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PARITY NONCONSERVATION IN pα SCATTERING

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Abstract: The elastic scattering of longitudinally polarized protons on ^{4}He at energy $E_p \lesssim 15$ MeV is considered. The difference of cross sections for protons of positive and negative helicities which is due to the weak interaction is calculated. The magnitude of the effect reaches $2\text{–}3 \times 10^{-7}$.

Experimental and theoretical investigations of parity nonconservation (PNC) effects can be divided into two groups. The first is the study of few-nucleon systems or one-particle transitions in heavy nuclei. The second group comprises investigations of PNC effects in the compound nucleus. A relatively complete set of references may be found in a recent review article [1]). Usually the PNC effects in the second group are significantly larger than in the first one. Nevertheless we find, however, that now the first group of studies could become more important because only here can one quantitatively interpret the effect in terms of the weak nucleon–nucleon interaction.

PNC in pα scattering at proton energy $E_p = 46$ MeV has been studied experimentally [2]). According to a recent report [2b]) the experimental value of the effect is in agreement with the theoretical one. Calculations of the parity-odd (*P*-odd) asymmetry for proton energies 15 and 40 MeV have been performed in refs. [3,4]). A somewhat more rough calculation was published in ref. [5]). In these papers the internal excitations of the α-particle were not taken into account (potential approximation). But now it is well known that the contribution of such excitations to the PNC effects can be enhanced significantly [see e.g. ref. [1])]. The threshold of α-particle excitation is 20 MeV and apparently at an energy of 46 MeV in pα scattering this contribution cannot be neglected. The agreement of the experimental value with the result of calculations using the potential approximation could be fortuitous. We think that for the calculation of the PNC effect this approximation is applicable only at energies essentially less than the threshold of α-particle excitation, i.e. at $E_p \lesssim 10\text{–}15$ MeV. The present paper is devoted to a consideration of this region.

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The method is similar to that used previously in ref. [6]) for the calculation of the neutron spin rotation in ^{4}He. We stress once again that the investigation of P-odd asymmetry at a proton energy $E_p \lesssim 10$–15 MeV is predominantly important from an experimental point of view because apparently only at these energies is the quantitative interpretation in terms of the NN interaction possible.

Without PNC, the amplitude of pα scattering in the centre-of-mass system is of the form [see e.g. ref. [7])]

$$f = A + B\boldsymbol{\nu} \cdot \boldsymbol{\sigma} . \tag{1}$$

Here $\boldsymbol{\sigma}$ is the proton Pauli matrix, $\boldsymbol{\nu} = [\boldsymbol{n} \times \boldsymbol{n}']/|[\boldsymbol{n} \times \boldsymbol{n}']|$, $\boldsymbol{n}$ is the initial direction of the proton motion and $\boldsymbol{n}'$ is the final one. The amplitudes A and B are as follows:

$$A = \frac{1}{2ik} \sum_l [(l+1)(e^{2i\delta_+} - 1) + l(e^{2i\delta_-} - 1)] P_l(\cos\theta) ,$$

$$B = \frac{1}{2k} \sum_l (e^{2i\delta_+} - e^{2i\delta_-}) P_l^{(1)}(\cos\theta) . \tag{2}$$

Here $k = \sqrt{2\mu E_{\mathrm{c.m.}}}$, and $\mu = m_p m_\alpha/(m_p + m_\alpha)$ is the reduced mass. If E_p is the proton energy in the laboratory system (α-particle rest system) then $k = \sqrt{2\mu(\mu/m_p)E_p}$. The phases δ_+ and δ_- correspond to scattering with total angular momentum $\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} = l \pm \frac{1}{2}$. It is convenient to separate from δ the Coulomb part σ,

$$\delta = \bar{\delta} + \sigma , \tag{3}$$

where σ is independent of j and is of the form

$$\sigma_l = \arg\ \Gamma(l + 1 + i/ka_C) = \sigma_0 + \varepsilon_l ,$$

$$\varepsilon_l = \sum_{n=1}^{l} \operatorname{arctg}(1/nka_C) . \tag{4}$$

Here $a_C = 1/(2\mu\alpha)$ is the Coulomb radius (we set $\hbar = c = 1$). The scattering phases $\bar{\delta}$ are due to the strong interaction. The values of $\bar{\delta}$ for pα scattering are presented, for example, in ref. [8]). The scattering amplitude can be presented in the form [7])

