

From the assumption that the bunch completely 'spreads' over the ellipse during the characteristic period of the loss in coherence ($\ell/\Delta\nu$), it is also possible to estimate the phase volume determining the maximum size of the bunch:

$$\Phi_{max} = \frac{(\Delta x)^2 L}{\ell_0^2} \cdot \frac{2+\sqrt{2}}{4} \left(\frac{\mathcal{E}_0}{\mathcal{E}_f}\right)^2 \left(\frac{\Delta\mathcal{E}}{\mathcal{E}}\right)_c$$

II. The coherent displacement of lenses is due to the ground vibrations when propagating the seismic waves, whose amplitudes and wavelengths can be very different. Note that the coincidence of the wavelengths of transverse oscillations of the bunch with the ground vibrations wavelength is possible to take place. Far from the resonance, the excited phase volume is as follows:

$$\Phi_\lambda \sim \frac{8}{2+\sqrt{2}} \cdot \frac{a_\lambda^2}{\lambda}$$

where a_λ is the amplitude of a seismic wave with the wavelength λ.

III. As is known, random turns of quadrupole lenses around the longitudinal axis lead, on the one hand, to the amplitude errors and, on the other hand, to the rotation of the bunch around the longitudinal axis [5]. Since the flat bunch with the relation between transverse sizes $\frac{a}{b} \gtrsim 100$ is used in the accelerator VLEPP, the bunch rotation effectively gives rise of the small size just because of the frequency spread. The increment in amplitude due to rotation of the lens by an angle $\Delta\varphi$ is equal

$$\Delta A = \Delta\varphi \cdot \frac{a}{2} \cdot \frac{\beta}{f}$$

and, correspondingly, the phase volume will constitute

$$\Phi_{\Delta\varphi} = \frac{\langle (a\Delta\varphi)^2 \rangle L}{\ell_0^2} \cdot \frac{2+\sqrt{2}}{8(1-\varkappa)} \cdot \left(\frac{\mathcal{E}_0}{\mathcal{E}_f}\right)^{2\varkappa}.$$

IV. Random errors in the alignment of accelerating sections may be represented as the displacements of the section as a whole and the inclination with respect to the accelerator axis. When displacing the section at a distance of , the bunch undergoes the action of the transverse force of radiation fields and acquires the transversal angle

$$x'_m = g Q \ell d / \mathcal{E},$$

where gQ is the gradient of transverse force.

At the end of acceleration the excited phase volume is equal to

$$\Phi = \left(\frac{g Q \ell_0}{\mathcal{E}_0}\right)^2 \langle d^2 \rangle \cdot \frac{2+\sqrt{2}}{\varkappa} L \left(\frac{\mathcal{E}_0}{\mathcal{E}_f}\right)^{2\cdot\varkappa}.$$

In the case of inclination, the ends of the section shift with respect to each other towards the transverse direction at the distance $d = \lambda\ell$, where λ is the angle of inclination and ℓ is the length of the section.

In this case, the axisymmetric accelerating field contributes, mainly, to the transverse force, and the acquired transversal angle equals

$$x'_m = \frac{1}{2} \frac{\Delta p_\parallel}{p} \lambda = \frac{\mathcal{E}_f \cdot d}{2\mathcal{E} L}$$

And, correspondingly, the final phase volume is equal to

$$\Phi = \frac{2+\sqrt{2}}{4\varkappa L} \langle d^2 \rangle \left(\frac{\mathcal{E}_0}{\mathcal{E}_f}\right)^{-\varkappa}$$

Thus, the basic mechanisms of increasing the phase volume of the bunch in a linac have been considered. As the quantitative relations for the VLEPP show, the stability of position of the lenses is the most significant: in order to achieve the required luminositv. it is required that the accuracy be at a level of fractions of a micron.

References

1. V.E.Balakin, G.I.Budker and A.N.Skrinsky. Proc. of the 6th National Conf. on Charged Particle Accelerators. Dubna, 1978, p. 27.
2. V.E.Balakin, I.A.Koop, A.V.Novokhatsky, A.N.Skrinsky and V.P.Smirnov. Proc. of the 6th National Conf. on Charged Particle Accelerators. Dubna, 1978, p. 143.
3. I.M.Kapchinsky, Usp. Fiz. Nauk (Sov.) 132, (1980) 639.
4. A.D.Vlasov, Theory of Linear Accelerators. Moscow, Atomizdat, 1965.
5. I.M.Kapchinsky, Particle dynamics in linear resonance accelerators. Moscow, Atomizdat, 1966.
6. Linear ion accelerators, ed.by B.P.Murin, vol. 1, Atomizdat, 1978.

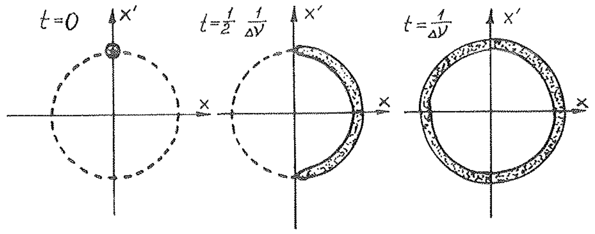

Fig.1. The time representation of the 'smearing' of a non-monochromatic bunch on the phase plane.

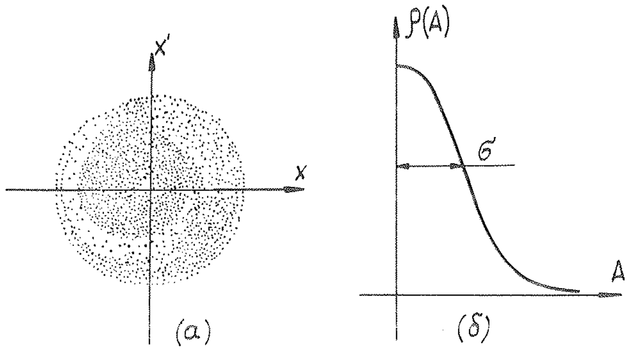

Fig.2: (a) 'stochastically' heated bunch on the phase plane,
(b) density distribution of the particles over the amplitude of oscillations.