POSSIBILITIES OF POLARIZED PROTON ACCELERATION UP TO AN ENERGY 1 TeV AND ABOVE

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One of the basic problems in setting up the experiments with polarized particles in accelerators and storage rings is a prevention of depolarization on spin resonances intersected upon beam acceleration. At the resonance points the frequency of spin precession ν coincides with the combination frequencies of orbital motion. It is known that a decrease of the degree of polarization by some resonance is small when the crossing through it is fast, or, otherwise, adiabatic. At high energies one does not managed to provide a sufficient rate of interaction of the main resonances. The methods which provide the adiabaticity during the entire acceleration process are more promising. The parameter adiabaticity depends on how much generalized frequency of spin precession (determined by all the structure of magnetic field with the perturbations taken into account) remains far from the resonance values. So, introduction of magnetic fields which change the sign of the vertical componen, of polarization in one or several parts of an orbit makes the value of the generalized precession frequency close to a constant (energy-independent) one, thereby allowing a maximal increase of the adiabaticity parameter. Using such magnetic structures, called "Siberian snakes", would make it possible to substantially increase an energy of polarized particles.

The schemes of acceleration with the help of one or two snakes have been previously considered /1.3/. The perturbating fields associated with the imperfectness of the main magnetic structure and with the spread of particle trajectories in the beam have assumed to cause only a small perturbation of the spin motion corresponding to the calculated closed orbit. These perturbations grow with increasing a maximum energy and the size of accelerators and their effect on the spin is comparable that of a snake. An analysis of the possibilities of conserving the polarization in such situations becomes in an energy range of the order of 1 TeV or higher, as, for example, in the UNK (SSSR) and FNAL (USA) projects.

At high energies the main perturbating influence on the spin is exerted by the radial field H_x. In accelerators without snakes, the powers of spin resonances w_K are determined by the correlation between a change of this field on a particle trajectory and by the spin precession in the vertical field H_z :

$$w_K = \frac{\nu_0}{\langle H_z \rangle} \cdot \left\langle H_x e^{i\nu_0 \alpha} \right\rangle_{\nu_0 = \nu_K} \tag{1}$$

where $\nu_0 = \gamma(g-2)/2$ is the spin precession frequency in the vertical field (in units of revolution frequency), $\alpha(\theta) = \int^{\theta} \mathcal{K}_z d\theta$ is the angle of rotation of a particle velocity on the orbit, $\mathcal{K}_z$ is the vertical magnetic field in units of its average value $\langle H_z \rangle$, θ is the total azimuth of the particle, the brackets $\langle ... \rangle$ denotes the averaging along the orbit (over the azimuth θ), ν_K is the integer combination of the frequencies of orbital motion. The quantity $2\pi w_K$ has a sense of the angle of spin rotation in the vertical plane per revolution of a particle in an accelerator under the stationary conditions in the resonance region (when $|\nu_0 - \nu_K| \ll |w_K|$).

The most powerful are resonances with free vertical oscillations:

$$\nu_0 = \pm \nu_z + pN \;, \qquad (p = 0, \pm 1, ...) \tag{2}$$

(ν_z is the betatron frequency of vertical oscillations, N is the number of periods of magnetic structure on the orbit which is assumed to be large) and also the integer resonances

$$\nu_0 = K \qquad (K = \pm \nu_z + pN) \tag{3}$$

connected with the vertical distortions of the equilibrium orbit.

For intrinsic resonances (2), the power w_K is equal to

$$w_K = \nu_0 \begin{cases} a_b \langle f'' e^{i\nu_K \alpha} \rangle \;, & \nu_0 = -\nu_z + pN \\ a_b^* \langle f^{*''} e^{i\nu_K \alpha} \rangle \;, & \nu_0 = \nu_z + pN \end{cases} \tag{4}$$

where $f(\theta) = f(\theta - 2\pi) exp(2\pi i \nu_z)$ is the normal solution. The Floque equations for vertical betatron oscillations :

$$z/R = a_b f + a_b^* f^* , \tag{5}$$

where $2\pi R$ is the orbit perimeter.

The system of natural resonances is so strongly rarefied because of large N that their powers are always, in practice, small compared to the distances between the neighbouring resonances. This means that the perturbation of vertical polarization is small when a particle passes a path of the order of one period of magnetic structure. However, we do not assume the smallness of a change in polarization for the period of particle revolution ($N \gg |w_k| \gtrsim 1$).