$$A = f_C + \frac{1}{2ik} \sum_l e^{2i\sigma_l} [(l+1)(e^{2i\bar{\delta}_+} - 1) + l(e^{2i\bar{\delta}_-} - 1)] P_l(\cos\theta) ,$$

$$B = \frac{1}{2k} \sum_l e^{2i\sigma_l} (e^{2i\bar{\delta}_+} - e^{2i\bar{\delta}_-}) P_l^{(1)}(\cos\theta) , \tag{5}$$

where

$$f_C = -\frac{1}{2k^2 a_C \sin^2 \frac{1}{2}\theta} e^{-(2i/ka_C)\ln\sin\frac{1}{2}\theta} e^{2i\sigma_0} .$$

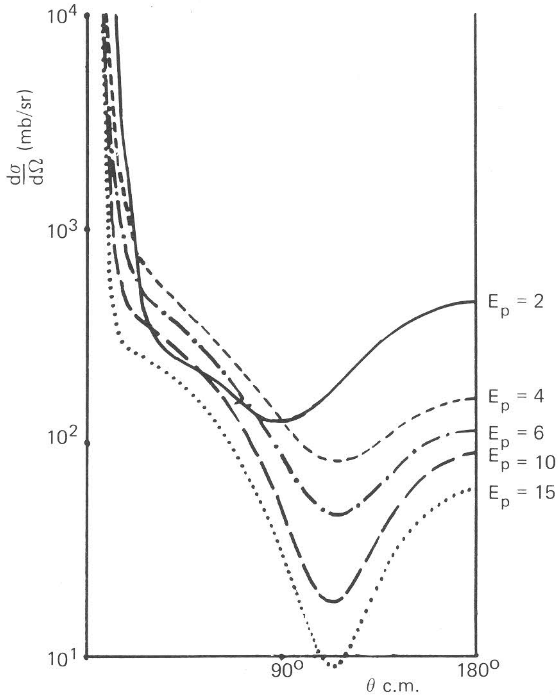

Fig. 1. Differential cross section of the pα reaction. E_p is given in units of MeV.

The differential cross section summed over the polarizations of the final proton is [7])

$$\frac{d\sigma}{d\Omega}=\left\langle\frac{d\sigma}{d\Omega}\right\rangle(1+R\boldsymbol{\nu}\cdot\boldsymbol{\xi}), \tag{6}$$

where

$$\left\langle\frac{d\sigma}{d\Omega}\right\rangle=|A|^2+|B|^2, \qquad R=\frac{2\,\mathrm{Re}\,AB^*}{|A|^2+|B|^2},$$

and $\boldsymbol{\xi}$ is the initial polarization of the proton. The quantities $\langle d\sigma/d\Omega\rangle$ and R have been measured in experiments and from these data the scattering phases have been determined [see references cited in ref. [8])]. Nevertheless we give in the present paper, for the sake of completeness, the plots of $\langle d\sigma/d\Omega\rangle$ (fig. 1) and $R(\theta)$ (fig. 2).

The part of the scattering amplitude which is due to the parity-violating weak interaction H_w is given by

$$f_{PNC}=-\frac{\mu}{2\pi}\langle\psi_f^{(-)}|H_w|\psi_i^{(+)}\rangle. \tag{7}$$

Here $\psi_i^{(+)}$ is the initial function which besides a plane wave contains the outgoing

