BEAM-BEAM EFFECTS FOR THE ELLIPTICAL BEAM WITH THE LARGE RATIO OF THE TRANSVERSE DIMENSIONS

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Achieving a maximal current together with a minimal beam size is one of the most important problems in colliding beam accelerator physics. As it is known, the main reason for an increase in the transverse sizes of the beams is the stochastic instability of the particles imposed by many interactions with nonlinear field of the colliding beam. Although the basic mechanism of this phenomenon is quite well understood, no satisfactory theory concerning its application to the colliding beam problem is available to date. The particular case of 'one-dimensional' models, in which only one degree of transverse particle motion is considered, has been quite thoroughly analysed, both analytically and numerically (see, e.g., Ref. 1). These models (in there simplest form) describe the interaction of a single particle with a beam whose cross section is round or flat.

However, in most existing facilities the beams have an elliptical cross section. Despite the fact that parameter $\varkappa = \sigma_x/\sigma_z$ is often large ($\varkappa \approx 10 \div 40$), it is questionable whether a one-dimensional approximation accounts for the specific features of the motion in beams with high ellipticity. Moreover, recent numerical experiments with more realistic models (Refs. 2-4) do not clarify the functional dependence of the critical values on the parameter $\varkappa$. It is known, for example, that the round beam with equal betatron frequencies ($\nu_x \approx \nu_z$) possesses some advantages (Ref. 5). On the other hand, some numerical results (Ref. 6) seem to indicate that the critical parameter ξ_{cr} for flat beam is larger than that of a round one. This is related with the peculiarities of the resonance structure in the phase plane. Therefore, the role of the beam ellipticity for the achievement of maximum ξ must be clarified.

In the present paper the structure of nonlinear resonances in the model of a strongly elliptical beam is analysed, and some characteristic features of this structure (which are directly related to the experimental data on the VEPP-2M facility) are investigated.

The e^+e^--model is described by a four-dimensional mapping including:

1) betatron oscillations between the collision points;

2) interaction with the strong beam in the usual thin-lens approximation;

3) modulation of the radial motion by synchrotron oscillations (with the Ψ -function not equal to zero at the collision points);

4) synchrotron radiation and quantum fluctuations related to this radiation (noise);

5) linear coupling to simulate the effect of quadrupole lenses turned by 45°.

All the parameter values have been chosen close to those of the VEPP-2M. The forces f_x and f_z from the strong beam with elliptical cross section (due to Gaussian distribution $\rho = \rho_0 \exp(-x^2/2\sigma_x^2 - z^2/2\sigma_z^2)$; $\varkappa = 20$) have been calculated according to exact formulae (Ref. 7) for discrete values (x_i, z_i) with linear interpolation between them.

An analytical study of similar systems with two degrees of freedom and with an external periodic perturbation turns out to be much more difficult than the 'one-dimensional' systems mainly because the potential of the elliptical beam is very complicated function of variables x and z. As a result, some important characteristics of nonlinear systems, such as the frequency dependence of the amplitude (or nonlinear tunes $\Delta\nu_x, \Delta\nu_z$) and the amplitude of resonance harmonics are difficult to describe analytically. In the case of numerical simulation some additional problems arise. They are associated with the fact that the phase space of the system is four-dimensional and therefore (unlike the "one-dimensional" models) the resonant structure is hard to visualize.

For this reason, in our research we have used the method suggested in Ref. 8. The main idea is to study the location of the most significant resonances in the amplitude space (A_x, A_z) of transverse motion rather than the resonance structure in the plane of the betatron frequencies. To do this, it is necessary, first of all, to find the tune shifts $\Delta\nu_x, \Delta\nu_z$ as a functions of the amplitudes A_x, and A_z. This dependence, obtained by numerical integration, is plotted in Fig. 1 for $\xi_x = \xi_z = 1$. The variation range of the amplitudes A_x and A_z, normalized over σ_x, σ_z, corresponds to the aperture of the VEPP-2M to $10\sigma_x$ in the x -direction and to $80\sigma_z$ in the z -direction, respectively. The real picture (for $\xi_x, \xi_z \neq 1$) is obtained by a suitable coordinate scaling. Regions in which $\Delta\nu_x$ is only slightly dependent on A_x (vertical lines $\Delta\nu_x \approx$ const), appear only for large $\varkappa$, and the motion in these regions is close to one-dimensional.

Fig. 1 gives the frequency-amplitude correspondence which makes it possible to map any resonant line from the frequency space into the amplitude space. In this way one can locate in the amplitude space (A_x, A_z) any given resonance $n_x\nu_x + n_z\nu_z + n_s\nu_s = k$ (here k, n_x, n_z, n_s are integers and ν_s is the synchrotron frequency).

Using the computer we investigated the resonance structure in amplitude space for various values ξ_x and ξ_z. A tipical example is illustrated in Fig. 2, where, the two scales are used in the z -direction.

