# BEAM-BEAM EFFECTS IN STORAGE RINGS WITH A MONOCHROMATOR SCHEME

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## Abstract

Beam-beam effects are considered for the case of electron-positron storage rings with a monochromator scheme, when the interaction of the beams is taking place in the presence of a large vertical energy dispersion at the interaction point. A limitation of luminosity in monochromatic experiments due to the decrease of a monochromaticity factor under the influence of the beam-beam effects is obtained.


## Introduction

In the Institute of Nuclear Physics at Novosibirsk the works are continued on the upgrade project of the storage ring VEPP-4 for the monochromatic experiments in $\Psi$, $Y$ -mesons energy region /1/. The important question for these studies is the estimate of the maximum luminosity, for which the effects of electromagnetic interaction of electrons and positrons (beam-beam effects) don't significantly affect the monochromaticity of the interaction energy of the particles.

It should be noted, that according to a suggested in the work /2/ scheme, the monochromaticity is obtained through the special method of performing the electron and positron beams collision. The beams at the interaction point are decomposed relative to the energy in the vertical direction, so that the size of decomposition is much larger than the r.m.s. betatron size. The gain of energy resolution due to the monochromator is determined for the unperturbed motion, when the particle distribution is gaussian, by a factor $\lambda = |\Psi_z|\sigma_\varepsilon/\sigma_{z\beta}$ where $\Psi_z$, $\sigma_{z\beta}$ are vertical dispersion function and betatron beam size at the interaction point, and $\sigma_\varepsilon$ is the relative energy spread in the beam. The interaction of particles of one beam with the space charge field of an opposite beam changes the distribution of particles. Even for the standard regime of electron-positron collision the number of particles in the distribution tails and the average size of the beam are growing with the increase of the beams intensity (see, for example, /3/). In our case one should expect a stronger manifestation of the effect because of a larger vertical dispersion at the interaction point. The broadening of the particles distribution function in $Z$ -direction leads to a stronger intra-beam mixing of different energies and monochromaticity deterioration.

The first studies of mechanisms of the possible loss of monochromaticity because of beam-beam effects were carried out in a one-dimensional model in the work /4/. In a present report we present the results of further studies, taking into account two-dimensional character of the motion.

## 1. Analytic estimates

For elliptical beams with a large aspect ratio $\varkappa = \sigma_x/\sigma_z >> 1$ the forces of interaction with the opposite beam $f_x$, $f_z$ can be introduced in a simple analytical form /5,6/. Further on, making use of a small values (for the regime with $\lambda >> 1$) of the amplitude $A_z$, normalized over a full vertical beam size; $A_z \sim \frac{1}{\lambda} << 1$, we perform a power decomposition of interaction forces $f_x, f_z$ in a "small parameter" $A_z$ and arrive to explicite formulas for nonlinear resonance characteristics in the regime with $\lambda >> 1$, $\varkappa >> 1$ /6/. Thus, for the betatron tune shift $\Delta\nu_z$ we obtain:

$$\Delta\nu_z = I_0\left(\frac{A_z^2}{4}\right)e^{-A_x^2/4}\cdot e^{-A_s^2/4}\left[I_0\left(\frac{A_s^2}{4}\right) - \frac{A_z^2}{8}\left(I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\cdot\left\{I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right\}\right)\right] \quad (1.1)$$

where $I_0$ and $I_1$ are the modified Bessel functions of zero and first orders. For the harmonic amplitude $V_{\ell mn}$ calculation we can use, for each given $m$, a corresponding power of $f_z$ decompozition in $A_z \cos\theta_z$ /5/. Thus, introducing normalized to the vertical betatron size amplitude $A_{z\beta} = \lambda A_z$, $\lambda = \sigma_z/\sigma_{z\beta}$, we arrive to the following expression for the resonance width:

$$\Delta A_{z\beta} = G_\ell(A_x)\cdot F_m(A_{z\beta}, A_s, \lambda) \quad (1.2)$$

