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On the mutual coherence of spontaneous radiation from two undulators separated by an achromatic bend

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The radiation from two undulators which are separated by the magnetic system that bends the electron beam by a certain angle is considered. The radiation coherence conditions are studied for the case of undulators. It is shown that the achromaticity is of importance. Some types of these bends are discussed. The layout of a device intended to observe the coherence is presented.

1. Introduction

In this article we are concerned with an ultrarelativistic electron beam that first goes through the first undulator, then through a magnetic system which bends the beam by a certain small angle θ, and finally through the second undulator (see fig. 1). Let θ be greater than the characteristic angular undulator-radiation divergence $\sqrt{\lambda/L}$ (λ is the wavelength of the fundamental harmonic of the forward radiation and L is the undulator length). Of interest in this case is the mutual coherence of the radiation generated by the undulators, i.e. the result of Young's experiment, where the holes in the screen were placed at the centers of the undulator radiation perpendicular to the radiation direction.

2. Coherence definition

Let us first consider the case of zero transverse dimensions and of zero angular and energy spreads in the electron beam. Assuming that the flight times of some electrons through our magnetic system are uncorrelated, it is easy to write down an expression for the real part of the degree of coherence [1]:

$$\gamma_{21}^{(r)} = \frac{\langle u_2(t+\tau)u_1(t)\rangle}{\sqrt{\langle u_1^2\rangle\langle u_2^2\rangle}} = \begin{cases} \left(1 - \dfrac{c\,|\tau-\Delta t|}{q\lambda}\right)\cos\omega(\tau-\Delta t), \\ \quad \dfrac{q\lambda}{c} > |\tau-\Delta t|, \\ 0, \quad \dfrac{q\lambda}{c} < |\tau-\Delta t|, \end{cases} \tag{1}$$

where $\langle\ \rangle$ denotes time averaging, u_1 and u_2 are the radiation fields of the first (along the beam path) and the second undulator, respectively, q is the number of periods in each undulator, $\omega = 2\pi c/\lambda$, c is the velocity of light, and Δt is the time lag of the radiation from the second undulator (for a "short" bending system $\Delta t \approx q\lambda/c$). From eq. (1) it follows that the corresponding fringe visibility of the interference bands is equal to

$$V = \begin{cases} 1 - \dfrac{c}{q\lambda}|\tau-\Delta t|, & |\tau-\Delta t| < \dfrac{q\lambda}{c}, \\ 0, & |\tau-\Delta t| > \dfrac{q\lambda}{c}. \end{cases} \tag{2}$$

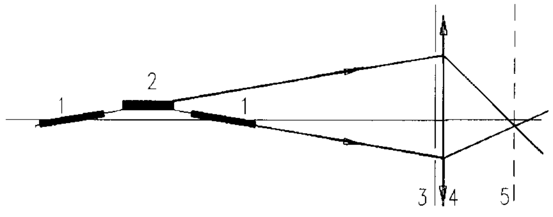

Fig. 1. The experimental arrangement to observe the coherent radiation from two undulators: 1 – undulators, 2 – bending system, 3 – screen with two holes, 4 – lens, 5 – plane of imaging of the centre of the bending system.

3. Calculations for a real electron beam

To take the finite sizes of the electron beam and the spreads in it into account, the expansion of Δt with respect to the initial deviations of the particle energy, its angles, and its coordinates from the corresponding equilibrium values in the middle of the first undulator (in this case, Δt occurs on the interval from the middle of the first to the middle of the second undulator) is presented in the form

$$\Delta t = (\Delta t)_0 + \left(\frac{\partial t}{\partial E}\right)_0 (E - E_0) + \left(\frac{\partial t}{\partial x}\right)_0 x + \left(\frac{\partial t}{\partial y}\right)_0 y + \left(\frac{\partial t}{\partial x'}\right)_0 x' + \left(\frac{\partial t}{\partial y'}\right)_0 y' + \cdots . \quad (3)$$