Integer resonances (3) are also grouped, over their power, near the intrinsic ones because the harmonics, which resonate with free oscillations, are most clearly expressed in distortions of the equilibrium orbit. A formula for vertical deviation Z_s can be written in the form similar to (5) with substitution

$$a_b \longrightarrow a_s = \frac{1}{2i} \int_{-}^{\theta} \mathcal{K}_x f^* d\theta \quad , \qquad (6)$$

where $\mathcal{K}_x$ is the radial magnetic field in terms of $\langle H_z \rangle$. In practice, at a large number of the magnetic structure elements the spectrum of perturbating field $\mathcal{K}_x$ is homogeneous. This enables one to consider the induced amplitude $a_s(\theta)$ as a function slightly varying on the period of betatron oscillations.

As long as the powers of the strongest resonances remain small ($|w_k| \ll 1$), the depolarizing effect of all the perturbations can be prevented by introducing one or two Siberian snakes. At a fairly high energy the smallness condition of w_k will not be satisfied ($|w_k| \gtrsim 1$). In this case, the influence of the perturbating radial fields is comparable with the snake effect and, therefore, introduction of one or two snakes does not guarantee the conservation of polarization upon acceleration. A comprehensive analysis needs to elucidate the possibility of avoiding depolarization such an analysis has to take into account the effects of adiabaticity violation by combination resonances of higher orders.

It is possible to suppress the influence of strong resonances, breaking the coherence of addition of spin perturbations along the orbit. This is performed by the methods described below.

1. Let us examine a situation when M pairs of symmetrically located snakes, which reverse the vertical polarization, are introduced into the straight sections of the periodical magnetic system of an accelerator. The rotation axes of polarization of the snakes in each pair constitute a certain energy-independent angle φ . In such a system, the equilibrium polarization $\vec{n}_0(\theta)$ is vertical outside the snakes and alters its sign after the passage of a particle through each snake. Here, the generalized spin frequency is equal to

$$\nu = M\varphi/\pi \quad . \qquad (7)$$

The spin perturbations described by parameters w_k cause the deviation of the precession axis, $\delta\vec{n} = \vec{n} - \vec{n}_0$, and shift the precession frequency, $\delta\nu \sim (\delta\vec{n})^2$. If in the process of acceleration the deviation of $\vec{n}$ remains small, one can turn out the precession frequency ν from all dangerous resonances by an appropriate choice of the angle φ .

The deviation of the precession axis is calculable according to formula /Ref. 4/:

$$\delta\vec{n} = \mathrm{Im}\; \vec{\eta} \int_{-}^{\theta} \vec{w}\vec{\eta}^* d\theta ,$$

where $\vec{\eta}(\theta+2\pi) = \vec{\eta}(\theta) e^{-i\nu\frac{2\pi}{M}}$ are the solutions orthogonal with respect to $\vec{n}_0$, which are written, outside the snakes, as follows:

$$\vec{\eta} = (-\vec{e}_x + i\vec{e}_y)\, e^{i\nu_0\alpha} \quad , \; -\frac{\pi}{M} < \theta < 0$$

$$\vec{\eta} = (\vec{e}_x + i\vec{e}_y)\, e^{-i\nu_0\alpha} \quad , \; 0 < \theta < \frac{\pi}{M}$$

where, for the snake of definiteness, it is assumed that the rotation axis of a snake spaced at $\theta = 0$, is directed along the velocity.

The deviation $\delta\vec{n}$ is maximum at the moments of acceleration process when

$$\nu_0(\gamma) = \pm \nu_z + pN \quad . \qquad (8)$$

Let us give a formula for deviation $\delta\vec{n}$ at the points (8) near the snake at (the point of maximum of $\delta\vec{n}(\theta)$ in our example):

$$\delta\vec{n}(\theta) = -\frac{\pi}{M} \mathrm{Im}\; \vec{\eta} \left[\frac{w_k}{1 - \exp\frac{2\pi i}{M}(\nu \pm \nu_z)} + \frac{w_k^*}{1 - \exp\frac{2\pi i}{M}(-\nu \pm \nu_z)} \right] (9)$$

Thus, the insertion of a large number of snakes suppresses the effect of characteristic resonances and provides the adiabaticity conditions upon acceleration: $|\delta\vec{n}| \ll 1$ at

$$M \gg \frac{\pi}{\sqrt{2}} |w_k| .$$

(of course, the choice of the angle φ , betatron frequency ν_z , and number of M is assumed to be optimum, i.e. such that the denominators in eq. (9) would not be small).