where

$$G_\ell(A_x) = \sqrt{\frac{I_{\ell/2}(A_x^2/4)}{I_0(A_x^2/4)}} \quad (1.3)$$

and $F_m$ for the lowest values of $m$ has explicit form:

$$F_1 = \frac{2\lambda^{3/2}}{\sqrt{A_{z\beta}}}\sqrt{\left|\frac{\frac{2}{n}A_s I_{\frac{n-1}{2}}\left(\frac{A_s^2}{4}\right) + I_{\frac{n+1}{2}}\left(\frac{A_s^2}{4}\right)}{I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left(I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right)}\right|}$$

$$F_m \sim \lambda\left(\frac{A_{z\beta}}{\lambda}\right)^{\frac{m}{2}-1} \qquad F_2 = 2\lambda\sqrt{\left|\frac{I_{n/2}(A_s^2/4)}{I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left(I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right)}\right|} \quad (1.4)$$

$$F_3 = 2\sqrt{\lambda A_{z\beta}}\sqrt{\left|\frac{A_s}{12}\,\frac{I_{\frac{n-1}{2}}\left(\frac{A_s^2}{4}\right) - I_{n/2}\left(\frac{A_s^2}{4}\right)}{I_0\left(\frac{A_s^2}{4}\right) - \frac{A_s^2}{2}\left(I_0\left(\frac{A_s^2}{4}\right) + I_1\left(\frac{A_s^2}{4}\right)\right)}\right|}$$

Obtained thus formulas for the resonance width $\Delta A_{z\beta}$ at the point $A_x, A_z$ show a number of peculiarities. The first of those is the same as in a conventional non-monochromatic regime independence of $\Delta A_{z\beta}$ either on $\varkappa$, or on $\xi_x, \xi_z$. The second peculiarity is the specific dependence of a relative magnitude of different harmonics resonances on the parameter $\lambda$. It is clear from the expressions (1.2-1.4), that the resonance widths for $|m| = 1,2,3$ and arbitrary $\ell$ and n are growing with the increase of $\lambda$, while the resonances width with $|m| = 4$ doesn't and the resonance widths with $|m| > 4$ are decreasing.

The applicability condition of the formulas (1.2)-(1.4) is the inequality $\Delta A_{z\beta} << A_{z\beta}$; which is fulfilled for $A_x, A_{z\beta}, A_s \sim 1$ only for high enough indices $|\ell|, |m|, |n|$. In spite of this limitation, however, the formulas (1.2)-(1.4) can be helpful as well for a comparison of relative magnitudes of different, and even low, order resonances (because a larger value of harmonic amplitude leads to a larger resonance size). Furthermore, in a conventional situation of having the working point in the tune region with a minimal beam "blow up", essential are high harmonic resonances, for which the formulas (1.2)-(1.4) are valid.

The information about the relative strength of different harmonic $\ell, m, n$ resonances is presented in the table I, showing the resonance width $\Delta A_{z\beta}$ for the lowest possible value of $|n|$ (zero for even m and 1 for odd m) and the first $|n|$, following it - 2 for even m and 3 for odd m). The width was calculated from the formulas (1.2)-(1.4) for $A_x = A_{z\beta} = A_s = 1$. Resonance widths $\Delta A_z$ for the standard, non-monochromatic regime $\lambda$ = 0 are shown for comparison. These quantities are nonzero only for even $\ell$ and m, and for zero n, because synchrotron modulations are absent in this case. The data shows, that the magnitudes of $\Delta A_{z\beta}(\lambda = 10)$ are considerably larger, than $\Delta A_z$ ( $\lambda$ = 0) magnitudes.

