Proceedings of the Eighth International Seminar

QUARKS '94

Editors

D Yu Grigoriev

V A Matveev

V A Rubakov

D T Son

A N Tavkhelidze

World Scientific

Proceedings of the Eighth International Seminar

QUARKS '94

Vladimir, Russia, 11 – 18 May 1994

Editors

D Yu Grigoriev

V A Matveev

V A Rubakov

D T Son

A N Tavkhelidze

Institute for Nuclear Research of Russian Academy of Sciences

THE SPIN PROPERTIES OF TOP QUARKS PRODUCED IN COLLISIONS OF POLARIZED PHOTONS IN THE THRESHOLD REGION

V.S. Fadin
Budker Institute for Nuclear Physics and Novosibirsk State University, 630090 Novosibirsk, Russia.

V.A. Khoze
Department of Physics, University of Durham, DH1 3LE, England and Institute for Nuclear Physics, 188350 St. Petersburg, Gatchina, Russia.

M.I. Kotsky
Budker Institute for Nuclear Physics, 630090 Novosibirsk, Russia.

Abstract

We study the spin properties of top quarks produced in collisions of polarized photons in the threshold region. For a relatively heavy top quark the influence of non-perturbative effects is small and its polarization parameters can be predicted in perturbative QCD. The measurements of the top polarization may allow a novel test of QCD in the $t\bar{t}$ system. In particular, they may provide a new way to determine the precise value of α_s and to study the properties of the top quark.

1. Introduction

Since the top mass m_t exceeds the W-mass, the t-quark can decay directly to W^+b and its decay width Γ_t is steeply increasing with its mass[1,2]. For $m_t \sim$ 130-150 GeV the t-quark width is typically in the GeV range[3] and the t generally decays before its hadronization occurs[1,2]. In such a situation, the t-quark spin proves to be a very useful tool. The point is that the t-polarization is not diluted since there is no hadronization and the bremsstrahlung gluons do not flip the top spin. Therefore the $t(\bar{t})$ quark is normally produced in a well-defined state of polarization which can be measured experimentally. This polarization is transmitted to the secondary particles produced in their decays and can be well reconstructed from their distributions (see e.g. Refs.[4-8]).

The new opportunities would be provided by a Photon Linear Collider (PLC). This is a facility[9-11] where high energy, high intensity photon beams are generated

at a linear e^+e^- collider via Compton backscattering using high-power lasers[12]. PLC provides highly polarized beams, large luminosity, and a variable luminosity spectrum. The potential of such a machine to explore top quark threshold production in

$$\gamma\gamma \to t\bar{t} \tag{1}$$

was discussed in Refs.[13-15]. Measurements of the top polarization in the process (1) may provide a novel probe of QCD dynamics in the $t\bar{t}$ system. Here one deals with the interference between the S-wave and P-wave production. These measurements may provide information not only about the overlap of S-wave and P-wave states of the $t\bar{t}$ system but also about the phase difference of the corresponding production amplitudes.

2. Effects of the final state Coulomb interaction

The striking features of the $t\bar{t}$ production in the threshold region have been discussed quite frequently in recent years (e.g.[6-8] and references therein). One faces here the unique situation that the QCD binding forces are strong but (because of the infrared cut-off provided by the large top width) are well under the control of perturbation theory[16-18]. The $t\bar{t}$ production near threshold is significantly enhanced because of the Coulomb gluon exchanges between the top quarks. To take full advantage of the polarization measurements one needs to understand the following questions. First, how strongly the top polarization is affected by the Coulomb threshold interaction. Second, how the top polarization can be exploited to obtain some new information on the $t\bar{t}$ interaction forces. Answering these questions is the main concern of this and the next section.

In what follows we shall use a non-relativistic Green's function formalism applied in the previous papers of two of the authors for finding the total cross-section[16-17] and the amplitude[17] for the $e^+e^- \to t\bar{t}$ process, see also Refs.[18-22].

Let us label the particle 4-momenta by $\gamma(k_1) + \gamma(k_2) \to t(p_+) + \bar{t}(p_-)$ and present the amplitude in the centre-of-mass frame in terms of the four-component quark wave functions $u(p_+)$ and $v(p_-)$ as

$$A_{fi}(\vec{p}) = 4\pi\alpha \cdot e_t^2 \bar{u}(p_+) M(\vec{p}) v(p_-), \tag{2}$$

where $e_t = \frac{2}{3}$ is the electric charge and $\vec{p} = \vec{p_+} = p\vec{n}_2$ is the momentum of top quark. Up to order $\frac{p^2}{m_t^2}$ terms one can rewrite the quark spinors in a form which simplifies the further treatment of the Dirac structure of the threshold amplitude

$$u(p_+) = \frac{(m_t + \hat{p}_+)}{\sqrt{2m_t}} \cdot \Lambda_+ u_0,$$

$$v(p_-) = \frac{(m_t - \hat{p}_-)}{\sqrt{2m_t}} \cdot \Lambda_- v_0 \tag{3}$$

with

$$u_0 = \begin{pmatrix} \psi \\ 0 \end{pmatrix}, \qquad v_0 = \begin{pmatrix} 0 \\ \chi \end{pmatrix} \qquad \Lambda_\pm = \frac{(1 \pm \gamma^0)}{2}. \tag{4}$$

Then A_{fi} takes the form

$$A_{fi} = 4\pi\alpha\, e_t^2 u_0^+ R(\vec{p}) v_0 \tag{5}$$

where

$$R(\vec{p}) = \frac{1}{2m_t}\Lambda_+ \cdot (m_t + \hat{p}_+) M(\vec{p})(m_t - \hat{p}_-) \cdot \Lambda_-. \tag{6}$$