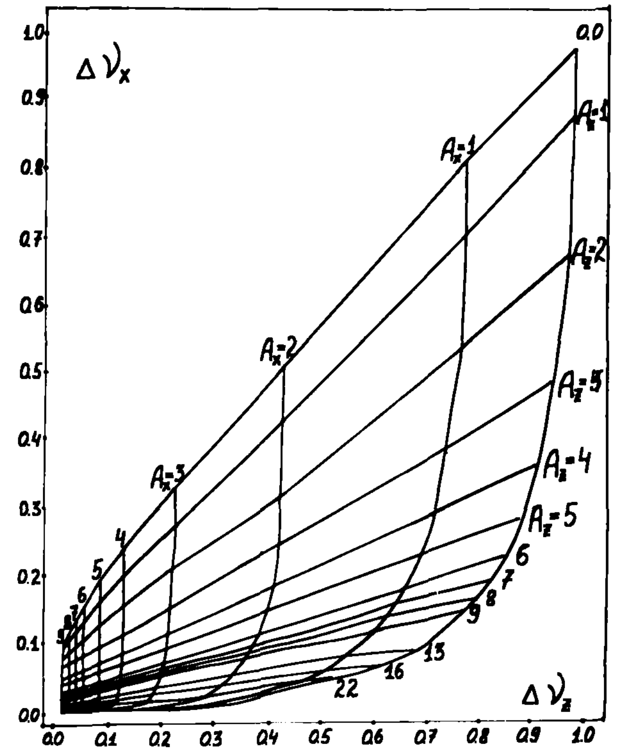

Fig. 1. Nonlinear tune shifts of elliptical beam for different values of the amplitudes A_x, A_z (for $\xi_x = \xi_z = 1$, $\ae = 20$).

For the sake of clarity, only the main resonances are represented, while we have neglected the multiplet splitting of every resonance caused by synchrotron oscillations ($\nu_s \neq 0$). The dotted lines correspond to the degenerated resonances which arise when the potential symmetry is broken (for example, in the case of constant displacement on X -direction, see, for example, Refs. 1, 6). However, as it has been shown in Refs. 5, 1, 9 and 10, the effect of synchro-betatron resonances can actually be significant, since the presence of these resonances can substantially decrease the stochasticity limit ξ_{cr}.

It is seen from the picture of resonance lines in the amplitude space that they are not uniformly spaced. We remark that it is not a consequence of having truncated the order of resonances ($N=|n_x|+|n_z| \leq 16$), but rather related to the specific properties of the coordinate change $A_{x,z} \longleftrightarrow \nu_{x,z}$ and the choice of the operating point ν_x^o, ν_z^o.

Usually the operating point is chosen near the strong lower-order resonance (in our case $\nu_x^o \approx \nu_z^o$). As seen in Fig. 2, the higher -order resonance lines are regrouped into families having common intersection points which lie on the coupling resonance line $\nu_x = \nu_z$. The "empty" regions in the amplitude plane close to these points can be of great importance, since here the motion is expected to be much more stable.

The most tipical peculiarity of the resonance structure is that the resonance li-

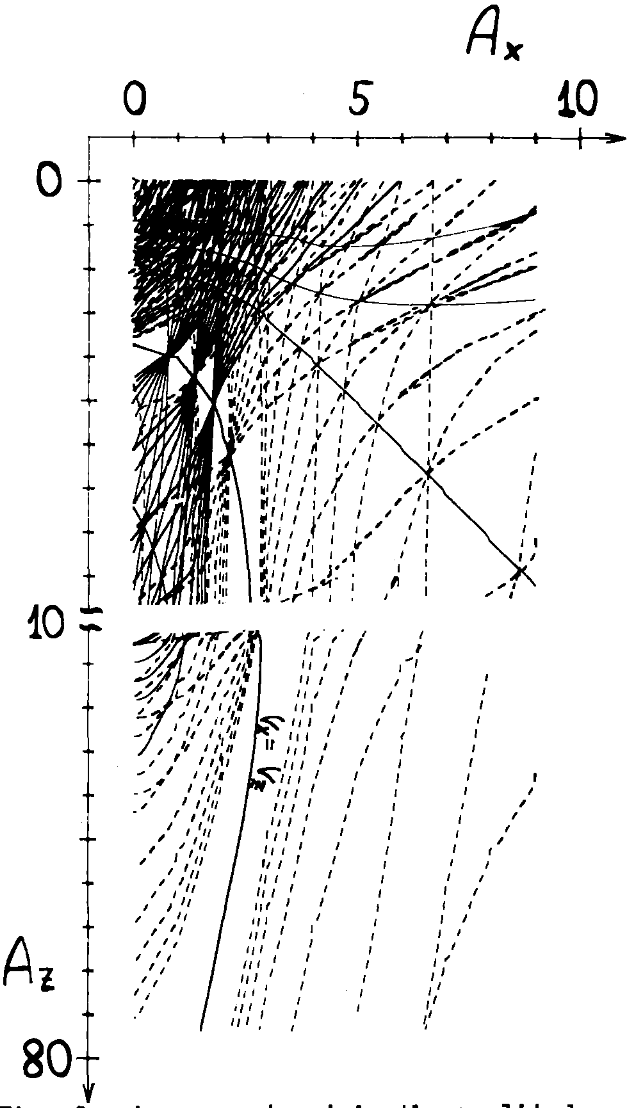

Fig. 2. A resonant net in the amplitude space (A_x, A_z) for elliptical beam. Two different scales in the z -direction are used: for $10 \leq A_z \leq 80$ the scale is 7 times larger than for $0 \leq A_z \leq 10$ ($\nu_x^o = 3.06$, $\nu_z^o = 3.08$, $\xi_x = 0.07$, $\xi_z = 0.14$. Maximum resonance order $N = 16$).

nes are considerably streched along z -direction. Therefore, whenever overlap occurs on x -direction the formation of a stochastic region which extents itself along z -direction is created.

