WHAT DO WE LEARN FROM ATOMIC PHYSICS ABOUT FUNDAMENTAL SYMMETRIES?

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Valuable information on interactions violating P- and T-invariance can be extracted from atomic experiments. The hypothesis of a large weak matrix element between single-particle states in heavy nuclei, ~ 100 eV, is ruled out by measurements of parity nonconservation in atoms. Experimental upper limit on the electric dipole moment of the ^{199}Hg atom strictly constrains parameters of CP-violation models. Upper limit on the T-odd, P-even admixture to nuclear forces is set: $\alpha_T < 10^{-11}$.

1 Is Large Weak Mixing in Heavy Nuclei Consistent with Atomic Experiment?

The scattering cross-sections of longitudinally polarized epithermal (1 – 1000 eV) neutrons from heavy nuclei at $p_{1/2}$ resonances have large longitudinal asymmetry. This parity nonconserving (PNC) correlation is the fractional difference of the resonance cross-sections for positive and negative neutron helicities. For a long time the most natural explanation of the effect was based on the statistical model of the compound nuclei. In fact, not only the explanation, but the very prediction of the huge magnitude of this asymmetry (together with pointing out the nuclei most suitable for the experiments) was made theoretically [1] on the basis of this model.

An obvious prediction of the statistical model is that after averaging over resonances, the asymmetry should vanish. However, few years ago it was discovered [2,3] that all seven asymmetries for ^{232}Th have the same, positive sign.

Most attempts [4,5,6,7] to explain a common sign, require the magnitude of the weak interaction matrix element, mixing opposite-parity nuclear levels, to be extremely large, ~ 100 eV. The same assumption seems to be necessary to explain unexpectedly large P-odd correlations observed in the Mössbauer transitions in ^{119}Sn and ^{57}Fe [8,9].

Such a large magnitude of the weak mixing can be checked in independent experiments. The recent proposal [10] is to measure PNC asymmetry in the M4 γ-transition between the states $1i\ 13/2^+$ and $2f\ 5/2^-$ in ^{207}Pb, which are believed to be predominantly single-particle ones. The experiment is in progress now [11]. Its sensitivity to the weak matrix element value is expected

to reach $5 - 13$ eV.

In [12] it was demonstrated that close upper limit on the weak mixing in ^{207}Pb follow already from the measurements of the PNC optical activity of atomic lead vapour [13]. The experiment was performed at the atomic M1 transition from the ground state $6p^2\ {}^3P_0$ to the excited one $6p^2\ {}^3P_1$. The nuclear spin of ^{207}Pb being $i = 1/2$, the total atomic angular momentum of the ground level is $F = 1/2$, and the upper level is split into two: $F' = 1/2,\ 3/2$. The following upper limit was established at the 95% confidence level for the relative magnitude of the nuclear-spin-dependent (NSD) part of the optical activity:

$$\frac{P_{NSD}}{P} < 0.02 \tag{1}$$

Here

$$P_{NSD} = P(F = 1/2 \to F' = 1/2) - P(F = 1/2 \to F' = 3/2)$$

and P is the main, nuclear-spin-independent, part of the PNC optical activity.

In heavy atoms the NSD P-odd effects were shown to be induced mainly by contact electromagnetic interaction of electrons with the anapole moment of a nucleus, which is its P-odd electromagnetic characteristic induced by PNC nuclear forces [14,15,16].

The electromagnetic PNC interaction of electrons with nuclear AM is of a contact type. It is conveniently characterized in the units of the Fermi weak interaction constant $G = 1.027 \times 10^{-5} m^{-2}$ (m is the proton mass) by a dimensionless constant κ.