In Born approximation

$$M^{(B)}(\vec{p}) = \frac{\hat{\epsilon}^{(1)}(\hat{p}_+ - \hat{k}_1 + m_t)\hat{\epsilon}^{(2)}}{2(k_1 p_+)} +$$

$$+ \frac{\hat{\epsilon}^{(2)}(\hat{p}_+ - \hat{k}_2 + m_t)\hat{\epsilon}^{(1)}}{2(k_2 p_+)} = \epsilon_\mu^{(1)}\epsilon_\nu^{(2)} T_{(B)}^{\mu\nu}(\vec{p}) \tag{7}$$

and $R(\vec{p})$ becomes

$$R^{(B)}(\vec{p}) = \Lambda_+ \left[a\gamma^0\gamma^5 + \frac{p}{m_t}\vec{b} \cdot \vec{\gamma}\right] \Lambda_-, \tag{8}$$

$$\gamma_5 = \begin{pmatrix} O & I \\ I & O \end{pmatrix}$$

The quantities a and b_i are related to the photon polarization vectors $\vec{\epsilon}^{(1)}, \vec{\epsilon}^{(2)}$ by

$$a = 2i(\vec{\epsilon}^{(1)} \times \vec{\epsilon}^{(2)})\vec{n}_1.$$

$$\vec{b} = 2[(\vec{\epsilon}^{(1)} \cdot \vec{\epsilon}^{(2)})\vec{n}_1 \cos\theta + (\vec{\epsilon}^{(1)} \cdot \vec{n}_2)\vec{\epsilon}^{(2)} + (\vec{\epsilon}^{(2)} \cdot \vec{n}_2)\vec{\epsilon}^{(1)}], \tag{9}$$

where $\vec{k}_1 = \frac{\sqrt{s}\vec{n}_1}{2}$ and $\vec{k}_2 = -\frac{\sqrt{s}\vec{n}_1}{2}$ are the momenta of the incoming photons in the lab ($\gamma\gamma$ centre-of-mass) frame ($\sqrt{s}$ is the total centre-of-mass energy), θ is the polar angle of the t with respect to $\vec{n}_1$.

Taking into account the final state interaction near threshold $R(\vec{p})$, analogously to Ref.[17], can be written as

$$R(\vec{p}) = \langle\vec{p}|(\hat{H} - E - i\Gamma_t)^{-1} R^{(B)}(\hat{\vec{p}})|\vec{r} = 0\rangle \cdot \left(\frac{\vec{p}^2}{m_t} - E - i\Gamma_t\right), \tag{10}$$

$$\hat{H} = \hat{H}_0 + V(r), \qquad \hat{H}_0 = \frac{\hat{\vec{p}}^2}{m_t}. \tag{11}$$

Here $\hat{\vec{p}} = -i\frac{\partial}{\partial\vec{r}}$ is the momentum operator and $V(r)$ is the nonrelativistic QCD potential for the $t\bar{t}$ system in the colour singlet state

$$V(r) = -\frac{4}{3}\frac{\alpha_s(r)}{r}. \tag{12}$$

$|\vec{p}\rangle$ denotes a state of definite momentum $\vec{p} = \vec{p}_+ = -\vec{p}_-$, and $|\vec{r}\rangle$ a state of definite relative coordinate (we use normalization $\langle\vec{p}|\vec{r}\rangle = e^{-i\vec{p}\cdot\vec{r}}$), $E = \sqrt{(p_+ + p_-)^2} - 2m_t$.

Let us define the functions $\tilde{G}(\vec{p}, E)$ and $\tilde{F}_i(\vec{p}, E)$ by

$$\tilde{G}(\vec{p}, E) = \langle\vec{p}|(\hat{H} - E - i\Gamma_t)^{-1}|\vec{r} = 0\rangle \qquad (a)$$

$$\tilde{F}_i(\vec{p}, E) = \langle\vec{p}|(\hat{H} - E - i\Gamma_t)^{-1}\hat{p}_i|\vec{r} = 0\rangle \qquad (b) \tag{13}$$

Then the modified expression for the $t\bar{t}$ production amplitude near threshold can be presented in the following form

$$A_{fi} = -4\pi\alpha \cdot e_t^2 \psi^+ \left[a\tilde{G}(\vec{p}, E) + \frac{p}{m_t}\vec{b} \cdot \vec{\sigma} \cdot F(\vec{p}, E)\right] \chi \cdot \left(E - \frac{\vec{p}^2}{m_t} + i\Gamma_t\right). \tag{14}$$

In Eq. (14) the a and $\vec{b}$ parameters are given by the relations (9) and

$$p^2 \tilde{F}(\vec{p}, E) = p_i \tilde{F}_i(\vec{p}, E). \tag{15}$$

Note that the functions $\tilde{G}$, $\tilde{F}_i$ coincide with those introduced in Ref.[20] for the description of the effects of the axial current in the $e^+e^- \to t\bar{t}$ reaction.

3. Top spin parameters

The amplitude A_{fi} allows one to derive the threshold expressions for both the unpolarized cross section $d\sigma_{\gamma\gamma}^{(0)}$ for the top production and the t-polarization vector. The threshold cross section $d\sigma_{\gamma\gamma}^{(\vec{\zeta}')}$ for production of the t-quark with polarization $\vec{\zeta}'$ can be written as

$$d\sigma_{\gamma\gamma}^{(\vec{\zeta}')} = \frac{\alpha^2 R_{\gamma\gamma}}{8\pi m_t^4} \cdot \Gamma_t d^3p \left[|a|^2 |\tilde{G}(\vec{p}, E)|^2 + \right.$$

$$\left. + \frac{2p}{m_t} Re(a^*(\vec{b} \cdot \vec{\zeta}') \cdot \tilde{G}^*(\vec{p}, E)\tilde{F}(\vec{p}, E))\right], \tag{16}$$

where $R_{\gamma\gamma} = N_c e_t^4$ ($N_c = 3$ is the namber of colours). An expression for the t-quark spin vector ζ is then

$$\vec{\zeta} = 2\frac{p}{m_t} Re(\zeta_0 \cdot D(\vec{p},E)),\ \zeta_0 = \frac{a^*\vec{b}}{|a|^2}, \qquad (a)$$

$$D(\vec{p},E) = \frac{\tilde{F}(\vec{p},E)}{\tilde{G}(p,E)}. \qquad (b) \qquad (17)$$

In the Born case $\vec{\zeta}_B = 2\frac{p}{m_t} \cdot Re\zeta_0$. For the highly polarized photon beams $|\zeta_0| \sim O(1)$. For instance, if two photons carry the same helicities then $|\zeta_0| = 1$.