Table 1

The value of resonance widths A for monochromatic and standard regimes

I) $\lambda = 10;\ A_x = A_{z\beta} = A_s = 1.$
II) $\lambda = 0;\ \Delta A_{z\beta} = \Delta A_z;\ A_x = A_z = 1.$

| $\lvert\ell\rvert$ / $\lvert m\rvert$ | | 2 | | 4 | | 6 | |
|---|---|---|---|---|---|---|---|
| | | I | II | I | II | I | II |
| 1 | $n = \pm I$ | 50. | 0. | 14. | 0. | 3. | 0. |
| | $n = \pm 3$ | 10. | 0. | 3. | 0. | 0.6 | 0. |
| 2 | $n = 0$ | 10. | 0.72 | 3. | 0.18 | 0.6 | 0.03 |
| | $n = \pm 2$ | 1. | 0. | 0.33 | 0 | 0.06 | 0 |
| 3 | $n = \pm 1$ | 1. | 0. | 0.33 | 0. | 0.06 | 0. |
| | $n = \pm 3$ | 0.33 | 0. | 0.11 | 0. | 0.02 | 0. |

An important question of the optimal regime choice in the ratio $\xi_x/\xi_z$ is the dependence of individual resonance widths on this parameter. In the formulas (1.2) the resonance width $\Delta A_{z\beta}$ doesn't depend on the parameters $\xi_x$, $\xi_z$, but the formulas themselves are valid only for $\frac{\xi_x^2}{\xi_z^2}\frac{\ell^2}{\text{æ} m^2} \ll 1$. When the ratio $\xi_x/\xi_z$ is of the order

$$\frac{\xi_x}{\xi_z} \sim \sqrt{\text{æ}}\left|\frac{m}{\ell}\right| \tag{1.5}$$

then the consideration similar to that of /5/, shows, that the decrease of $\Delta A_{z\beta}$ in respect to (1.10) can be estimated, for $A_x$, $A_{z\beta} \sim 1$, by the following expression:

$$\Delta A_{z\beta} \sim \frac{\Delta A^{\circ}_{z\beta}}{\sqrt{1+\xi_x^2\,\ell^2/\xi_z^2 m^2 \text{æ}}} \tag{1.6}$$

Thus the magnitude of the amplitude oscillation on the individual resonances, is decreasing for increasing ratio $\xi_x/\xi_z$, what lead us to the conjecture of the advantage of a regime with $\xi_x \gg \xi_z$ in respect to the conventional regime $\xi_z \gtrsim \xi_x$.

## 2. Simulation

In the model of computer simulation of beam-beam effects the following features were included: betatron and synchrotron oscillations in transverse and longitudinal directions, noise and damping in all coordinates, beam-beam kick. The position of the opposite beam centre was modulated with a synchrotron oscillations of a particle according to the formula:

$$\bar{P} = P + \xi \cdot f(X, Z_s) \tag{2.1}$$

where the full displacement $Z_s$ is the sum of synchrotron and betatron displacements, normalized to the full vertical size of the beam:

$$Z_s = \frac{Z+\lambda E}{\sqrt{1+\lambda^2}} \tag{2.2}$$

$f(a,b)$ are the normalized forces of the interaction with the opposite beam, depending on the normalized to the corresponding sizes of the beam coordinates; X - the coordinate $x$ normalized to the horizontal size of the beam; Z - the coordinate $z$, normalized to the vertical betatron size of the beam; E - the energy coordinate, normalized to the energy spread magnitude: $\lambda$ - the ratio of synchrotron and betatron beam sizes in vertical direction.

Moreover, in our model the affect of the longitudinal oscillations of the particles on the tune $\nu_z$ and linear tune shift $\xi_z$ was taken into account (see /7/):

$$\nu_z = \nu_{z0} + \delta\nu_z \cdot E \qquad \xi_z = \xi_{z0}\sqrt{1+A^2E^2} \tag{2.3}$$

where the modulation amplitudes are given by

$$\delta\nu_z = \nu_s A \qquad A = \ell/\beta_z \tag{2.4}$$

with $\ell$ standing for the beam length, and $\beta_z$ - for the beta function at the interaction point.