Now we must replace Δt in eq. (1) by eq. (3) and average $\gamma_{21}^{(r)}$ over the particle distribution in the beam. Averaging over the energy should be performed in the same way as is done in the theory of an optical klystron [2,3]. The only difference in the averaging arises from the fact that the bending angle θ is zero in the case of an optical klystron. As a result, $\gamma_{21}^{(r)}$ and the fringe visibility V are still multiplied by

$$\exp\left[-\frac{1}{2}\left(\omega \frac{\partial t}{\partial E} \sigma_E\right)^2\right],$$

where σ_E is the rms energy spread. For a "short" bending system

$$\frac{\partial \Delta t}{\partial E} \approx 2q \frac{\lambda}{cE}.$$

The fringe visibility at

$$\frac{\sigma_E}{E} < \frac{1}{4\pi q}$$

remains the same, which coincides with the condition when there is no broadening of the undulator radiation spectrum.

Let the bending by the angle θ take place in the horizontal plane, it is known for that case that [4]

$$\frac{\partial \Delta t}{\partial y} = \frac{\partial \Delta t}{\partial y'} = 0$$

and

$$\frac{\partial \Delta t}{\partial x} x + \frac{\partial \Delta t}{\partial x'} x' = \frac{1}{c} \int_{S_1}^{S_2} \frac{x(S)}{\rho(S)} \mathrm{d}S, \quad (4)$$

where S_1 and S_2 are the longitudinal coordinates of the middles of both undulators ($x(S_1) = x$, $x'(S_1) = x'$), and $\rho(S)$ is the curvature radius of the equilibrium trajectory ($\int_{S_1}^{S_2} \mathrm{d}S/\rho(S) = 0$). Estimating the integral (4) to be $x_m\theta/c$ (x_m is the electron coordinate in the bending system) and taking into account the luminosity preservation condition $\omega\sigma(\Delta t) < 1$, we obtain the restriction on the bending angle θ:

$$\theta < \frac{\lambda}{2\pi\sigma_x}.$$

Here σ_x is the horizontal size of the electron beam. We thus come to a quite evident result: the bending angle θ should not be larger than the angle of spatial coherence of a source whose size is σ_x. If we remember that we are interested in the case $\theta > \sqrt{\lambda/L}$, we shall have an upper bound on the size of the electron beam:

$$\sigma_x < \frac{1}{2\pi}\sqrt{\lambda L},$$

coupled with the evident limitation on the angular spread $\langle x'^2 \rangle < \lambda/L$; the latter leads to the known fundamental condition for the horizontal emittance.

To overcome the above bounds the integral (4) should be zero, i.e. the linear dependence of the trajectory length on the horizontal angles and coordinates must be eliminated. It is known [4] that for an arbitrary trajectory the condition that the integral (4) is zero coincides with the condition that there is no dispersion (i.e. the achromaticity) in the magnetic bending system.

4. Achromatic bend

A great variety of nondispersive systems are known in accelerating technology. Recall that a nondispersive bend means that, when deflection from the equilibrium energy occurs, the particles in the bend that have no deflection with respect to the angle and energy leave the bend without having the proper deflections from the equilibrium trajectory. The simplest illustration of an achromatic bend is a pair of short magnets, each bending the beam by an angle $\theta/2$, with a focusing lens between them; note that its focal length is equal to one-fourth of the magnet-to-magnet distance. Since it is desirable that the length of the bending system is not too large (for example, no longer than the undulator length L), the bending system proves to be difficult to focus, which complicates the optimization of the beam envelopes in the undulators. The situation can be improved by applying a system which incorporates four short magnets and a focusing lens (see fig. 2). Assuming that the second and third magnets and the lens are equally spaced, close to each other, between the first and the fourth magnets, each bending the beam by an angle $\alpha + \theta/2$ (the second and third bend the beam by $-\alpha$), we obtain that for zero dispersion the focal distance of the lens should be equal to

$$F = \frac{l}{4}\left(1 + \frac{2d}{\theta}\right), \quad (5)$$

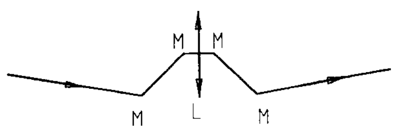

Fig. 2. The layout of the achromatic bend: M – bending magnets, L – lens.