To calculate, κ we present the effective P-odd potential for an external nucleon in a contact form in the spirit of the Landau-Migdal approach:

$$W = \frac{G}{\sqrt{2}} \frac{g}{2m} \sigma[\mathbf{p}\rho(r) + \rho(r)\mathbf{p}\,]. \tag{2}$$

Here σ and $\mathbf{p}$ are respectively spin and momentum operators of the valence nucleon, $\rho(r)$ is the density of nucleons in the core normalized by the condition $\int dr \rho(r) = A$ (the atomic number is assumed to be large, $A \gg 1$). A dimensionless constant g characterizes the strength of the P-odd nuclear interaction. It is an effective one and includes already the exchange terms for identical nucleons. This constant includes also additional suppression factors reflecting long-range and exchange nature of the P-odd one-meson exchange, as well as the short-range nucleon-nucleon repulsion.

Under some simplifying assumptions the anapole constant κ can be estimated for a heavy nucleus even analytically with the following result [15]:

$$\kappa = \frac{9}{10}\, g\, \frac{\alpha\mu}{m r_0} A^{2/3}. \tag{3}$$

Here μ is the outer nucleon magnetic moment, $r_0 = 1.2$ fm. The enhancement $\sim A^{2/3}$ compensates to a large extent the small fine structure constant $\alpha = 1/137$. That is why the nuclear AM is perhaps the main source of the nuclear-spin-dependent PNC effects in heavy atoms [14,15]. This formula predicts for lead

$$\kappa(^{207}Pb) = -0.08\, g_n. \quad (4)$$

More serious numerical calculations using a realistic description of the core density and the Woods-Saxon potential including the spin-orbit interaction, give [15,17]

$$\kappa(^{207}Pb) = -0.105\, g_n. \quad (5)$$

Recently it was demonstrated[18] that various many-body corrections, taken together, do not change essentially this result.

On the other hand, atomic calculations predict the magnitude of the NSD optical activity in lead at given κ with the accuracy about 20% [19,20]. At the experimental value of P obtained in [13] this prediction for the ratio (1) constitutes

$$0.023\, \kappa(^{207}Pb). \quad (6)$$

Combining the experimental result (1) with this theoretical one, we get the following upper limit for the anapole constant:

$$\kappa(^{207}Pb) < 1, \quad (7)$$

and for the effective neutron PNC constant:

$$g_n < 10. \quad (8)$$

Close upper limits on the effective constant g_p for an outer proton can be extracted from the optical experiments with atomic cesium [21] and thallium [22,23]. Less strict bound on g_p follows from the experiment [24] with bismuth.

A simple-minded estimate for the weak mixing matrix element, based on formula (2), leads to its following value:

$$2\, g \text{ eV}.$$

More sophisticated calculations based on the Woods-Saxon potential with the spin-orbit interaction, give for the concrete matrix element of interest for the proposed experiment with ^{207}Pb

$$\langle 3d\, 5/2^+ | W | 2f\, 5/2^- \rangle = 1.4\, g_n \text{ eV} \quad (9)$$

in a reasonable agreement with the results of other single-particle nuclear calculations cited in [10]. Combining (8) and (9), we get the following upper limit on this matrix element

$$\langle 3d\, 5/2^+ | W | 2f\, 5/2^- \rangle < 14 \text{ eV} \quad (10)$$

which is close to the expected accuracy of the experiment discussed in [10,11].

Nevertheless, this experiment would be obviously both interesting and informative. As to the hypothesis itself, according to which the value of the weak mixing matrix element is as high as 100 eV, it does not agree with the results of the atomic PNC experiments.

2 *CP*-Violation without Strangeness, or Electric Dipole Moments

Up to now *CP*-violating effects have been observed only in the decays of the neutral *K*-mesons, and their nature remains mysterious. At present the searches for the electric dipole moments (EDM) in neutron and atomic physics are practically the only other source of the information on *CP*-violation. Even without discovering the effects sought for, the neutron and atomic experiments have ruled out most models of *CP*-violation suggested to explain the effects in *K*-meson decays; in fact, one can argue that the neutron EDM experiments have ruled out more theoretical models than any other experiment in the history of physics. As to the mechanism of *CP*-violation incorporated into the standard model of electroweak interactions, which is most popular at present, its predictions for the EDMs are many orders of magnitude below the present experimental bounds.