Thus the top polarization could be strongly modified by the threshold effects and its measurement in collisions of polarized photons would provide a challenging opportunity to probe experimentally the new characteristics of the interquark potential, $Re(\tilde{G}^* \cdot \tilde{F})$ and $Im(\tilde{G}^* \cdot \tilde{F})$. The latter quantity looks especially intriguing since it cannot be measured by the other methods which are now under consideration, see e.g.[20]. To some extent, however, this should be taken with a pinch of salt. The point is that the correlation $Im(a^*\vec{b}\vec{\zeta})$ is constructed from the linear polarizations of the incoming photons. But at PLC one can expect the high degree only for the circular polarization (about 95% for polarized electron beams[1]). The degree of linear polarization will not be as high as the circular one (probably around 30%). Moreover, the monochromaticity of the beams is highest just in the case of circular polarization and as is well known, the monochromaticity is one of the main concerns in the discussions of the threshold $t\bar{t}$ production. However, we do not expect that the top polarization gets very much diluted because of the beam energy spread and, in principle, better monochromaticity could be achieved at the cost of luminosity. Once more, one has to sacrifice more luminosity in the linear case than in the circular case. It would be rather premature (and for these authors also quite inappropriate) to make some more definite statements about the prospects of exploiting the advantages of the linear polarization as well as about the effects of the energy resolution for the beams that might be achievable ten years from now.

4. Discussion

In the threshold region the ratio D (see Eq. (17)) exhibits a quite intricate structure which is strongly dependent on the top quark mass and the precise value of α_s. To elucidate some specific consequences of the Coulomb physics let us include in the consideration the width of the top keeping α_s fixed. Using Meixner representation

[1]VAK is indebted to D. Borden for the discussion of the polarization properties of the beams at PLC.

for $G_E(\vec{r},\vec{r}' = 0)$[23] we obtain

$$\tilde{G}(\vec{p},E) = \frac{m_t}{(\kappa^2+p^2)}[1 + 4p_s\kappa \cdot \int_0^1 \frac{dx\, x^{-\frac{p_s}{\kappa}}}{\kappa^2(1+x)^2 + p^2(1-x)^2}], \qquad (18)$$

$$D(\vec{p},E) = 2 + (\kappa^2+p^2)\frac{d}{dp^2}\ell n\tilde{G}(\vec{p},E), \qquad (19)$$

where $p_s = \frac{2}{3}\alpha_s m_t$ and $\kappa = \sqrt{-m_t(E+i\Gamma_t)}$. For $\Gamma_t > \frac{m_t\alpha_s^2}{2\sqrt{3}}$ representation (18) is valid for all real values of E; in the opposite case it can be used in the region

$$E^2 + \Gamma_t^2 > \frac{2}{9}m_t\alpha_s^2(\sqrt{E^2+\Gamma_t^2} - E). \qquad (20)$$

For values of E outside this region $D(\vec{p},E)$ can be obtained by analytical continuation.

It is clear from (18),(19), that $D(\vec{p},E) \simeq 1$ at $p \gg p_s$. This is in accord with the region of applicability of Born approximation. For running α_s it can be written as

$$\frac{p}{m_t} \gg \alpha_s(p) \qquad (21)$$

In the region (21) the influence of the final state interaction on the top polarization is negligible. In the opposite case of small p one can obtain from (18),(19)

$$D(\vec{0},E) = 8\frac{\int_0^1 \frac{dx\, x^{1-\frac{p_s}{\kappa}}(1-x)}{(1+x)^5}}{\int_0^1 \frac{dx\, x^{-\frac{p_s}{\kappa}}(1-x)}{(1+x)^3}}. \qquad (22)$$

This shows that outside the region (21) $D(\vec{p},E)$ strongly depends on α_s. This conclusion is confirmed by the numerical calculations for the realistic case of a running α_s. We have performed the calculations by a method similar to that of Ref.[18,20,21], using the same parameters as in Ref.[21].

5. Conclusion

We have studied the influence of the spectacular threshold effects on the polarization of top quarks produced in the collisions of polarized photons. Without loss of generality one may consider some other parity violating angular asymmetries in the distribution of the secondary particles. For relatively heavy top quark ($m_t >$ 130 GeV) the decay width Γ_t provides an infrared cut-off for the strong forces between quarks and antiquarks and perturbative QCD suffices to treat the threshold phenomena.

We have demonstrated that the final state interaction between t-quarks induces two major modifications of the Born result for the top polarization above threshold. First, in the collisions of linearly polarized photons order α_s (T-odd) effects may appear which would allow one to measure the relative phase of the low energy S-wave and P-wave scattering of t-quarks. Second, the degree of polarization arising in collisions of circularly polarized photons can be seriously affected because of the running of the QCD coupling. For relatively heavy top quark its polarization below threshold is quite sizeable, order α_s. The resulting polarization effects are expected to survive the averaging over the luminosity distributions in the incoming beams. Optimistically, we anticipate that these polarization measurements would open new prospects in studying the interquark dynamics. In combination with the threshold measurements of the cross-sections these polarization results may provide us with an efficient systematic cross-check on the accuracy with which the values for m_t and α_s will have been determined.