The forces from the opposite beam $f_x$, $f_z$ were computed with a linear interpolation from the grid in X,Z plane, and the values of forces on the grid were calculated with a numerical integration. The main results of simulation were the values of the "monochromaticity factor" $K_m$ and specific luminosity $L_{sp}$, defined by the expressions

$$L_{sp} = \frac{1}{N_L}\iint \rho_s(X,Z_s)\, e^{-X^2/2}\cdot e^{-Z_s^2/2}\, dX\, dZ \tag{2.5a}$$

$$K_m = \frac{1}{N_K}\iint \rho(X,Z)\, e^{-Z^2/2}\, dX\, dZ \tag{2.5b}$$

where $\rho(X,Z)$, , $\rho_s(X,Z_s)$ are the equilibrium distribution functions in (X,Z) and $(X,Z_s)$ planes, and normalization constants $N_L$, $N_K$ are determined from the condition, that in the absence of an opposite beam, when $\xi_x = \xi_z = 0$ and $\rho(X,Z) = \exp(-X^2/2 - Z^2/2)$, $\rho_s = \exp(-X^2/2 - Z_s^2/2)$ the quantities $L_{sp}$ and $K_m$ equal unity. Thus defined "monochromaticity factor" $K_m$ doesn't depend on the horizontal beam size, and it's dependence on the vertical beam size is the same, as the corresponding dependence of specific luminosity so that $K_m \sim 1/\sigma_z$ (for a gaussian distribution $\rho(X,Z) \sim \exp(-Z^2/2\sigma_z^2)$. Therefore, the real energy resolution gain due to monochromatization will equal, with the beam-beam effects in account, to $K_m\lambda$. The distribution functions $\rho(X,Z)$ and $\rho_s(X,Z_s)$ were computed in the simulation programs as the density distribution of all the particles of the simulation at each iteration step.

The values of the model parameters, constant in all the simulation runs (and corresponding to the planned monochromatic regime of VEPP-4), were: synchrotron frequency $\nu_s$ = 0.02; frequency modulation amplitude $\delta\nu_z$ = 0.06; beam intensity modulation amplitude A = 1; aspect ratio of the opposite beam æ = 30; damping time, measured in the number of collizions N = 3000; synchrotron/ betatron beam sizes ratio $\lambda$ = 10; collizions number in the ring equals one.

The goal of the simulation was the determining of the specific luminosity and monochromaticity factor dependence on the opposite beam current for the optimum tunes $\nu_x$, $\nu_z$ and ratio $\xi_x/\xi_z$. It doesn't seam possible, however, to carry out a direct optimization in a large number parameters in consideration with a direct computation of a necessary quantities because of an amount of computations required. So the problem of optimization in $\xi_x/\xi_z$ was solved with a help of a special methodics of fast computation of an auxiliary quantity, connected qualitatively with a monochromaticity factor. For such a quantity we took the value of the oscillations of the coordinate Z at $P_z \approx 0$ moments, rather than the oscillations of amplitude themselves. Thus, we computed the quantity $\Delta z$ :

$$\Delta_z = \max_i \left[ \left(Z_{max}\right|_{\frac{P_z}{\sqrt{Z^2+P_z^2}}<0.1} - \left(Z_{min}\right|_{\frac{P_z}{\sqrt{Z^2+P_z^2}}<0.1} \right] \tag{2.6}$$

where $Z_{max}$ and $Z_{min}$ refer to the maximal and minimal values of Z under the condition $|\rho_z|/\sqrt{Z^2+\rho_z^{2'}}$ $< 0.1$ and for Z, belonging to a single trajectory $i$, and max means taking a maximum from different trajectories (initial conditions).

Examples of level curves of the quantity $\Delta_z$ in the plane $\nu_x, \nu_z$ are shown at the Figs. 1,2.