But does it mean that the EDM experiments are of no serious interest for the elementary particle physics, are nothing else but mere exercises in precision spectroscopy? Just the opposite. It means that these experiments now, at the present level of accuracy are extremely sensitive to possible new physics beyond the standard model, physics to which the kaon decays are insensitive. Examples of this type will be discussed in more detail below.

2.1 *Phenomenological CP-Odd Nuclear Potential and the Schiff Moment*

The most strict upper limit on the EDM of anything is obtained in the recent experiment [25] with ^{199}Hg atom:

$$d(^{199}\text{Hg})/e < 9 \cdot 10^{-28} \text{ cm}. \quad (11)$$

The electronic shells of this diamagnetic atom are closed. Therefore, the effect is due to the nuclear EDM. To be more precise, because of the electrostatic screening, the atomic dipole moment (11) is induced not by the nuclear EDM itself, but by the so-called Schiff moment, which is proportional to the difference between the nuclear dipole moment form-factor and nuclear charge form-factor (see, e.g., book [16]).

It has been demonstrated in [26] that nuclear CP-odd electromagnetic moments, the Schiff moment included, are most efficiently induced by T- and P-odd nuclear forces, but not by the nucleon EDM.

So, let us start with discussing the T- and P-odd nucleon-nucleon potential. If we assume for simplicity the interaction to be local and limit ourselves to first-order terms in the nucleon velocities p/m_p, then to this approximation the most general form of the effective potential (in the spirit of the Landau-Migdal approach) is

$$W_{ab} = \frac{G}{\sqrt{2}}\frac{1}{2m}\ \{(\xi_{ab}\sigma_a - \xi_{ba}\sigma_b)\,\nabla\,\delta(\mathbf{r}_a - \mathbf{r}_b)$$
$$+\,\xi_{ab}[\sigma_a \times \sigma_b]\{(\mathbf{p}_a - \mathbf{p}_b)\,\delta(\mathbf{r}_a - \mathbf{r}_b) + \delta(\mathbf{r}_a - \mathbf{r}_b)\,(\mathbf{p}_a - \mathbf{p}_b)\}\} \quad (12)$$

The dimensionless constants ξ, characterizing the strength of the interaction in units of the Fermi constant G, are supplied with subscripts in order to distinguish between protons and neutrons. These are effective constants and include already the exchange terms for identical nucleons. In a detailed theory the constants should also include additional suppression factors reflecting long-range and exchange nature of the realistic interaction, as well as the short-range nucleon-nucleon repulsion. As it should be, the potential (12) is invariant under the Galilean transformations.

According to detailed numerical calculations with the Woods-Saxon potential, including the spin-orbit interaction, the Schiff moment of ^{199}Hg nucleus is [28]:

$$S(^{199}\mathrm{Hg})/e = -\,1.8\cdot 10^{-7}\,\xi_{np}\ \mathrm{fm}^3 \quad (13)$$

On the other hand, the Hartree-Fock calculations relate the mercury atomic dipole moment $d(^{199}\mathrm{Hg})$ to the nuclear Schiff moment $S(^{199}\mathrm{Hg})$ as follows [29,28]:

$$d(^{199}\mathrm{Hg}) = -\,3\cdot 10^{-18}\left(\frac{S(^{199}\mathrm{Hg})}{\mathrm{fm}^3}\right)\ \mathrm{cm}. \quad (14)$$

Combining relations (13) and (14) with the experimental result (11), we obtain the following upper limit on the CP-odd nuclear interaction:

$$\xi_{np} < 1.7\cdot 10^{-3}. \quad (15)$$

There are strong reasons to believe that the CP-odd NN interaction in nuclei is dominated by the π^0-exchange. This mechanism, considered first in [27], is singled out by the large value, 13.6, of the strong πNN constant and by the small π-meson mass. The derivative occurring at one of the vertices (in this case at the strong one) arises inevitably in the case of a P-odd interaction and does not lead to a relative suppression of the corresponding contribution. Finally, a charged particle exchange is suppressed as compared to neutral exchange in the nuclear shell model.