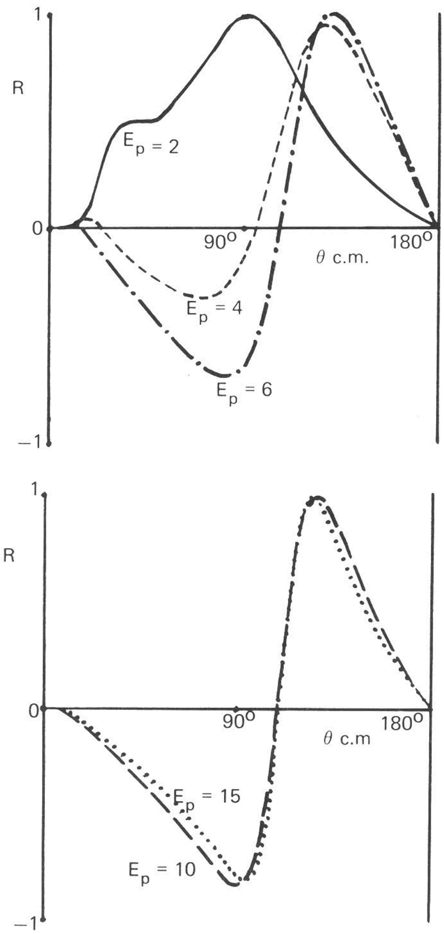

Fig. 2. Right–left asymmetry $R(\theta)$ for pα scattering. E_p is given in units of MeV.

spherical one:

$$\psi_i^{(+)} = e^{i\boldsymbol{k}_i \cdot \boldsymbol{r}} + \frac{f}{r} e^{ikr}, \qquad r \to \infty . \tag{8}$$

The final function $\psi_f^{(-)}$ contains the ingoing spherical wave:

$$\psi_f^{(-)} = e^{i\boldsymbol{k}_f \cdot \boldsymbol{r}} + \frac{f^*}{r} e^{-ikr}, \qquad r \to \infty . \tag{9}$$

The decompositions of $\psi_i^{(+)}$ and $\psi_f^{(-)}$ into partial waves are as follows:

$$\psi_i^{(+)} = \frac{4\pi}{kr} \sum_{lj} i^l e^{i\delta_{lj}} R_{ljk} \sum_m \Omega_{jlm}(\boldsymbol{N}) \Omega_{jlm}^\dagger(\boldsymbol{n}) \chi_i ,$$

$$\psi_f^{(-)} = \frac{4\pi}{kr} \sum_{lj} i^l e^{-i\delta_{lj}} R_{ljk} \sum_m \Omega_{jlm}(\boldsymbol{N}) \Omega_{jlm}^\dagger(\boldsymbol{n}') \chi_f . \tag{10}$$

The radial functions $R_{ljk}(r)$ are normalized in such a way that at $r \to \infty$

$$R \to \sin\left(kr + \frac{1}{ka_C}\ln 2kr - \tfrac{1}{2}\pi l + \delta_{lj}\right),$$

χ_i and χ_f are the initial and final spin wave functions, and Ω_{jlm} is a spherical spinor. The unit vectors are as follows: $\boldsymbol{N} = \boldsymbol{r}/r$, $\boldsymbol{n} = \boldsymbol{k}_i/k$, $\boldsymbol{n}' = \boldsymbol{k}_f/k$.

The hamiltonian of the parity-violating NN weak interaction in the standard parametrization is of the form [9])

$$\begin{aligned} H_w = {} & i\frac{f_\pi g_\pi}{2\sqrt{2}\,m_p}\frac{1}{2}[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[(\boldsymbol{p}_1-\boldsymbol{p}_2), F_\pi] \\ & -\frac{g_\rho}{2m_p}\{h^0_\rho\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + h^1_\rho\tfrac{1}{2}(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)_z + \tfrac{1}{2}\sqrt{\tfrac{1}{6}}\,h^2_\rho(3\tau^z_1\tau^z_2-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\} \\ & \times\{(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{(\boldsymbol{p}_1-\boldsymbol{p}_2), F_\rho\} + i(1+\mu_V)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot[(\boldsymbol{p}_1-\boldsymbol{p}_2), F_\rho]\} \\ & -\frac{g_\omega}{2m_p}\{h^0_\omega + h^1_\omega\tfrac{1}{2}(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)_z\} \\ & \times\{(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\{(\boldsymbol{p}_1-\boldsymbol{p}_2), F_\omega\} + i(1+\mu_S)(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot[(\boldsymbol{p}_1-\boldsymbol{p}_2), F_\omega]\} \\ & -\frac{1}{2m_p}(g_\omega h^1_\omega - g_\rho h^1_\rho)\tfrac{1}{2}(\boldsymbol{\tau}_1-\boldsymbol{\tau}_2)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\{(\boldsymbol{p}_1-\boldsymbol{p}_2), F_\rho\} \\ & -\frac{i}{2m_p}g_\rho h'^1_\rho\tfrac{1}{2}[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot[(\boldsymbol{p}_1-\boldsymbol{p}_2), F_\rho], \end{aligned}$$