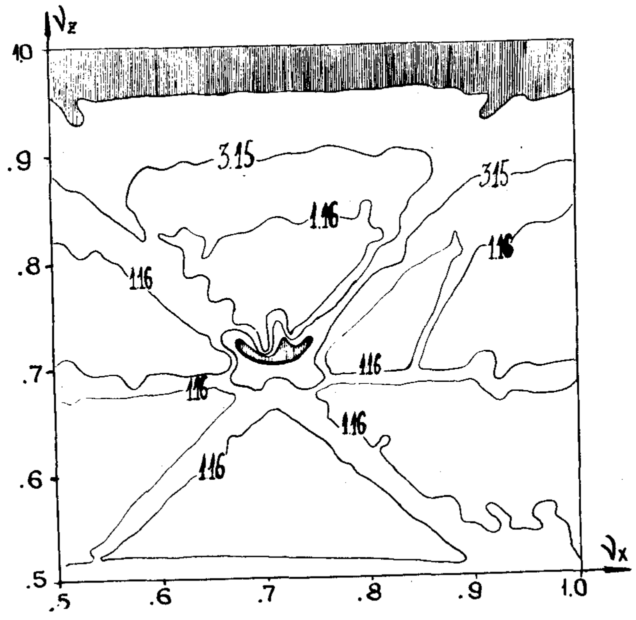


Fig. 1. Level curves of the quantity $\Delta_z$ in a plane $\nu_x, \nu_z$ for a standard regime: $\lambda = 0$, $\xi_x = 0.05$, $\xi_z = 0.10$. The regions with $\Delta_z > 10$ are marked with a shading.

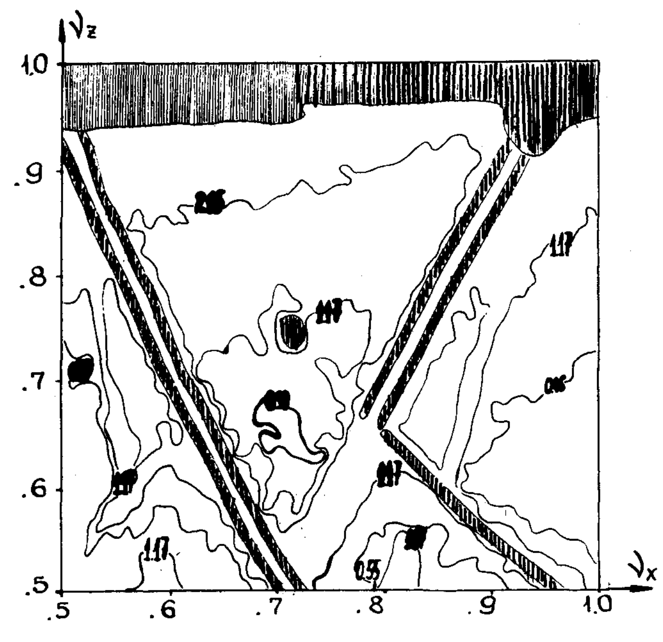


Fig. 2. Level curves of the quantity $\Delta_z$ in a plane $\nu_x, \nu_z$ for monochromatic regime: $\lambda = 10$, $\xi_x = 0.05$, $\xi_z = 0.01$. The regions with $\Delta_z > 10$ are marked with a shading.

Comparing Figs. 1 and 2, one sees the principal difference of a standard ($\lambda = 0$, Fig. 1) and monochromatic ($\lambda = 10$, Fig. 2) regimes. So, the strongest in Fig. 1 are the coupling resonances $\nu_x = \nu_z$ and $\nu_x + \nu_z = 1$, while the strongest in Fig. 2 are the double lines (synchrotron sidebands $n = \pm 1$) of synchrotron resonances $2\nu_x + 2\nu_z = \pm\nu_s$ and $2\nu_x - \nu_z = \pm\nu_s$, what agrees well with a Table 1 data.

The pictures of Fig. 3 and 4 illustrate the assertion of a $\xi_x / \xi_z \gg 1$ regime preferability, distinct from conventional regime $\xi_z/\xi_x \sim 3$.