To simplify the discussion, we will confine ourselves to the limit of zero momentum transfer in the NN interaction. In this way the π^0-exchange induces an effective operator

$$\frac{G}{\sqrt{2}}\,\xi\,(\bar{N}\,i\gamma_5 N)(\bar{N}'N') \quad (16)$$

with a dimensionless constant

$$\xi = \frac{g_{\pi NN}\,\bar{g}_{\pi NN}\sqrt{2}}{G\,m_\pi^2}. \quad (17)$$

In the nonrelativistic reduction, expression (16) generates in the coordinate representation interaction (12). Then the result (15) can be formulated as an upper limit on the effective CP-odd neutral pion constant:

$$\bar{g}^0_{\pi NN} < 2\cdot 10^{-11}. \quad (18)$$

2.2 *Quark Chromoelectric Dipole Moment and Constraints on Models of CP-violation*

The last upper limit is most efficiently employed for constraining models of CP-violation in the following way. Let us consider the effective operator of the CP-odd quark-gluon interaction

$$H_c = \frac{1}{2}\,d^c\,\bar{q}\gamma_5\sigma_{\mu\nu}t^a q\,G^a_{\mu\nu}. \quad (19)$$

where $t^a = \lambda^a/2$ are the generators of the colour $SU(3)$ group. This is a close analogue of the EDM interaction with the electromagnetic field, so it is only natural to call the constant d^c in expression (19) the quark chromoelectric dipole moment (CEDM). The CP-odd $\pi^0 NN$ vertex generated by operator (19) transforms by the PCAC technique:

$$<\pi^0 N\,|\,g\,\bar{q}\gamma_5\sigma_{\mu\nu}t^a q\,G^a_{\mu\nu}\,|\,N> = \pm\,\frac{i\sqrt{2}}{f_\pi} < N\,|\,g\,\bar{q}\sigma_{\mu\nu}t^a q\,G^a_{\mu\nu}\,|\,N>; \quad (20)$$

the plus and minus in the lhs refer to the u- and d-quark CEDM, respectively. The QCD sum rule estimate for the last expectation value is $7\,\mathrm{GeV}^2$

[30]. Combining it with (18), we obtain the upper limit for the quark CEDM:

$$d^c < 2.4 \cdot 10^{-26}\,\text{cm}. \tag{21}$$

Let us consider now the model of spontaneous CP-violation in the Higgs sector. Its old version, with light Higgs bosons, has been ruled out by the experimental upper limit on the neutron EDM. We will consider therefore its more "natural" version, with heavy Higgs bosons. Of course, in this case the model is responsible for only a small portion of CP-violation in kaon decays. It would be new physics, a new source of CP-violation, supplemental to that generating the effects already observed.

The estimate for the quark CEDM obtained in the "natural" version of the model, under the assumption that the Higgs mass is about the same as that of the t-quark, is [31,32]:

$$d^c(q) \sim 3 \cdot 10^{-25}\,\text{cm}. \tag{22}$$

As it is pointed out in[33], this prediction is 12 times larger than the experimental upper limit (18).

Thus, very special assumptions concerning the parameters of the model of spontaneous CP-violation in the Higgs sector (such as large mass of the Higgs boson, small values of the CP-violating parameters, etc) are necessary to reconcile the predictions of this model with the experimental upper limits on the neutron electric dipole moment.

The same situation takes place in the supersymmetric $SO(10)$ model [34].

Let us mention here that the atomic experiment constrains the parameters of both model more strongly than the upper limit on the neutron EDM.

3 What Do We Know in Fact about T-odd, but P-even interactions?

Direct experimental information on the T-odd, P-even (TOPE) interactions is rather poor. Best limits on the relative magnitude of the corresponding admixtures to nuclear forces lie around 10^{-3} [35,36,37,38]. We will relate below all interactions to the Fermi weak interaction constant G. Since the nuclear scale of weak interactions is $Gm_\pi^2 \sim 2 \cdot 10^{-7}$, those limits can be formulated as $10^4\,G$. Most advanced experimental experimental proposals aim at improving these limits by three orders of magnitude.

