On Mutual Coherency of Spontaneous Radiation from Two Undulators Separated by Achromatic Bend

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***Abstract*—The radiation from two undulators separated by the magnetic system bending the electron beam by a certain angle is considered. The radiation coherency conditions are studied for the case of undulators. It has been revealed that the achromaticity is of significance and some types of such bends are discussed. The layout of a device intended to observe the coherency is presented.**

I. Introduction

We deal here with an ultrarelativistic electron beam passing sequentially through the first undulator, a magnetic system which bends the beam by a certain small angle ∂, and the second undulator (see Fig. 1). Let θ be larger than the characteristic angular undulator–radiation divergence $\sqrt{\lambda/L}$. λ is the wavelength of the fundamental harmonic of modulator radiation and L is the undulator length. In this paper we present the question of mutual coherency of the radiation generated by the undulators, i.e., the result of Young's experiment, where the holes on a screen were placed within the central core of the angular distribution of the undulator radiation.

Let us first consider the case of zero transverse dimensions and of zero angular and energy spreads of electrons. Assuming that the come-in times of electrons into our magnetic system are uncorrelated, it is easy to write down an expression for the real part of the coherency degree [1]:

$$\gamma_{21}^{(r)} \equiv \frac{\langle u_2(t+\tau)u_1(t)\rangle}{\sqrt{\langle u_2^2\rangle\langle u_1^2\rangle}} = \begin{cases} \left(1 - \dfrac{c|\tau-\Delta t|}{q\lambda}\right)\cos\omega(\tau-\Delta t), & \dfrac{q\lambda}{c} > |\tau-\Delta t| \\ 0, & \dfrac{q\lambda}{c} < |\tau-\Delta t| \end{cases} \tag{1}$$

where $\langle\ \rangle$ denotes the time averaging, u_1 and u_2 are the radiation fields of the first (along the beam path) and second undulators, q is the number of periods in each undulator, $\omega = 2\pi c/\lambda$, c is the velocity of light, and Δt is the time lag of the radiation from the second undulator (for a "short" bending system $\Delta t \approx q\lambda/c$). It follows from (1) that the corresponding fringe visibility of the interference bands are equal to

$$v = \begin{cases} 1 - \dfrac{c}{q\lambda}|\tau-\Delta t|, & |\tau-\Delta t| < \dfrac{q\lambda}{c} \\ 0, & |\tau-\Delta t| > \dfrac{q\lambda}{c}. \end{cases} \tag{2}$$

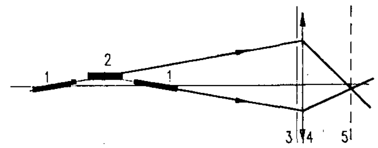

Fig. 1. The experimental agreement to observe the coherent radiation from two undulators: 1-undulators, 2-bending system, 3-screen with two holes, 4-lens, 5-plane of imaging of the centre of the bending system.

II. Coherency Conditions

One can use a Taylor expansion of the transit time Δt:

$$\Delta t = (\Delta t)_0 + \left(\frac{\partial \Delta t}{\partial E}\right)_0 (E - E_0) + \left(\frac{\partial \Delta t}{\partial x}\right)_0 x + \left(\frac{\partial \Delta t}{\partial y}\right)_0 y + \left(\frac{\partial \Delta t}{\partial x'}\right) x' + \left(\frac{\partial \Delta t}{\partial y'}\right) y' + \cdots$$

where $(\Delta t)_0$ is the equilibrium electron transit time from the middle of the first undulator to the middle of the second undulator to take into account the electron beam energy, angular, and transverse position spread. Now we have to substitute (3) into (1) and to average $\gamma_{21}^{(r)}$ over the particle distribution in the beam. Averaging over the energy should be performed similarly to that which is done in the theory of an optical klystron [2], [3]. This averaging is different from our case only in the fact that the bending angle θ is zero. As a result, $\gamma_{21}^{(r)}$ and the fringe visibility V are multiplied by

$$\exp\left[-\frac{1}{2}\left(\omega \cdot \frac{\partial \Delta t}{\partial E}\sigma_E\right)^2\right]$$

where σ_E is the rms energy spread. For a "short" bending system

$$\left(\frac{\partial \Delta t}{\partial E} \approx -2q\frac{\lambda}{cE}\right)$$

Manuscript received May 7, 1991; revised August 1, 1991.
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IEEE Log Number 9103658.