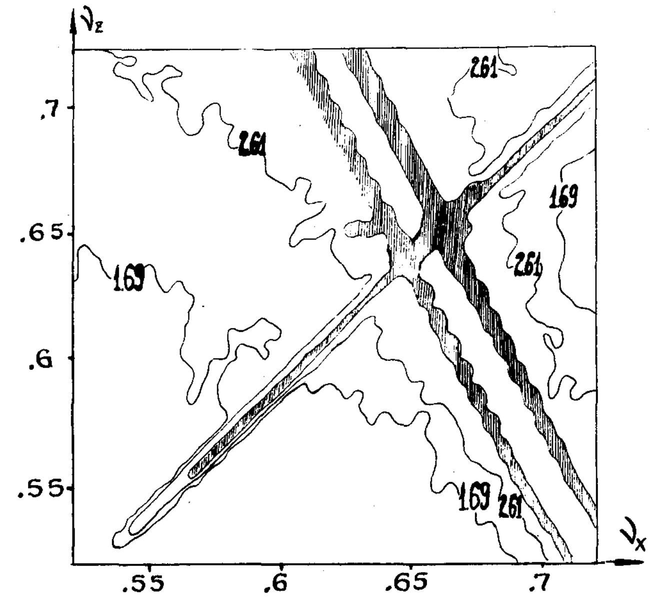


Fig. 3. Level curves of the quantity $\Delta_z$ in a plane $\nu_x, \nu_z$ for monochromatic regime: $\lambda = 10$, $\xi_x = 0.015$, $\xi_z = 0.04$. The regions with $\Delta_z$ 10 are marked with a shading.

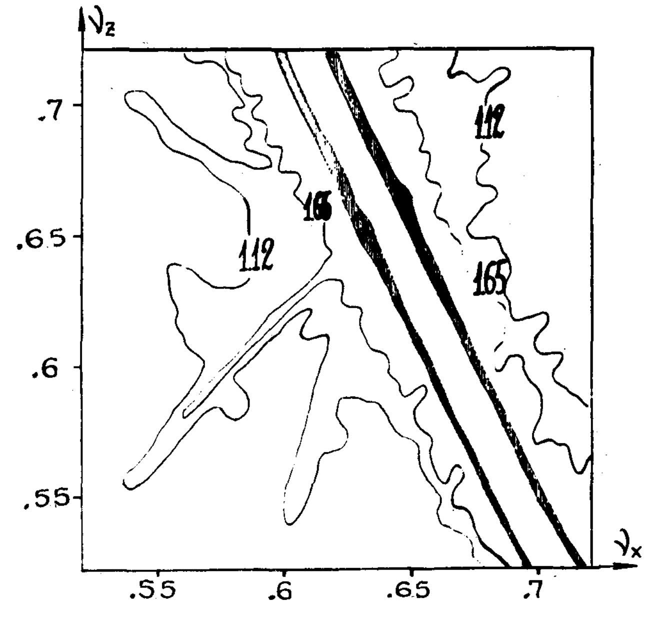


Fig. 4. Level curves of the quantity $\Delta_z$ in a plane $\nu_x, \nu_z$ for monochromatic regime: $\lambda = 10$, $\xi_x = 0.04$, $\xi_z = 0.015$. The regions with $\Delta_z > 10$ are marked with a shading.

The reduction of a ( $\ell = 2$, $m = -2$) resonance for $\xi_x/\xi_z \sim 3$ is clearly observed. Moreover, the background height of $\Delta_z$, generated presumably by nonresonance oscillations, is also lower for a larger $\xi_x/\xi_z$. So, with the data at hand, we choose the ratio $\xi_x/\xi_z$ to be equal to 5.