Experimental information on TOPE electron-nucleon interaction is practically absent. In [39] an atomic experiment was suggested which can hopefully reach an accuracy about $\sim 3 \cdot 10^4 G$ (see also [40]). Higher accuracy is aimed at in the recent experimental proposal [41].

As to the TOPE electron-electron interaction, its possible manifestations in positronium were discussed in [42].

Much better upper limits on the TOPE interactions can be obtained as follows. Radiative corrections, due to the P-odd part of the electroweak interaction, transform the T-odd, but P-even fermion-fermion interaction into a T-odd and P-odd one. The experimental information about T-odd, P-odd effects is sufficiently rich to obtain in this way new limits, much better than direct ones, on the parameters of T-odd, P-even electron-electron, electron-nucleon and nucleon-nucleon interactions.

3.1 *Long-Distance Effects*

Let us point out first that the predictions of all modern renormalizable theories of CP-violation (and not only the standard model!) cannot exceed $(10^{-3} - 10^{-4})\,G$. The reason is obvious. Parity violation is an intrinsic property of all these models, and therefore T-odd, P-even effects should be roughly of the same order of magnitude as T-odd, P-odd ones.

An even stronger result was obtained recently in [43]. In any renormalizable theory, TOPE flavour conserving quark-quark interactions are absent to second order in the electroweak coupling. This conclusion holds to all orders both in the chromodynamic and electromagnetic interactions, if the θ-term is neglected.

Therefore, the investigations in this field are in fact the search for an essentially new physics, well beyond the modern theories. That is why we will describe the TOPE interactions phenomenologically, using effective quark-quark operators.

There is only one (up to the interchange $1 \leftrightarrow 2$) such operator [46], which can be presented as

$$\frac{G}{\sqrt{2}}\frac{q_1}{2m}\,\bar{\psi}_1 i\gamma_5\sigma_{\mu\nu}(p_1' - p_1)_\nu \psi_1 \bar{\psi}_2\gamma_\mu\gamma_5\psi_2. \tag{23}$$

We measure the interaction discussed in the units of the Fermi weak interaction connstant G; the choice of m as the necessary dimensional parameter being also a matter of convention; q_1 is dimensionless.

A hint at the kind of limits that can be obtained by combining this interaction with the P-odd one, is given by the following argument, close in spirit to the corresponding estimates from [44,45]. (From now on we sacrifice the purity of style, and use freely the results of neutron experiments in line with atomic ones.) Let us consider the contribution to the neutron EDM from the combined action of the usual P-odd, T-even weak interaction and the discussed

T-odd and P-even interaction, the strength of the latter being q times smaller than that of the previous one. The contribution constitutes obviously

$$d(n)/e \sim \frac{1}{m_p}(Gm_\pi^2)^2 q \qquad (24)$$

From the comparison with the experimental upper limit for the neutron EDM [47,48]

$$d(n)/e < 10^{-25}\,\mathrm{cm}, \qquad (25)$$

we obtain the limit $q < 10^2$, which is about two orders of magnitude better than the direct limits mentioned above. This estimate is obviously of a very crude nature. In particular, the dipole moment arises here at least in one-loop approximation which leads to a small geometrical factor. So, it is better perhaps to accept for this limit a more cautious estimate

$$q < 10^2 - 10^3, \qquad (26)$$

which can be otherwise formulated as an upper limit for the relative magnitude α_T of the TOPE admixture to nuclear forces:

$$\alpha_T < 10^{-5} - 10^{-4}. \qquad (27)$$

Analogous estimates for the electron-nucleon interaction were made in [49,50] (as cited in [46]) and [51]. Let us mention also recent elaborate investigations [52,53,54,55] of the long-distance interplay between TOPE and usual P-odd interactions in the hadronic sector. They are of a certain interest for the theoretical nuclear physics, but none of them resulted in a serious improvement over the simple-minded estimate (26).