6. References

1. J.H. Kühn, *Acta Phys. Aust. Suppl.* **24** (1982) 203.
2. I.I. Bigi et al., *Phys. Lett.* **181B** (1986) 157.
3. For recent Standard Model update of the top quark width see M. Ježabek and J.H. Kühn, *Phys. Rev.* **D** (to be published).
4. J.H. Kühn and K.H. Streng, *Nucl. Phys.* **B198** (1982) 71; J.H. Kühn, *Nucl. Phys.* **B237** (1984) 77.
5. T. Arens and L.M. Sehgal, *Nucl. Phys.* **B393** (1993) 46.
6. Proceedings, "*Physics and Experiments with e^+e^- Linear Colliders*", Saariselkä 1991, R. Orava, P. Eerola and M. Nordberg eds., World Scientific, Singapore 1992.
7. Proceedings, *e^+e^- Collisions at 500 GeV: The Physics Potential*, Munich-Annecy-Hamburg 1991, P.M. Zerwas ed., DESY 92-123 A,B.
8. P. Zerwas, *Talk at LC92, Workshop on e^+e^- Linear Colliders*, Garmisch-Partenkirchen, Germany, 1992; DESY 93-001, Jan. 1993.
   J.H. Kühn and P.M. Zerwas, in *Advanced Series on Directions in High Energy Physics, "Heavy Flavours"*, eds. A.J. Buras and M. Lindner, World Scientific, Singapore 1992, p. 434.

9. I.F. Ginzburg et al., *Nucl. Inst. Meth.* **205** (1983) 47; *Nucl. Inst. Meth.* **219** (1984) 5.
10. D.L. Borden, D.A. Bauer and D.O. Caldwell, SLAC-PUB-5715, Jan. 1992; *Phys. Rev.* **D48** (1993) 4018.
11. V.I. Telnov in Ref.[9], p. 739.
12. F.R. Arutyunian and V.A. Tumanian, *Phys. Lett.* **4** (1963) 176; R.H. Milburn, *Phys. Rev. Lett.* **10** (1963) 75.
13. J.H. Kühn, E. Mirkes and J. Steegborn, *Z. Phys.* **57** (1993) 615.
14. I.I. Bigi, F. Gabbiani and V.A. Khoze, *Nucl. Phys.* **B406** (1993) 3.
15. O.J.P. Éboli et al., *Phys. Rev.* **D47** (1993) 1889.
16. V.S. Fadin and V.A. Khoze, *JETP Lett.* **46** (1987) 525; *Sov. J. Nucl. Phys.* **48** (1988) 309.
17. V.S. Fadin and V.A. Khoze, *Proc. 24th LNPI Winter School*, Leningrad 1989, **V. 1**, p. 3.
18. M.J. Strassler and M.E. Peskin, *Phys. Rev.* **D43** (1991) 1500.
19. V.S. Fadin and V.A. Khoze, *Sov. J. Nucl. Phys.* **53** (1991) 692; I.I. Bigi, V.S. Fadin and V.A. Khoze, *Nucl. Phys.* **B377** (1992) 461.
20. H. Murayama and Y. Sumino, *Phys. Rev.* **D47** (1992) 82.
21. Y. Sumino et al., *Phys. Rev.* **D47** (1992) 56; Y. Sumino, Thesis, UT-655 (1993).
22. M. Ježabek, J.H. Kühn and T. Teubner, *Z. Phys.* **C56** (1992) 653; M. Ježabek and T. Teubner, Karlsruhe preprint, TTP 93-11.
23. Meixner J., *Math.Z.* **36** (1933) 677.

where
$$\lambda_R = \frac{\lambda}{1 + \frac{N\lambda}{8\pi}t}$$
is just the renormalized singlet scattering amplitude. One sees that for a small number of final particles, $n \ll N$, leading order in $1/N$ result is given just by the tree–level formula where the coupling constant is replaced by the renormalized one. This is precisely the result of ref.[10] in the particular case of (2+1) dimensions.

However, eq.(18) is valid only when $n \ll N$. If the number of final particles is comparable to N, the effect of loops is obviously not a simple renormalization of the coupling constant. One finds from eq.(16) that the correction to the large–N result, eq.(17), is proportional to n^2/N. When the number of final particles becomes comparable to the number of their spieces, the $1/N$ expansion becomes unreliable. One can expect that the breakdown of the $1/N$ expansion is not a peculiar feature of (2+1) dimensions but holds also in (3+1)- and higher-dimensional theories.

5. So, we see that the renormalization group is a poweful mean for investigating the multiparticle amplitudes in (2+1) dimensional scalar field theory at and around the threshold. The exact formula for the amplitude, if ever be found, must incorporate the information obtained here by making use of the renormalization group equations.

The authors are indepted to M. Voloshin for bringing to our attention the possibility of summing leading logarithms in (2+1)d. We thank Yu. Makeenko, E. Mottola, L. Yaffe for discussions of the results.

References

1. J. M. Cornwall, *Phys. Lett.* **B243** (1990) 271.
2. H. Goldberg, *Phys. Lett.* **B246** (1990) 445.
3. M. B. Voloshin, *Nucl. Phys.* **B383** (1992) 233.
4. L. S. Brown, *Phys. Rev.* **D46** (1992) 4125.
5. E. N. Argyres, R. H. P. Kleiss and C. G. Papadopoulos, *Nucl. Phys.* **B391** (1993) 42.
6. M. B. Voloshin, *Phys. Rev.* **D47** (1993) 357.
7. B. H. Smith, *Phys. Rev.* **D47** (1993) 3518.
8. E. N. Argyres, R. H. P. Kleiss and C. G. Papadopoulos, *Phys. Lett.* **B308** (1993) 292.
9. A. S. Gorsky and M. B. Voloshin, *Phys. Rev.* **D48** (1993) 3843.
10. Yu. Makeenko, *Exact Multiparticle Amplitude at Threshold in Large–N Component ϕ^4 Theory*, Niels Bohr Inst. preprint NBI-HE-94-25 (1994).
11. C. Thorn, *Phys. Rev.* **D6** (1979) 39; see also K. Huang, *Quarks Leptons and Gauge Fields*, §10.8 (World Scientific, 1982).