To check the methodics of a qualitatitve beam size estimate with a fast computed quantity $\Delta_z$ and to have an exact data about the monochromaticity factor $K_m$, a numerical simulation was conducted in a model with a noise and damping switched on. The quantity $K_m$ was computed from the formula (2.5b) with a help of numerical integration. Simultaniously the specific luminosity $L_{sp}$ was computed from the formula (2.5a). The results of the computations are presented in Fig.5, where the level curves of $K_m$ are shown in a plane $\nu_x$, $\nu_z$, and the regions with $K_m < 0.38$ and $0.38 < K_m < 0.68$, in contrast to the region $K_m > 0.68$, are marked with a shading.

*tion, but doesn't affect the monochromaticity factor* $K_m$ value. Further on it should be noted, that in a close to the integer resonance region of tunes, as it is known from the experiment, the beam size grows under the strong influence of a "machine" resonance, the existence of which is not taken into account in our model.

Thus, in monochromatic regime, the best in respect to beam-beam effects is the region of close to semi-integer resonce $\nu_x = 0.5$ and not too high tunes $\nu_z < 0.7$. An example of monochromaticity factor $K_m$ and specific luminosity $L_{sp}$ dependence on the opposite beam space charge parameter $\xi_z$ in the considered to be the best working point $\nu_x = 0.525$, $\nu_z = 0.63$ is given in Fig. 6. One sees, that with an increase of $\xi_z$ the monochromaticity factor $K_m$ rapidly decreases. The specific luminosity $L_{sp}$ is decreasing at the same time much more slowly.

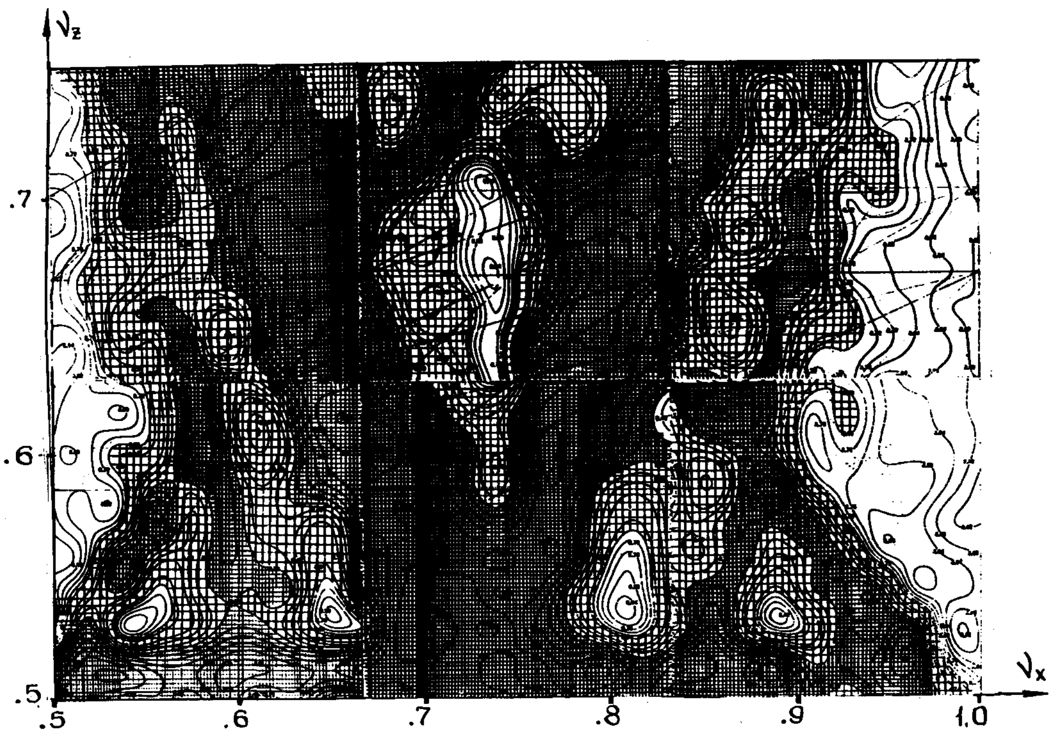


Fig. 5. Level curves of the quantity $K_m$ in a plane $\nu_x, \nu_z$. The regions with $K_m < 0.38$ are marked with a dense shading, the regions with $0.38 < K_m < 0.68$ - with a sparce shading. Regions with $K_m > 0.68$ are blank.