The influence of perturbations, which are due to the imperfectness, is determined by a formula similar to (9) where the amplitude of free betatron oscillations a_b should be substitutes for the induced a_s (see eqs. (4) and (6)). This is justified by a slight variation in the amplitude $a_s(\theta)$ on lengths of the order of the distance between the snakes because of the resonant nature of the distortions of the closed orbit. In view of this, a sufficient number of snakes suppresses the perturbation of polarization by the imperfectness of the magnetic system as well.

2. One can suggest the other effective method of suppressing the powers of characteristic resonances (2),(3) by means of a specially arranged correlation between the spin flips in magnets and betatron oscillations inside the period of magnetic system. Let us demonstrate the principle of this method for resonances $\nu_0 \approx -\nu_z + pN$. The powers of resonances are proportional to the factor

$$F = \int_0^{2\pi/N} f'' exp(i\nu_0\alpha)\, d\theta \quad . \qquad (10)$$

This factor can be decreased down to a fairly low value by an appropriate choice of the magnetic structure. As an example, such a magnetic structure is given in Fig. 1.

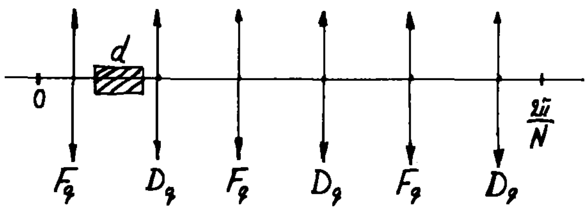

Fig. 1. The structure of magnetic system period:

F_q, D_q are the focusing and defocusing quadrupoles,
d is the bending magnet.
The focusing system period is equal to $2\pi/3N$.

Integrating over the parts of (10), one obtains

$$F = f'_{2\pi/N} \cdot exp(\frac{2\pi i\nu_0}{N}) - f'_0 + f'_d\left[1 - exp(\frac{2\pi i\nu_0}{N})\right],$$

where $2\pi\nu_0/N$ is the angle of spin rotation by a dipole magnet d, f'_d is the value of the derivative of the Floque solution in the dipole. The first two terms are cancelled at the points of spin resonances ($\nu_0 = -\nu_z + pN$) and, hence,

$$F \Rightarrow f'_d\left[1 - exp\frac{2\pi i\nu_z}{N}\right]$$

Choosing ν_z near N, one can reduce the powers of characteristic resonances approximately by a factor of $N/|\nu_z - N|$ (and the powers of resonances $\nu_0 \approx \nu_z + pN$), too). It is noteworthy that the point $\nu_z = N$ does not mean a resonance for betatron motion but lies near the stability center so that the number of focusing system periods is 3N. Therefore, it is possible, in principle, to eliminate completely the characteristic resonance effect.

Note that the reduction in the powers of the main resonances by the method described above permits one to confine oneself to the use of two snakes only in order to remove depolarization.

3. Once more possible method is based on breaking the correlation between the spin precession and the vertical oscillations of particles via modulation of the angles of spin flip by dipole magnets*. Let the value of field in dipoles be modulated according to the law

$$\mathcal{H}_z = 1 + h\cos\ell\theta \ ,$$

where h = const is the modulation amplitude, ℓ is an integer. The modulation frequency ℓ must be high enough in order that the spin perturbation by radial fields be small on the length $2\pi/\ell$. With these conditions the characteristic resonances are splitted into series of modulational ones

$$\nu_0 = \pm\nu_z + pN + q\ell \quad , \quad (q = 0, \pm 1, \ldots)$$

with powers decreased by $J_q(\nu_0 h/\ell)$ times (J_q is the Bessel function). For a substantial decrease of powers (by $\sqrt{\nu_0 h/\ell}$ times), it is necessary to satisfy the condition $\nu_0 h \gg \ell$.

The described methods allow one to considerably increase maximally attainable energies of the beam at which the adiabaticity conditions necessary to conserve the polarization in the acceleration process are satisfied.

References

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*) The idea of this method has been suggested by V.I.Balbekov