$$F_a = \frac{e^{-m_a r}}{4\pi r}, \qquad r = |\boldsymbol{r}_1 - \boldsymbol{r}_2|. \tag{11}$$

Here $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are spin and isotopic spin Pauli matrices, and $\boldsymbol{p}$ is the momentum operator. Indices 1, 2 correspond to the first and second nucleon. The nucleon mass is denoted by m_p, and m_π, m_ρ, m_ω are the π-, ρ- and ω-meson masses. The brackets [,] mean commutator, and { , } anticommutator; $\mu_S = -0.12$, $\mu_V = 3.7$ are the isoscalar and isovector anomalous magnetic moments of the nucleon. The strong nucleon–meson constants are $g_\pi = 13.45$, $g_\rho = 2.79$, $g_\omega = 8.37$. Finally f_π and $h^i_{\rho,\omega}$ are the weak nucleon–meson constants; estimates for them are presented in ref. [9]).

In the matrix element of H_w in eq. (7) both direct and exchange terms contribute as shown in fig. 3. The line N corresponds to one of the α-particle nucleons, jl and $j\tilde{l}$ ($\tilde{l} = 2j - l$) correspond to the external nucleon in the $|jl\rangle$ and $|j\tilde{l}\rangle$ states. The isospin of the α-particle is equal to zero; thus, the exchange by ω-meson only is possible in the direct graph. The exchange graph is more important since the π-exchange term has a relatively small denominator $\sim 1/m_\pi^2$, and the ρ-exchange term is enhanced by the factor $(1+\mu_V)$.

Fig. 3. Contributions to the parity-violating amplitude.

A simple calculation of the matrix element with the functions (10) gives

$$\begin{aligned}\langle\psi_{\mathrm{f}}^{(-)}|H_{\mathrm{w}}|\psi_{\mathrm{i}}^{(+)}\rangle=&\frac{k}{m_{\mathrm{p}}}\langle\chi_{\mathrm{f}}|\boldsymbol{\sigma}\cdot(\boldsymbol{n}+\boldsymbol{n}')|\chi_{\mathrm{i}}\rangle\\ &\times\sum_j \tfrac{1}{2}(2j+1)\,\mathrm{e}^{i(\delta_{lj}+\delta_{\tilde{l}j})}\{-2g_\omega(h_\omega^0+h_\omega^1\tau_z)J_\omega^j\\ &-\sqrt{\tfrac{1}{2}}f_\pi g_\pi\tau_z I_\pi^j+g_\rho(3h_\rho^0+h_\rho^1\tau_z)(1+\mu_{\mathrm{V}})I_\rho^j\\ &+g_\omega(h_\omega^0+h_\omega^1\tau_z)(1+\mu_{\mathrm{S}})I_\omega^j+g_\rho h_\rho'^1\tau_z I_\rho^j\}\\ &\times\frac{P_l(\cos\theta)+P_{\tilde{l}}(\cos\theta)}{1+\cos\theta.}\,,\end{aligned}\qquad(12)$$

where $l=j-\frac{1}{2}$, $\tilde{l}=j+\frac{1}{2}$, and J^j and I^j are direct and exchange integrals:

$$\begin{aligned}J_\omega^j=&\frac{2\pi}{k^3}\int \mathrm{d}r_1\,\mathrm{d}r_2\,\mathrm{d}\cos\theta_{12}F_\omega(r_{12})|\varphi(r_{12})|^2|R_\alpha(r_2)|^2\\ &\times\left\{\frac{\mathrm{d}R_{\tilde{l}}(r_1)}{\mathrm{d}r_1}R_l(r_1)-R_{\tilde{l}}(r_1)\frac{\mathrm{d}R_l(r_1)}{\mathrm{d}r_1}+\frac{2j+1}{r_1}R_{\tilde{l}}(r_1)R_l(r_1)\right\},\\ I_a^j=&-\frac{4\pi}{k^3}\int \mathrm{d}r_1\,\mathrm{d}r_2\,\mathrm{d}\cos\theta_{12}P_l(\cos\theta_{12})F_a(r_{12})|\varphi(r_{12})|^2\\ &\times R_\alpha(r_2)R_l(r_2)\left\{\frac{\mathrm{d}}{\mathrm{d}r_1}[R_\alpha(r_1)R_{\tilde{l}}(r_1)]+\frac{j-\frac{1}{2}}{r_1}R_\alpha(r_1)R_{\tilde{l}}(r_1)\right\}.\end{aligned}\qquad(13)$$

Here R_α is the radial wave function of the α-particle nucleon. The Jastrow function $\varphi(r)$ is introduced in the direct and exchange integrals to take into account the short-range repulsion of the nucleons:

$$\varphi(r)=1-a\exp(-dr^2)\,.$$

The parameters of this function for the system $\mathrm{N}+\alpha$ are given in refs. [10,11]): $a=0.6$, $d=3\ \mathrm{fm}^{-2}$. The matrices $\boldsymbol{\tau}$ and $\boldsymbol{\sigma}$ in eq. (12) correspond to the external nucleon. For forward neutron scattering in the definite helicity state, eq. (12) coincides with the corresponding formula from ref. [6]). From eqs. (7) and (12) we have

$$f_{\mathrm{PNC}}=C\boldsymbol{\sigma}\cdot(\boldsymbol{n}+\boldsymbol{n}')\,,\qquad(14)$$

where

$$C=-\frac{\mu}{m_{\mathrm{p}}}\frac{k}{2\pi}\sum_j \tfrac{1}{2}(2j+1)\,\mathrm{e}^{i(\delta_{lj}+\delta_{\tilde{l}j})}$$
$$\times\{-2g_\omega(h^0_\omega+h^1_\omega)J^j_\omega-\sqrt{\tfrac{1}{2}}f_\pi g_\pi I^j_\pi+g_\rho(3h^0_\rho+h^1_\rho)(1+\mu_{\mathrm{V}})I^j_\rho$$
$$+g_\omega(1+\mu_{\mathrm{S}})(h^0_\omega+h^1_\omega)I^j_\omega+g_\rho h'^1_\rho I^j_\rho\}\frac{P_l(\cos\theta)+P_{\tilde{l}}(\cos\theta)}{1+\cos\theta}\,. \tag{15}$$

Similar to the basic part of the scattering amplitude (1), f_{PNC} in (14) is presented as an operator in spin space. This means that the matrix element of eq. (14) between the initial and final spin states needs to be calculated.

We will here put $I_\rho=I_\omega$ because the ω- and ρ-meson masses are very close. We calculated the proton wave functions in the continuum spectrum using the Woods-Saxon potential presented in ref. [8]). This potential gives a good fit of the scattering phases. The wave function of the bound nucleon we found as $R_\alpha=\sqrt{4\pi\rho(r)r^2}$, where $\rho(r)$ is the experimental charge density of the α-particle [12]):

$$\rho(r)=Q(1+w(r/c)^2)/(\mathrm{e}^{(r-c)/z}+1)\,,$$
$$w=0.445\,,\qquad c=1.01\ \mathrm{fm}\,,\qquad z=0.327\ \mathrm{fm}\,. \tag{16}$$