A good qualitative agreement of a Fig. 5 picture with a bottom half of a Fig. 2 picture confirms the validness of a conjecture of a qualitative connection of $K_m$ decrease with a $\Delta_z$ growth and justify the $\Delta_z$ - methodics implementation.

The most powerful in a Fig. 5, the same as in Fig. 2, are the synchrobetatron resonances $\ell = 2$, $m = \pm 1$, $n = \pm 1$, while the following, and approximately equal in strength are the resonances $\ell = 4$, $m = \pm 1$, $n = \pm 1$ and $\ell = 2$, $m = \pm 2$, $n = 0$.

With a help of an additional analysis of $L_{sp}$ values, which were computed simultaneously with $K_m$, it was cleared out, that the horizontal beam size was strongly enlarged in all the $\nu_x, \nu_z$ regions with high $K_m$ values in Fig. 5, except the regions, close to the integer and semi-integer resonances $\nu_x = 0.5$ and $\nu_x = 1$. This leads to a luminosity and (possibly) beam lifetime deterioration. This is quite natural, because the luminosity decrease occurs only in a situation, when the vertical betatron beam size is of the order of magnitude of the synchrotron size and consequently, has to be enlarged many times ($\sim 10$).

The "monochromaticity factor" cutoff in a storage rings with a monochromator scheme impose a severe limitation of a maximum $\xi$, and, consequently, a maximum luminosity, values. The value of $\xi_z$ in monochromator experiments, according to Fig. 6, if we wouldn't allow the energy resolution gain to decrease more, than 20%, can not exceed 0.01 (for $\xi_x/\xi_z = 5$).

Finally, we would like to stress, that the strong modulation of the "force" of the interaction with the opposite beam turns out to be a strong effect, presumakly dominating all the others, not included in the model.

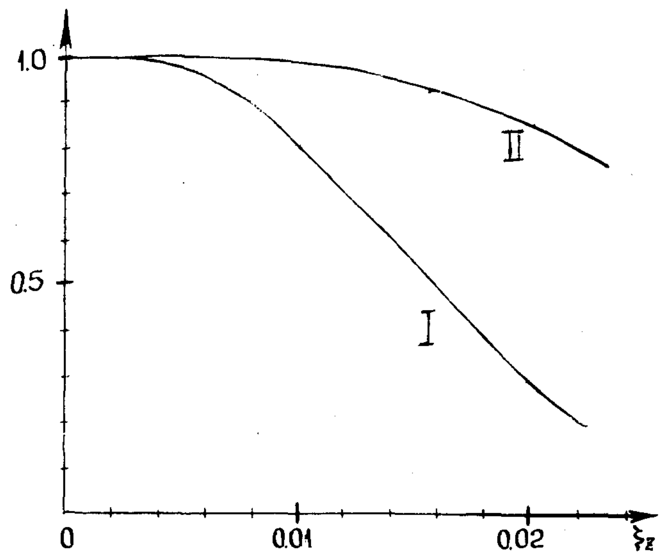


Fig. 6. The dependence of the monochromaticity factor $K_m$ (curve I) and specific luminosity $L_{sp}$ (curve II) on the parameter $\xi_z$ for a constant ratio $\xi_x/\xi_z$ = 5 and tunes $\nu_x$ = 0.525, $\nu_z$ = 0.63.

In such a situation, we hope, that the results of our simulation are fairly reliable, contrary to the simulation of a customary, non-monochromatic beam-beam interaction, where a large number of approximately equal in strength effects are present.

Over a considerable period of time the authors frequently discussed the problems of a present study with F.M.Izrailev and G.M.Tumaikin. A substantial help in a simulation methodics choice was given by A.B.Temnikh. To all of them we wish to express our sincere gratitude.

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