Instanton–Antiinstanton pair induced Asymptotics of Perturbation Theory in QCD

P.G.Silvestrov
Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia

Abstract

The instanton–antiinstanton pair induced asymptotics of perturbation theory expansion for QCD correlators is considered. It is argued that though the true asymptotics is dominated by renormalon, the instanton-induced contribution may dominate in the intermediate asymptotics $n = 5 \div 15$.

Obtained asymptotic formulae are valid for $N_f \le N_c$. For $N_f = N_c$ the finite nonperturbative expression for instanton–antiinstanton contribution was also found.

At $q^2 < 0$ the imaginary part of correlators in the case $N_f = N_c$ is suppressed like $1/\log(q/\Lambda)$, but the present accuracy of instanton calculations allows to fix it unambiguously.

The series of corrections to the instanton induced asymptotics of the order of $\sim (\log(n)/n)^k$ is found.

1 Introduction. Renormalon–Instanton

The aim of this note is to try to show, what the important role may play the instanton for large order terms of perturbation theory in QCD. During last few years a lot of papers appears (see e.g. [1–7]) considering the asymptotic behaviour of perturbation theory series in QCD and QED. However the main attention was paid to the so called renormalon asymptotics. Two renormalons were considered, the ultraviolet and infrared. The usual form of the ultraviolet renormalon contribution to, for example, $R_{e^+e^-\to hadrons}$ perturbative expansion is

$$a_n \sim \left(-b\frac{\alpha_s}{4\pi}\right)^n n! \ , \tag{1}$$

where $b = 11/3N_c - 2/3N_f \approx 10$. In QCD the series (1) is sign alternating and at least the Borel sum of the series is well defined. The problems with summation of ultraviolet renormalon appears in QED, where all terms of series have the same sign.

Another kind of asymptotics is the infrared renormalon:

$$a_n \sim \left(\frac{b}{2}\frac{\alpha_s}{4\pi}\right)^n n! \ . \tag{2}$$

At large n this series turns to be much smaller than (1). The keen interest in infrared renormalon was caused by the fact that nobody knows how to sum the series (2). The Borel sum of this series is ill defined (up to arbitrary $\sim 1/q^4$ correction).

Both ultraviolet and infrared renormalons are associated with a certain chains of Feynman graphs. However since the works of L.N.Lipatov[8] another approach is developed for large order perturbation theory estimates. In this approach one has nothing to do with Feynman graphs, but tries to find the specific ("classical") large fluctuations in the functional space making the main contribution to the high order terms of perturbative expansion. The natural example of such important fluctuations in QCD is the instanton–antiinstanton pair,but up to now only in the paper of I.I.Balitsky[9] the instanton asymptotics for $R_{e^+e^-\to hadrons}$ was considered. In present work we try to clarify further the role of instanton–antiinstanton pair effects for QCD correlators, especially in the interesting case $N_f = N_c$ (the author of[9] at $N_f = N_c$ have not found the perturbative asymptotics and use the ambiguous regularization prescription to find nonperturbative corrections).

The generic form of instanton–induced asymptotics appears to be:

$$a_{nIA} \sim \left(\frac{\alpha_s}{4\pi}\right)^n (n+4N_c)! \ . \tag{3}$$

The overall numerical factor in a_{nIA} may also be sufficiently large. It is seen that though the renormalons (1,2) do dominate at very large n ($n > 15$), the instanton-induced contribution may dominate in the intermediate asymptotics $n = 5 \div 15$. If so, the pure renormalon behaviour (1) will hardly be observed in directly calculated terms of perturbation theory due to a strong competition with the instanton contribution.

Of course the exactly known $3 \div 4$ terms of perturbation theory series (for β–function of QCD or $R_{e^+e^-\to hadrons}$) are much smaller than the estimate (3) and the question at what number n the perturbative series could reach the full strength (3) is open now. Moreover the corrections which are formally $\sim 1/n$ at large n may also change the value of a_n in many orders at small n. For example

$$(n+4N_C)! = n^{4N_C} n! (1 + O(1/n)) \ . \tag{4}$$

In this paper we have also found (and summed up) the subseries of corrections to a_{nIA} (3), which behave like $(\log(n)/n)^k$ (see (24)).

Like for the infrared renormalon (2) all terms of the series (3) have the same sign. Nevertheless the problem of summation the series (3) seems not so hopeless as the summation of renormalon. Following G.'t Hooft[10] the author of[9] proposed to rewrite the integral over the instanton–antiinstanton pair in the Borel form by considering the action as a collective variable. The well–separated instanton–antiinstanton pair is responsible for the singular part of Borel function, while the ambiguous strongly interacting instanton and antiinstanton contribute to its smooth part. On the other hand

the best way to describe the smooth part of the Borel transform is to calculate exactly the few first terms of perturbative expansion. The divergent singular part of the Borel integral corresponds to almost non-interacting pseudoparticles. The accurate subtraction from the singularity of dilute gas contribution in the toy model (double well oscillator) allowed to find the finite nonperturbative instanton–antiinstanton contribution[11]. In QCD at $N_f = N_c$ the Borel integral diverges only logarithmically and the total nonperturbative contribution from instanton–antiinstanton pair may be found by cutting the instanton size at $\rho \ll 1/\Lambda$.

At $N_f = N_c$ the imaginary part of correlators at negative q^2 cancels in the one loop approximation. Nevertheless the invariance of the instanton contribution under the renormalization group transformations allows to find the imaginary parts.