where l is the total length of the achromatic bend. As α increases, the longitudinal dispersion of the system also increases:

$$cE\frac{\partial \Delta t}{\partial E} \approx l\alpha\left(\alpha + \frac{\theta}{2}\right), \tag{6}$$

which limits the increase of the focal distance (5). For this case, the longitudinal undulator dispersion $2q\lambda$ is a natural scale. Equalizing expression (6) to this quantity, we obtain an estimation for the maximum field distance (at $\alpha \gg \theta$):

$$F_{max} \approx \frac{1}{\theta}\sqrt{\frac{q\lambda L}{2}} \tag{7}$$

or for the maximum angle of divergence

$$\theta_{max} \approx \frac{1}{F}\sqrt{\frac{q\lambda L}{2}} = \sqrt{\frac{q}{2}}\frac{L}{F}\sqrt{\frac{\lambda}{L}}. \tag{8}$$

The condition $\theta_{max} > \sqrt{\alpha/L}$ transforms into

$$\sqrt{\frac{q}{2}}\frac{L}{F} > 1.$$

which is easily accomplished.

Thus, the achromatic bend of the given system provides mutual coherence of the radiation generated by two undulators. It is clear that, when designing the real bending systems, more complex schemes may turn out to be useful, such as MDMFMDM (M stands for a magnet, D and F for defocusing and focusing quadrupoles).

5. Young's experiment

In the preceding discussion we have not touched upon the bounds on the sizes of the diffractometer, assuming them to be rather small. Here its scheme will be considered in detail. Let D be the distance from the bending system to the screen with holes, d the diameter of the holes, and f the focal distance of a lens near the screen. We will consider the interference pattern in the imaging plane (see fig. 1). In this case, the centre of this pattern is shifted from the optical axis (passing through the centres of the undulators and the lens) by $(Z\theta/4) \times (f/D)$, where Z is the distance which separates the centres of the undulators. The space period of the interference band is $(\lambda/\theta)(f/D)$.

If the diameter of the holes is less than the size of the spatial coherence region, $d < D\lambda/(\pi\sigma_x)$, then the diameter of the illuminated spot in the imaging plane is equal to $(\lambda/d)f$, while the total number of bands is correspondingly $D\theta/d$. The maximum fringe visibility will be observed at $\tau = \Delta t$, i.e. at a distance $(c\Delta t/\theta) \times (f/D)$ from the centre of the interference pattern. This is possible only at

$$\frac{c\Delta t}{\theta}\frac{f}{D} < \frac{\lambda}{d}f,$$

i.e.,

$$d < \frac{\theta D\lambda}{c\Delta t}. \tag{9}$$

The quantity $c\Delta t$ in relation (9) can be increased by observing the interference pattern through a filter whose spectral width is smaller than the width of the undulator radiation line. Another way of overcoming limitation (9) is compensation of the time lag Δt. This can be done, for example, by placing two identical plane-parallel glass plates and by bending a plate through which the radiation generated by the first undulator passes.

With the time lag compensated and with the electron beam of rather small transverse sizes, $\sigma_x < \sqrt{\lambda L/2\pi}$, we can remove the screen with holes. In this case, the diameter of the diffraction spot becomes of the order of $\sqrt{\lambda L/2\pi}(f/D)$, and the total number of interference bands is $\theta\sqrt{L/2\pi\lambda}$. Note that the number of the bands in question is, in order of magnitude, equal to the ratio of the bending angle θ to the angular divergence of the undulator radiation $\sqrt{\lambda/L}$, i.e. it characterizes the degree of divergence.

6. Summary

The experiment described above was performed in April 1989 on the storage ring VEPP-3 (INP, Novosibirsk). Interference was observed on a wavelength of 0.6 μm with a telescope. The conditions required to observe the interference were the achromatic nature of the bend and the composition of the time lag by plane-parallel glass plates (or the lengthening of the radiation packet by means of an interference filter). The description of the experiment is beyong the scope of the present paper and will be presented elsewhere [5].

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