Better limits on TOPE effects are obtained by a simple and elegant argument presented in a recent paper [56]. By dimensional reasons, the TOPE 4-fermion effective interaction of dimension seven can be written as

$$\frac{C_7}{\Lambda^3}\bar{\psi}_1 i\gamma_5\sigma_{\mu\nu}(p_1' - p_1)_\nu \psi_1 \bar{\psi}_2\gamma_\mu\gamma_5\psi_2. \qquad (28)$$

It is only natural to assume that the momentum scale Λ in this operator exceeds that of the electroweak theory, i.e.,

$$\Lambda > 100\,\mathrm{GeV}.$$

As to the dimensionless number C_7, it is natural to assume that it is about unity. Then the dimensional estimate for the magnitude of TOPE effects on the hadronic scale of momenta $p \sim 1$ GeV, is

$$\left(\frac{p}{\Lambda}\right)^3 < 10^{-6}. \qquad (29)$$

This line of reasoning applies not only to 4-fermion operators, but, for instance, to a quark-gluon-photon operator of the form [56]

$$\frac{C_7'}{\Lambda^3}\bar{q}\sigma_{\mu\nu}t^a q\, G^a_{\mu\rho}F_{\nu\rho}. \qquad (30)$$

Let us mention here also the TOPE photon-fermion scattering amplitudes. They belong to higher dimensions (starting from 10) [57], and are constrained more strongly in this way.

3.2 *TOPE Fermion-Fermion Interactions. One-Loop Approach*

A serious advance in the problem is due to the observation that the electroweak corrections to TOPE fermion-fermion operators are controlled mainly not by the large-distance effects, but by short-distance ones. Therefore, they are of the order α/π (up to some chiral suppression factor which is quite essential), but not of the order Gm_π^2 [46].

We will concentrate here and below on the corrections due to the Z-boson exchange. These can be calculated self-consistently in the sense that the result is independent of the choice of the gauge for the Z-boson propagator.

A consistent, gauge-independent calculation of the W-boson exchange contribution to the induced T- and P-odd amplitudes is much more model-dependent and will not be discussed here in detail. It can be expected however to be even larger than that of the Z-exchange, due to small numerical values of the neutral weak charges. These small values are responsible in particular for the well-known relative suppression of the neutral-current cross-sections as compared to the charged-current ones. So, the Z-exchange contribution serves as an estimate from below for the effects discussed. As to the Higgs boson exchange, in the standard model it conserves parity and is therefore of no interest to us.

The result of the transformation of this effective operator into T- and P-odd one is

$$\frac{G}{\sqrt{2}}\frac{\alpha}{3\pi}\log\frac{\Lambda^2}{M^2}\frac{q_1}{m}\{\,2m_2v_1\,[\,3a_2\bar{\psi}_1\psi_1\,\bar{\psi}_2 i\gamma_5\,\psi_2 - (a_1+a_2)\bar{\psi}_1\psi_1\bar{\psi}_2 i\gamma_5\psi_2\,]$$

$$-a_1v_2\,[\,m_2\bar{\psi}_1 i\gamma_5\sigma_{\mu\nu}\psi_1\,\bar{\psi}_2\sigma_{\mu\nu}\psi_2 - \bar{\psi}_1 i\gamma_5\sigma_{\mu\nu}(p_1'-p_1)_\nu\psi_1\,\bar{\psi}_2\gamma_\mu\psi_2\,]\,\}. \qquad (31)$$

Here M is the Z-boson mass. The dependence of the result on the cut-off parameter Λ is due to nonrenormalizability of the TOPE interaction. But trying to be as conservative as possible in our numerical estimates, we will assume the log to be of the order of unity. $m_{1,2}$ are the masses of the first and second fermions, respectively. $v_{1,2}$ and $a_{1,2}$ are their weak neutral vector and axial charges. In particular, for the electron, u-, and d-quarks they are:

$$v_e = -\frac{1}{2}(1 - 4\sin^2\theta) \approx -0.04, \quad a_e = -\tfrac{1}{2},$$
$$v_u = \frac{1}{2}(1 - \frac{8}{3}\sin^2\theta) \approx \frac{1}{6}, \quad a_u = \tfrac{1}{2},$$
$$v_d = -\frac{1}{2}(1 - \frac{4}{3}\sin^2\theta) \approx -\frac{1}{3}, \quad a_d = -\tfrac{1}{2}. \tag{32}$$