2 The ansatz

The most popular puzzle for applying the high order estimates is the calculation of $R_{e^+e^-\to hadrons}$, which is connected with the Euclidean correlator of two electromagnetic currents

$$\Pi_{\mu\nu} = \int e^{iqx} d^4x < j_\mu(x) j_\nu(0) > \ , \tag{5}$$

where $j_\mu = \sum_{flavours} e_f \Psi_f^\dagger \gamma_\mu \Psi_f$. Calculation of instanton–induced contribution to (5) requires considerable algebraic efforts (e.g. the fermionic Green function in the pseudoparticle background must be used). Therefore for the sake of simplicity we will examine the correlator of two scalar currents

$$j(x) = \frac{3\alpha_s}{4\pi}\left[G^a_{\mu\nu}(x)\right]^2 \tag{6}$$

(the notations of[12] are used). This correlator, which may be useful for the gluebal physics, is simple enough so the reader can check almost all steps of the calculation. Moreover the correlator of two currents (6) reproduces all the interesting features of the correlator of electromagnetic currents.

As we have said above, the strongly interacting instanton and antiinstanton correspond to a smooth part of the Borel function. It was shown by Balitsky[9] that the instanton–antiinstanton configuration relevant for the large orders of perturbation theory is a small instanton inside the very large antiinstanton (or vice versa). The size of small instanton is regulated by the internal momentum of correlator (5) $q\rho_I \sim 1$. The size of antiinstanton (as well as the distance between the pseudoparticles $\rho_A \sim R$) becomes parametrically large as we consider the higher terms of perturbation theory.

Now let us specify the ansatz for the gauge fields. We are interesting in the instanton–antiinstanton interaction in the leading nontrivial approximation. Therefore the simple sum of instanton and antiinstanton may be used:

$$A_\mu = U_A A^A_\mu U^+_A + U_I A^I_\mu U^+_I \tag{7}$$

where U_A, U_I are the constant $SU(N)$ matrices. For small instanton the singular gauge seems to be preferable

$$A^I_\mu = \frac{\overline{\eta}_{\mu\nu}(x-x_I)_\nu \rho_I^2}{(x-x_I)^2((x-x_I)^2+\rho_I^2)} \ , \ \overline{\eta}_{\mu\nu} \equiv \tau^a \overline{\eta}^a_{\mu\nu} \ . \tag{8}$$

Before we add the antiinstanton to (8) the singularity at $x = x_I$ is pure gauge singularity. Therefore in order to remove the singularities from all the physical quantities one may choose A^A_μ in any regular gauge which satisfy the equality $A^A_\mu(x = x_I) = 0$. For example one can rotate the BPST antiinstanton

$$A^A_\mu = S\left[\frac{\overline{\eta}_{\mu\nu}(x-x_A)_\nu}{(x-x_A)^2+\rho_A^2}\right]S^+ + iS\partial_\mu S^+ \ , \tag{9}$$

$$R_\mu = (x_A - x_I)_\mu \ , \ S = \exp\left\{i\frac{\overline{\eta}_{\mu\nu}R_\nu}{R^2+\rho_A^2}(x-x_I)_\mu\right\} \ .$$

It is easy to show that any other smooth matrix function $S(x)$, which allows to cancel the antiinstanton field at $x = x_I$ will lead to the same correlator.

After direct calculation the classical action of the instanton–antiinstanton configuration may be found with the usual dipole–dipole interaction of pseudoparticles

$$S_{IA} = \frac{4\pi}{\alpha_s}\{1-\xi h\} \ \ , \ \xi = \frac{\rho_I^2\rho_A^2}{(R^2+\rho_A^2)^2} \ , \ \ h = 2|TrO|^2 - TrOO^+ \ , \tag{10}$$

and O is the upper left 2×2 corner of the matrix $U = U^+_A U_I$ (7).

Another part of the problem, extremely sensitive to the instanton–antiinstanton interaction, is the fermionic determinant. It may be shown, that for each flavor of massless fermions the two anomalously small eigenvalues of Dirac operator $\hat{D}$ appears

$$\lambda_{1,2} = \pm\frac{2\rho_I\rho_A}{(\rho_A^2+R^2)^{3/2}}|TrO| \ . \tag{11}$$

3 Calculation of correlator

After we have defined the gauge field configuration it is easy to write down the instanton–antiinstanton contribution to the correlator of two currents (6). Everywhere it is possible the notations of[9] are used.

$$\Pi(q) = \int e^{iqx} d^4x < j(x)j(0) >= \tag{12}$$
$$= 2\int j_I(x)j_I(0)e^{iqx}[4\xi^{3/2}|TrO|^2]^{N_f}\exp\left\{\frac{4\pi}{\alpha_s}\xi h\right\}\frac{d(\rho_I)}{\rho_I^5}\frac{d(\rho_A)}{\rho_A^5}dx\,dx_I\,dx_A\,d\rho_I\,d\rho_A\,dU \ ,$$

where

$$j_I(x) = \frac{36}{\pi^2}\frac{\rho^4}{((x-x_I)^2+\rho_I^2)^4} \ , \tag{13}$$

and the instanton density[13]

$$d(\rho) = A\left(\frac{2\pi}{\alpha_s(\rho)}\right)^{2N_c}\exp\left(-\frac{2\pi}{\alpha_s(\rho)}\right) \ . \tag{14}$$

The factor 2 in front of the integral in (12) accounts for the equal contribution from small antiinstanton and large instanton.

We will also use the well known two–loop formula

$$\frac{4\pi}{\alpha_s(q)} = b\log\left(\frac{q^2}{\Lambda^2}\right) + \frac{b'}{b}\log\left(\log\left(\frac{q^2}{\Lambda^2}\right)\right) + \dots \ . \tag{15}$$

In the most interesting case $N_f = N_c = 3$ $\ b = 9$ and $b' = 64$.

Before passing to the formal computations let us say a few words about the existence of integral (12) as a whole. The most ambiguous part of the problem is the integration over large antiinstanton coordinates ρ_A and $R = x_A - x_I$. There are two competing effects. The factor $d(\rho_A) \sim \rho_A^b$ tends to make the integral over ρ_A divergent. On the other hand, the almost zero fermionic modes (11) tends to suppress the large ρ_A and R contribution. If $N_f \le N_c$ the first effect dominates and the integral (12) diverges at large ρ_A. Nevertheless just in this case the well defined instanton induced asymptotics of perturbation theory may be extracted from (12). For nonperturbative calculation of the whole integral (12) at $N_f \le N_c$ the new physical income is necessary. Below we will show how to perform this integration for $N_f = N_c = 3$.