Q is the normalization constant ($\int\rho\,\mathrm{d}^3r=1$). Of course it is possible to use the function R_α calculated in the Woods-Saxon potential. The integrals J and I are practically the same. The calculated integrals I^j_π are presented in fig. 4. The calculation shows that the ratios of integrals I^j_ρ/I^j_π and J^j_ω/I^j_π are practically independent of the proton energy at $E_{\mathrm{p}}\lesssim 15$ MeV and equal†

$$I^j_\rho/I^j_\pi\approx 0.082\,,\qquad J^j_\omega/I^j_\pi\approx -0.10\,.$$

Therefore the quantity C (eq. (15)) can be rewritten in the following form:

$$C(\theta)=0.8\frac{k}{2\pi}\times 1.73y\sum_j\tfrac{1}{2}(2j+1)I^j_\pi\,\mathrm{e}^{i(\delta_{lj}+\delta_{\tilde{l}j})}$$
$$\times\frac{P_l(\cos\theta)+P_{\tilde{l}}(\cos\theta)}{1+\cos\theta}\,,$$
$$y=5.5f_\pi-1.80h^0_\rho-0.60h^1_\rho-1.20(h^0_\omega+h^1_\omega)-0.13h'^1_\rho\,. \tag{17}$$

We use in eq. (17) the values of the integral ratios and of the constants $g_{\rho,\omega,\pi}$. The quantity y is separated in such a way that it practically coincides with the constant $X^{\mathrm{p}}_{\mathrm{N}}$ introduced in refs. [13,9]). We use another symbol to stress that there is still some difference, but really this difference is within the accuracy of calculation. From the

† Of course we calculate the integrals with the Jastrow correction. Without this correction the integrals I^j_π are larger by $\simeq$33%, and the integrals I^j_ρ, J^j_ω are larger by a factor 2.

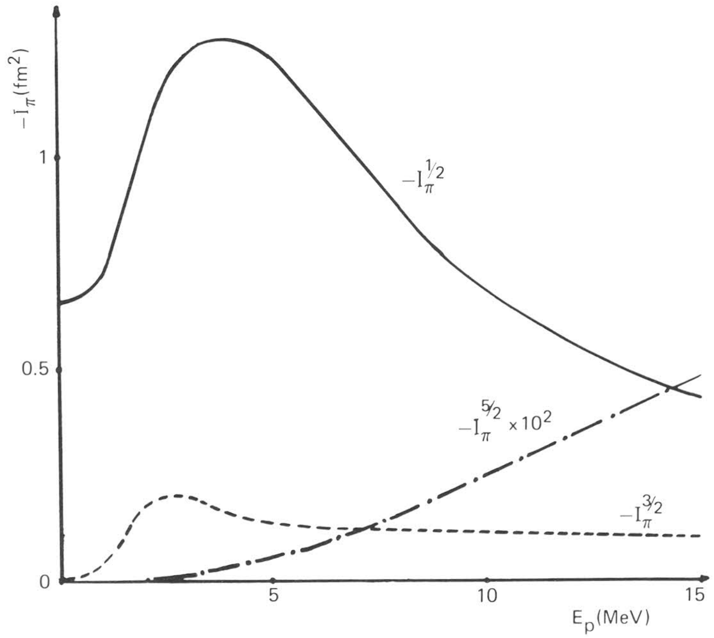

Fig. 4. Values of the pion-exchange integrals $-I_\pi^{1/2}$, $-I_\pi^{3/2}$ and $-I_\pi^{5/2}$ in units of fm^2.

analysis of the experimental data presented in refs. [13,9])

$$y \simeq X_N^p \simeq 4 \times 10^{-6}. \tag{18}$$

Thus due to eqs. (1) and (14)

$$f = A + B\boldsymbol{\nu} \cdot \boldsymbol{\sigma} + C\boldsymbol{\sigma} \cdot (\boldsymbol{n} + \boldsymbol{n}'). \tag{19}$$

Let us consider the relative difference of the cross sections for the positive- and negative-helicity protons due to the parity violation

$$p(\theta) = \frac{\mathrm{d}\sigma_+/\mathrm{d}\Omega - \mathrm{d}\sigma_-/\mathrm{d}\Omega}{\mathrm{d}\sigma_+/\mathrm{d}\Omega + \mathrm{d}\sigma_-/\mathrm{d}\Omega}. \tag{20}$$

Using eq. (19) one can easily verify that

$$p(\theta) = \frac{2\,\mathrm{Re}\,[(1+\cos\theta)AC^* + i\sin\theta BC^*]}{|A|^2 + |B|^2}. \tag{21}$$

This function calculated with y from eq. (18) is plotted in fig. 5. The contribution of $j = \frac{5}{2}$ ($d_{5/2} - f_{5/2}$ mixing) does not exceed a few percent even at $E_p = 15$ MeV. The $j = \frac{3}{2}$ contribution ($d_{3/2} - p_{3/2}$ mixing) is also not very important.