Here θ is the electroweak mixing angle,

$$\sin^2\theta \approx 0.23.$$

Let us perform the concrete estimates for the electron-nucleon interaction. In this case the induced electron-quark operator is

$$\frac{G}{\sqrt{2}}\frac{\alpha}{3\pi}\log\frac{\Lambda^2}{M^2}\, v\,\{\, q_{eq}\,[\frac{m}{m}\,\bar{e}\, i\gamma_5\sigma_{\mu\nu} e\, \bar{q}\,\sigma_{\mu\nu} q - \frac{1}{2m}\,\bar{e}\, i\gamma_5\sigma_{\mu\nu}(p_1' - p_1)_\nu e\, \bar{q}\,\gamma_\mu q\,]$$
$$+ q_{qe}\,\frac{m_e}{m}\,[\,(1-2a)\,\bar{e}\, i\gamma_5 e\,\bar{q}\, q \;-\; 3\,\bar{e}\, e\,\bar{q}\, i\gamma_5 q\,]\,\}. \tag{33}$$

Here m and m_e are the quark and electron masses, v and a are the quark vector and axial charges, q_e and q are the dimensionless constants in the T-odd, P-even operators with the explicit momenta belonging to electrons and quarks, respectively. We have neglected here the contribution proportional to the electron vector charge v_e which is numerically small (see (32)). Operator (33) should be summed over u- and d-quarks, and its expectation value should be taken first over a nucleon and then over a nucleus.

In the static approximation for nucleons, the only term in (33) that depends on both electron and nucleon spin is $\gamma_5\sigma_{\mu\nu} \times \sigma_{\mu\nu}$. The dimensional estimate for the nucleon expectation value of the operator $\bar{q}\sigma_{\mu\nu}q$ is $\bar{N}\sigma_{\mu\nu}N$. Then the dimensionless effective constant of the T- and P-odd tensor electron-neutron interaction is

$$k_2 = \frac{\alpha}{3\pi}\log\frac{\Lambda^2}{M^2}\frac{m_d}{2m}\, v_d q_{eq} \sim 10^{-6} q_{eq}. \tag{34}$$

The upper limit on the constant k_2 obtained in [25] leads to the following result:

$$q_{eq} < 10^{-2}. \tag{35}$$

For the d-quark mass we assume here the value $m_d = 7$ MeV.

Upper limits on the level 0.1 – 1 can be extracted in this way from atomic experiments for the constants q_{qe} referring to the electron-quark interaction with the derivatives in the quark vertex.

The analogous estimates for various quark-quark constants, as derived from the results both for the atomic and neutron EDMs, lead to the upper limits

$$q_{qq} < 1. \tag{36}$$

We do not go here into details since in the next subsection much better upper limits will be obtained for all these constants.

3.3 TOPE Fermion-Fermion Interactions. Two-Loop Approach

The idea of the next improvement of the upper limits on the constants discussed [58,59], can be conveniently explained for the case of hadrons. The previous advance from (26) to (36) was obtained by going over from the long-distance effects of the usual weak interaction to the short-distance ones. It allowed us to get rid of one small factor Gm_π^2 in formula (24), trading it for α/π with some extra chiral suppression. But can we get rid of the second factor Gm_π^2 in that formula? The answer is: yes, we can. Up to now we computed one-loop radiative correction which transformed the interaction discussed into T- and P-odd effective operator. Then we estimated in fact the long-distance contribution of this operator to the neutron dipole moment. Now we are going to make the next step: to calculate a completely short-distance two-loop contribution of the TOPE interaction times the weak interaction, directly to the quark EDM. And the latter at least does not significantly exceed the neutron dipole moment. We gain in this way even more than at the first step since now there is no more chiral suppression factor m_q/m_p.