The important problems in numerical simulation to be clarified are both the choice of an optimal ratio ξ_x/ξ_z and the study of the influence of the aperture sizes on the limiting value of the current. It is usually assumed that only the z -direction is of significance for the aperture limitation. However, the experiments performed at the VEPP-2M (Ref. 11) have shown that for the conditions of this storage ring (where the radial aperture is approximately 8 times larger than the vertical one) the limitation to the lifetime stems from the particle departure out of the radial aperture rather than the vertical one.

Numerical simulation has been carried

out with the following parameters, close to the parameters of the VEPP-2M: $\nu_x^0 = 3.06$, $\nu_z^0 = 3.08$ and $\varkappa = \sigma_x/\sigma_z = 20$; the number of collision points is $m = 2$, the damping time is $\tau_0 \approx 10^5$ iterations, the modulational amplitude (connected with the radial dispersion function) $A_s = 0.5\sigma_x$ and the modulational frequency is $\nu_s = 0.01$.

The results of numerical simulation have been represented as the histograms of motion in the amplitude plane A_x, A_z. In a detailed numerical study of the behaviour of individual particles for various initial data, the general features of the motion was found to be accounted for by the resonance structure (Fig. 2). For example, a rapid growth of the amplitude in the z - direction was observed: $z_{max} \approx 55$ for $\xi_x \approx 0.055$ and $\xi_z = 0.11$. At the same time numerical simulation shows that the width of stochastic region in the x -direction not too small and increases up to the $10\sigma_x$ at large times. This fact can explain above mentioned experimental data (Ref. 11). It should be noted also that on the edge of stochastic region only the highest-order resonances are present and, therefore, their effect in restricting the lifetime of the beam is likely to be significant. The typical example of such motion is given in Fig. 3.

Conclusions

1) For elliptical beams with large ratio $\sigma_x/\sigma_z \gg 1$, the resonances in the amplitude space are stretched in the z -direction while their overlap occurs, mainly, in the x -direction. As a result a large stochastic region, elongated along z, appears. The numerical simulation confirms this conclusion.

2) Although the increase in the amplitude of betatron oscillations is most significant in the z -direction, in the case of insufficiently large radial aperture it is the radial x -direction which is responsible for the limitation of the maximal currents in colliding beams.

3) Numerical simulation shows the presence of a weak diffusion in the x and z -directions for large time scales ($t \gtrsim 10\tau_0$) which in turn causes a small increase of the stochastic area. This fact may explain some experimental data on VEPP-2M, where lifetime of the beams even for large $z \gg \sigma_z$ is slightly dependent on the aperture limitation.

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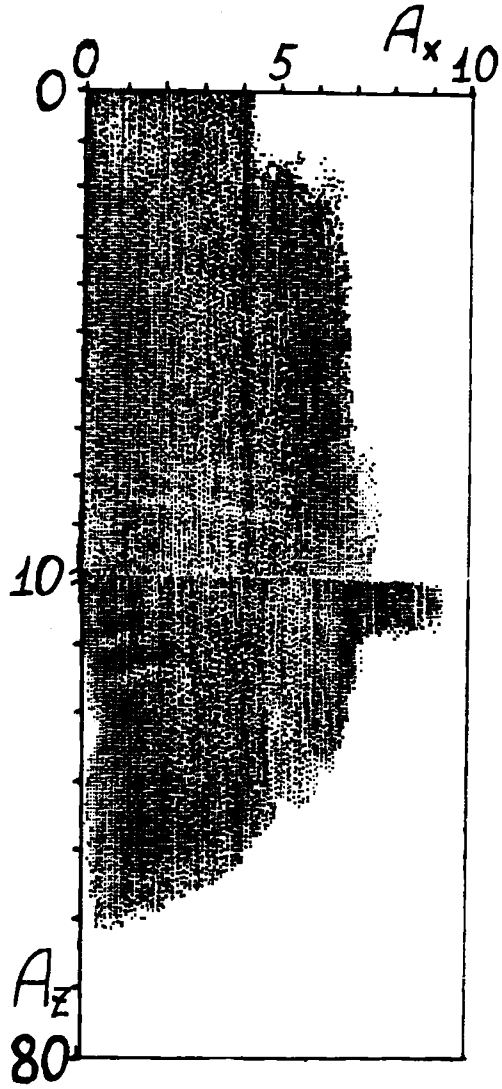

Fig. 3. Typical distribution of the trajectory in the A_x, A_z-space for $t = 90\tau_0$ (The same parameters as for Fig. 2).