The calculation of the P-odd asymmetry in the angular distribution for $E_p = 15$ and 40 MeV has been performed in ref. [4]). All calculations (including ours) are carried out in a potential approximation. As we mentioned in the introduction this

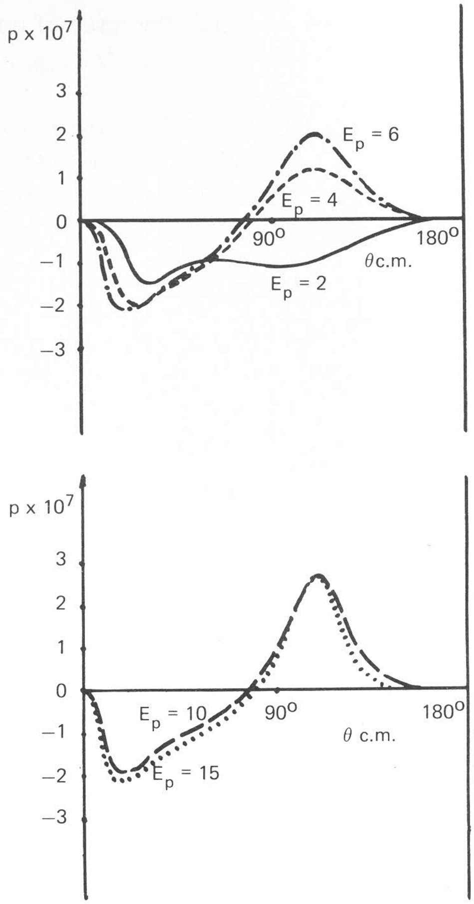

Fig. 5. The *P*-odd asymmetry $p(\theta)$ for pα scattering. E_p is given in units of MeV.

approach is unjustified for energies higher than 20 MeV. This is the reason why we limited ourselves to an energy of 15 MeV. In ref. [4]) only averaged values of $p(\theta)$ for two intervals of laboratory angles are presented: $\langle p\rangle_{5°-180°} = -0.038X_N^p$, $\langle p\rangle_{25°-70°} = -0.044X_N^p$ at $E_p = 15$ MeV. Our calculation gives: $\langle p\rangle_{5°-180°} = -0.025X_N^p$, $\langle p\rangle_{25°-70°} = -0.028X_N^p$. (One should remember that in the figures we present the angle in the c.m. system.) Thus, our calculation gives values smaller by a factor 1.5 than those in ref. [4]). We cannot point out the reason for the disagreement because no details of the calculations are presented in ref. [4]). But it should be mentioned that if one does not take into account the Jastrow correction then the asymmetry would increase by a factor 1.3–1.5.

In ref. [3]) Coulomb effects were neglected and only the *P*-odd asymmetry in the total cross section was calculated. Our parametrisation of the short-range part of the PNC interaction differs from that expected in ref. [3]). Therefore we can compare only the π-meson contribution with this work. The result of ref. [3]) for an energy of 15 MeV is $\langle p \rangle_{\text{total}} = -0.09 f_\pi$. Our calculation gives $\langle p \rangle_{5^\circ-180^\circ} = -0.14 f_\pi$. The value from ref. [4]) is $\langle p \rangle_{5^\circ-180^\circ} = -0.20 f_\pi$.

In conclusion we want to stress once more that experimental investigations of the P-odd asymmetry for low proton energies ($E_p \lesssim 10$–15 MeV) is very important. Here the contribution of internal α-particle excitations to the effect is small and a quantitative interpretation in terms of the nucleon–nucleon interaction is possible.

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