0018-9197/91$01.00 © 1991 IEEE

the fringe visibility at

$$\frac{\sigma_E}{E} < \frac{1}{4\pi q}$$

remains the same, which coincides with the condition when there is no broadening of the undulator radiation spectrum.

Let a bending by the angle θ take place in the horizontal plane, it is then known that [4]

$$\frac{\partial \Delta t}{\partial y} = \frac{\partial \Delta t}{\partial y'} = 0$$

and

$$\frac{\partial \Delta t}{\partial x} x + \frac{\partial \Delta t}{\partial x'} x' = \frac{1}{c} \int_{s_1}^{s_2} \frac{x(s)}{\rho(s)} ds \qquad (4)$$

where s_1 and s_2 are the longitudinal coordinates of the middles of both undulators ($x(s_1) = x$, $x'(s_1) = x'$), $\rho(s)$ is the curvature radius of the equilibrium trajectory ($\int_{S_1}^{S_2} ds/\rho(s) = \theta$). Estimating the integral (4) as $x_m \theta / c$ (x_m is the electron coordinate in the bending system) and taking into account the luminosity preservation condition $\omega\sigma(\Delta t) < 1$, we obtain the restriction on the bending angle θ:

$$\theta < \frac{\lambda}{2\pi\sigma_x}$$

where σ_x is the horizontal size of the electron beam. We thus come to a quite evident result: the bending angle θ should not be larger than the angle of spatial coherency of a source whose size is σ_x. If we remember that we are interested in the case $\theta > \sqrt{\lambda/L}$, we shall have an upper bound on the size of the electron beam:

$$\sigma_x < \frac{1}{2\pi} \sqrt{\lambda L}$$

coupled with the evident limitation on the angular spread $\langle x'^2 \rangle < \lambda/L$ the latter leads to the known fundamental condition for the horizontal emittance: $\epsilon_x < \lambda/2\pi$.

To avoid the above limitations the integral (4) should be matched to zero, i.e., the linear dependence of the trajectory length on the horizontal angles and coordinates must be eliminated. It is known [4] that for an arbitrary trajectory $x(s)$ the matching-to-zero condition of the integral (4) coincides with the zero-dispersivity condition (i.e., the achromaticity) of the magnetic bending system.

III. Achromatic Bend

A great variety of zero-dispersive (achromatic) systems are known in accelerating technique.

In achromatic bend particles with the same initial transverse position and angle

$$x = x_{in} \qquad x' = x'_{in}$$

but with small energy deviations from equilibrium one ($E = E_0 + \delta E$) are turned for the same angle without any displacement in transverse direction:

$$x_{out}(E_0 + \delta E) = x_{out}(E_0) \qquad x'_{out}(E_0 + \delta E) = x'_{out}(E_0).$$

The simplest illustration of an achromatic bend is a pair of short magnets, each bending the beam by an angle $\theta/2$, with a focusing lens between them; note that its focal length is equal to the one-fourth of the magnet-to-magnet distance. Since it is desirable that the length of the bending system be not too large (for example, no longer than the undulator length L) the bending system proves to be hard focusing, which complicates the optimization of the beam envelopes in undulators. The situation can be improved by applying a system which incorporates four short magnets and a focusing lens (see Fig. 2). Assuming that the second and third magnets and the lens are equally spaced, closely to each other, between the first and fourth magnets, each bending the beam by an angle $\alpha + \theta/2$ (the second and third bend is by $-\alpha$), we obtain that for the zero-dispersivity the focal length of the lens should be equal to

$$F = \frac{l}{4}\left(1 + \frac{2\alpha}{\theta}\right) \qquad (5)$$

where l is the total length of the achromatic bend. As α increases, the longitudinal dispersion of the system also increases:

$$cE \frac{\partial \Delta t}{\partial E} \approx l\alpha\left(\alpha + \frac{\theta}{2}\right) \qquad (6)$$

which limits an increase of the focal distance (5). For this case, the longitudinal undulator dispersion $2q\lambda$ is a natural scale. Equalizing the expression (6) to this quantity, we obtain an estimation for the maximum focal length (at $\alpha >> \theta$):

$$F_{max} \approx \frac{1}{\theta} \sqrt{\frac{q\lambda L}{2}} \qquad (7)$$

or for the maximum angle of divergence

$$\theta_{max} \approx \frac{1}{F} \sqrt{\frac{q\lambda L}{2}} = \sqrt{\frac{q}{2}} \frac{L}{F} \sqrt{\frac{\lambda}{L}}. \qquad (8)$$

The condition $\theta_{max} > \sqrt{\alpha/L}$ transforms into $\sqrt{q/2}\,(L/F) > 1$, which is easily accomplished.

Thus, the achromatic bend of the given system provides the mutual coherency of the radiation generated by two undulators. It is clear that when designing the real bending systems the more complex schemes may turn out to be useful, such as MDMFMDM (M stands for a magnet, D and F are for defocusing and focusing quadrupoles).

IV. Spatial Coherency

In the above discussion we haven't touched upon the bounds on the hole of the diffractometer, assuming them rather small. Here its scheme will be considered in detail. Let D be the distance from the bending system to the holed screen, d the diameter of the holes, and f be the focal distance of a lens near the screen. We will consider the

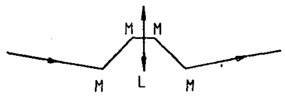

Fig. 2. The layout of the achromatic bend: M-bending magnets and L-lens.

interference picture in the imaging plane (see Fig. 1). In this case, the center of this picture is shifted from the optical axis (passing through the centers of the undulators and the lens) by $(Z\theta/4)(f/D)$, where Z is the distance which separates the centers of the undulators. The space period of the interference band is $(\lambda/\theta)(f/D)$.

If the diameter of the holes is less than the size of the spatial coherency region, $d < (D\lambda/\pi\sigma_x)$, then the diameter of the illuminated spot in the imaging plane is equal to $(\lambda/d)f$, while the total number of the bands is correspondingly $D\theta/d$. The maximum fringe visibility will be observed at $\tau = \Delta t$, i.e., at the distance $(c\Delta t/\theta)(f/D)$ from the center of the interference picture. This is possible only if $(c\Delta t/\theta)(f/D) \le (\lambda/d)f$, i.e.,

$$d < \frac{\theta D\lambda}{c\Delta t}. \tag{9}$$

The quantity $c\Delta t$ in (9) can be increased by observing the picture through a filter whose spectral width is smaller than the width of the undulator radiation line. Another way of overcoming the limitation (9) is a compensation of the time lag Δt. It can be done, for example, by placing two identical plane-parallel glass plates and by bending a plate through which the radiation generated by the first undulator passes.

With the time lag compensated and with the electron beam of rather small transverse sizes, $\sigma_x < \sqrt{\lambda L/2\pi}$, we can remove the holes screen. In this case, the diameter of the diffraction spot becomes of the order of $\sqrt{\lambda L/2\pi}\, f/D$, and the total number of the interference bands is $\theta\sqrt{L/2\pi\lambda}$. Note that the number of the bands in question is equal, in order of magnitude, to the ratio of the bending angle θ to the angular divergence of the undulator radiation, $\sqrt{\lambda/L}$, i.e., it characterizes the degree of divergence.

V. Conclusion

The experiment described above was performed in April 1989 on the storage ring VEPP-3 (INP, Novosibirsk). Interference was observed on a wavelength of 0.6 μm. The conditions required to observe the interference were the achromatic nature of the bend, the compensation of the time lag by plane-parallel glass plates (or the lengthening of the radiation packet by means of an interference filter). The description of the experiment is beyond the scope of the present paper and will be presented elsewhere.

Acknowledgment

The authors are thankful to academician A. N. Shrinsky for helpful discussions and to E. S. Gluskin for useful recommendations.

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