If the number of massless flavours is sufficiently large ($N_f > N_c$) the attraction due to fermionic zero modes prevails. As a result the approximation of almost noninteracting pseudoparticles breaks down ($\rho_A \sim R \sim \rho_I$) and the instantonic approach itself became ambiguous.

Formulae (14), (15) allow to extract the ρ_A dependent part from (12)

$$d(\rho_A) = \phi(\rho_I^2/\rho_A^2)\left(\frac{\rho_I}{\rho_A}\right)^b d(\rho_I) \ \ , \ \ \phi(x) = \left[1 + b\frac{\alpha_s}{4\pi}\log(x)\right]^{2N_c - \frac{b'}{2b}} \tag{16}$$

Everywhere below we suppose $\alpha_s = \alpha_s(q) \simeq \alpha_s(\rho_I)$. For calculation of leading perturbative asymptotics one may assume $\phi(x) \equiv 1$ (as it was done in[9]), but in order to calculate the nonperturbative value of correlator (12) the function $\phi(x)$ of the form (16) should be used. Moreover the corrections $\sim (\alpha_s \log(x))^k$ contained in the $\phi(x)$ leads to an important subseries of preasymptotic corrections $\sim (\log(n)/n)^k$ to the leading asymptotics of the perturbative expansion. Therefore below we shell use the function $\phi(x)$, though suppose that its argument is small enough $|\log(x)| \gg 1$.

Now we would like to integrate over ρ_A and $R = x_A - x_I$ for a fixed value of ξ

$$\int \phi(\rho_I^2/\rho_A^2)\rho_A^{b-5}\delta\left(\frac{\rho_I^2\rho_A^2}{(R^2+\rho_A^2)^2} - \xi\right) d\rho_A d^4R = \frac{\pi^2}{2(b-2)(b-1)}\frac{\rho_I^b}{\xi^{b/2+1}}\phi(\xi) \ . \qquad (17)$$

The part of (12) depending on ρ_I, x and x_I in the leading approximation over α_s gives:

$$\int j_I(x)j_I(0)e^{iqx}\rho^{2b-5}d^4x d^4x_I d\rho = 9\frac{2^{2b-3}}{q^{2b-4}}\frac{\Gamma(b+2)\Gamma^2(b)\Gamma(b-2)}{\Gamma(2b)} \ . \qquad (18)$$

After all the correlator (12) reads

$$\Pi(q) = Const\ q^4 d^2(1/q)\int |TrO|^{2N_f}\exp\left\{\frac{4\pi}{\alpha_s}\xi h\right\}\frac{\phi(\xi)d\xi}{\xi^{\frac{b-3N_f}{2}+1}}dU \ . \qquad (19)$$

The last step, which allows to rewrite the correlator in the form of Borel integral is to introduce the variable $t = 1 - \xi h$:

$$\begin{aligned}\Pi(q) \;=\;& 9\pi^2 2^{2(b+N_f-2N_c)-3}\frac{(b+1)!(b-1)!(b-3)!^2}{(2b-1)!}A^2q^4\left(\frac{4\pi}{\alpha_s(q)}\right)^{4N_c} \qquad (20)\\ & \left\{\int_0^1 dt\phi(1-t)(1-t)^{\frac{3N_f-b}{2}-1}e^{-\frac{4\pi}{\alpha_s}t} < |TrO|^{2N_f}\Theta(h)h^{\frac{b-3N_f}{2}} > + \right.\\ & \left. + \int_1^\infty dt\phi(t-1)(t-1)^{\frac{3N_f-b}{2}-1}e^{-\frac{4\pi}{\alpha_s}t} < |TrO|^{2N_f}\Theta(-h)(-h)^{\frac{b-3N_f}{2}} > \right\} \ .\end{aligned}$$

Here Θ–function equals 0 or 1 in accordance with sign of its argument and $< \ ... \ >$ means averaging over the orientation of the matrix U.

Essentially the same expression as (20) (except for the overall power of q and numerical factors and with ϕ replaced by 1) was found in[9] for the correlator of electromagnetic currents (5).

4 Analyzing the result

The first conclusion which is to be done is that the result (20) may be used only for $N_f \le N_c$ because the integral over orientations of the matrix U diverges at $h = 0$ if

$N_f > N_c$. This means that our method can not be applied without strong modification , for example, to calculation of $\Gamma_{Z_0 \to hadrons}$ ($N_c = 3$, $N_f = 5$).

Another interesting application for asymptotic formulae will be the case $N_f = N_c = 3$. The averaging over U for $SU(3)$ group gives

$$< |TrO|^6 > \ = \ \frac{7}{5} \ ,$$
$$< |TrO|^6\Theta(h) >= 1.37 \quad ; \quad < |TrO|^6\Theta(-h) >= 0.03 \ . \qquad (21)$$

Here the averages with Θ–function are the numerical estimates. The accuracy of the last (small) value is expected to be not worse than 20%.

Thus the final expression for instanton–antiinstanton pair contribution at $N_f = N_c = 3$ reads

$$\begin{aligned}\Pi(q) \;=\;& \frac{9}{\pi^2}e^{5/3}\frac{10!7!6!^2}{17!}q^4\left(\frac{4\pi}{\alpha_s}\right)^{12} \qquad (22)\\ & \left\{1.37\int_0^1 \phi(1-t)\frac{\exp(-\frac{4\pi}{\alpha_s}t)}{1-t}dt + 0.03\int_1^\infty \phi(t-1)\frac{\exp(-\frac{4\pi}{\alpha_s}t)}{t-1}dt\right\} \ .\end{aligned}$$

This expression is enough to find the leading asymptotics of perturbative expansion for $\Pi(q)$:

$$\Pi(q) = q^4\sum \Pi_n\left(\frac{4\pi}{\alpha_s}\right)^n \quad , \quad \Pi_n = \frac{9}{\pi^2}e^{5/3}\frac{10!7!6!^2}{17!}1.37(n+12)! \ . \qquad (23)$$

We see that instanton–antiinstanton induced contribution to the series of perturbation theory at $n \sim 10$ do has a huge enhancement $(n+12)!$ compared with $b^n n!$ for renormalon. The complete calculation of the $\sim 1/n$ corrections to the leading asymptotics requires a considerable efforts even in the simple toy model[11]. Nevertheless one may try to find any particular corrections which are enhanced in some way. The set of such enhanced $\sim \log(n)/n$ corrections appears from the expansion of $\phi(1-t)$ in (22) in powers of α_s. Let us remind, that physically with $\phi(x)$ one takes into account the running of the coupling $\alpha_s(\rho_A)$ which describes the large antiinstanton. It seems very unlikely if one can find any other effect which lead to such a large corrections $\sim (\log(n)/n)^k$. Under this assumption we find:

$$\Pi_n = 176\left[1 - 9\frac{\log(n)}{n}\right]^{22/9}(n+12)! \ . \qquad (24)$$

Of course this result may be used only if $n \gg \log(n)$. Though the expression (22) provides us with the asymptotic of perturbative series, both integrals in (22) diverge at $t = 1$. In the configuration space these divergences are related to the integration over almost noninteracting instanton and antiinstanton. Because the divergence is only logarithmic one can try to use the physical intuition in order to restrict the range of integration in (22). Anyway the natural cut for ρ_A seems to be $\rho_A \ll 1/\Lambda_{QCD}$, or in terms of t

$$|t-1|_{min} \sim \left(\frac{\rho_I}{\rho_{Amax}}\right)^2 \ll \frac{\Lambda^2}{q^2} \tag{25}$$

If so the nonperturbative part of (22) may be found explicitly (up to corrections $\sim \alpha_s$).

$$\Pi(q) = 129 q^4 \left(\frac{4\pi}{\alpha_s}\right)^{12} \left\{ 1.37 P \int_0^\infty \phi(|1-t|) \frac{\exp(-\frac{4\pi}{\alpha_s}t)}{1-t} dt + \frac{7}{155}\left(\frac{4\pi}{\alpha_s}\right) \exp(-\frac{4\pi}{\alpha_s}) \right\} . \tag{26}$$

Here P means the principal value integral. Let us stress that if one replaces $\phi(x)$ by 1 in (22), the nonperturbative part of (26) would be 31/9 times larger. Effectively the integration over t in (22),(26) may be thought as the integration over the size of large antiinstanton. The size of antiinstanton also may be determined through the coupling constant $\alpha_s(\rho_A)$ (the logarithmic scale $\alpha_s(\rho_A)^{-1} \sim -\log(\rho_A \Lambda)$). The remarkable feature of our result is that all the values of $\alpha_s(\rho_A)$ contribute to the nonperturbative part (26) in the whole range $\alpha_s(\rho_I) < \alpha_s(\rho_A) \ll 1$.

5 Imaginary part

In order to find the physical quantities such as the inclusive widths and cross-sections one have to consider the imaginary part of correlators, which appears after the analytic continuation to Minkovsky momentum $Im(\Pi(-q^2 + i\varepsilon))$. In the lowest order in α_s both the singular part and the nonperturbative corrections in (26) behave like $(\Lambda/q)^{2b}$. Thus for $N_f = N_c = 3$ ($b = 9$) the imaginary part cancels. In this case the second term of the expansion of α_s should be considered (15) and the analytic continuation of (26) gives:

$$\frac{1}{q^4} Im\big(\Pi(-q^2 + i\varepsilon)\big) = \tag{27}$$

$$= 2.4\,10^4 \left(\frac{4\pi}{\alpha_s}\right)^{11} P \int_0^\infty \psi(|1-t|) \frac{\exp(-\frac{4\pi}{\alpha_s}t)}{1-t} dt + 0.9\,10^3 \left(\frac{4\pi}{\alpha_s}\right)^{12} \exp(-\frac{4\pi}{\alpha_s}) ,$$

$$\psi(x) = \left(1 + \frac{43}{6}\frac{4\pi}{\alpha_s}\log(x)\right)\left(1 + 9\frac{4\pi}{\alpha_s}\log(x)\right)^{13/9}$$

Very similar expression may be found for $R_{e^+e^- \to hadrons}$. Although the considerable additional efforts are necessary for this calculation.

6 References

1. A.H. Mueller, *in* "QCD- Twenty Years Later", Aachen, (1992)

2. G. B. West, Phys. Rev. Lett. **67** (1991)1388.

3. L. S. Brown and L. G. Yaffe, Phys. Rev. **D45** (1992) R398; L. S. Brown, L. G. Yaffe, and C. Zhai, Phys. Rev. **D46** (1992) 4712.

4. V.I. Zakharov, Nucl. Phys., **B385** (1992) 452.

5. M. Beneke and V.I. Zakharov, Phys. Rev. Lett. **69** (1992) 2472.

6. G. Grunberg, Phys. Lett. **B304** (1993) 183.

7. D. J. Broadhurst, Z. Phys **C58** (1993) 339.

8. L.N. Lipatov Zh. Eksp. Teor. Fiz. **72** (1977) 411.

9. I.I. Balitsky, Phys. Lett., **B273** (1991) 282.

10. G.'t Hooft, in: *The why's of subnuclear physics*, (Erice, 1977), ed. A. Zichichi, (Plenum New York 1977).

11. S.V. Faleev and P.G. Silvestrov, *Instanton–antiinstanton interaction and asymptotics of perturbation theory expansion in double well oscillator*, Preprint BUDKERINP 94-24 (1994).

12. V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys., **B191** (1981) 301.

13. C. Bernard, Phys. Rev. **D 19** (1979) 308.