To regularize, at least partly, the related Feynman integrals, it is convenient to introduce explicitly an axial boson of mass μ mediating the TOPE interaction. It was pointed out long ago [60] that the amplitudes (23) could arise through the exchange by a neutral pseudovector boson if its vertices contain the mixture of the "normal" axial operator $\gamma_\mu\gamma_5$ and the "anomalous" one $i\gamma_5\sigma_{\mu\nu}(p'-p)_\nu$ of opposite CP-parity. We consider however the introduction of this boson only as a convenient way to soften the ultraviolet divergence of the integral from the quadratic to logarithmic one, without any serious discussion of this particle by itself. Thus the TOPE fermion-fermion amplitude can be presented as

$$\frac{4\pi\beta}{\mu^2 - k^2}\frac{1}{2m}\,\bar{\psi}_1 i\gamma_5\sigma_{\mu\nu}(p_1' - p_1)_\nu\psi_1\;\bar{\psi}_2\gamma_\mu\gamma_5\psi_2. \tag{37}$$

Here β is the dimensionless coupling constant analogous to $\alpha = 1/137$ in QED. Consider now the contribution of the two-loop diagram 1a to the fermion EDM. Here the dashed line represents the propagator of the axial boson X.

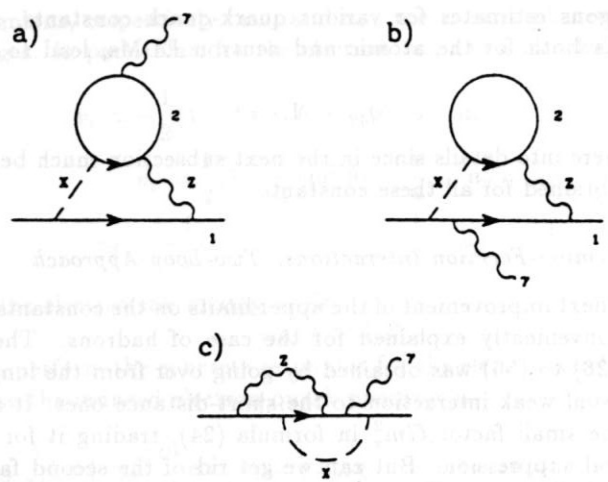

Figure 1: Two-loop diagrams.

The Z-boson exchange (the wavy line) introduces the parity nonconservation necessary to induce the EDM.

It can be easily seen that the contribution proportional to the mass of fermion 1, is small, $\sim G$. Therefore, this fermion will be taken as massless. Since the dipole moment interaction changes the fermion chirality, the lower vertex of the X-line should be $i\gamma_5\sigma_{\mu\lambda}k_\lambda$ and the upper one, correspondingly, $\gamma_\mu\gamma_5$. Then according to the Furry theorem for the fermion loop, the upper vertex of the Z line is γ_ν, and the lower one, respectively, $\gamma_\nu\gamma_5$. To simplify the calculations and result, we will neglect the mass of the fermion propagating in the upper loop as well (though for the heavy t-quark there are no special reasons to do so). Then the expression for the fermion loop is [61]

$$\frac{i}{8\pi^2}F_{\alpha\beta}[\epsilon_{\alpha\beta\mu\nu} + \frac{1}{k^2}\epsilon_{\alpha\beta\kappa\lambda}k_\kappa(\delta_{\mu\lambda}k_\nu + \delta_{\nu\lambda}k_\mu)]; \tag{38}$$

here $F_{\alpha\beta}$ is the strength of the external electromagnetic field.

When considering the lower (Compton) block of diagram 1a, one should also include the contact term

$$\frac{e}{2\sin\theta\cos\theta}\,2\,a\,i\,\sigma_{\mu\nu}, \tag{39}$$

originating from the vertex

$$i\gamma_5\sigma_{\mu\lambda}(p'-p)_\lambda$$

via the substitution

$$p_\mu\psi \rightarrow [p_\mu - \frac{e}{2\sin\theta\cos\theta}(v + a\gamma_5)\,Z_\mu]\,\psi$$

which makes this vertex gauge invariant with respect to the Z-field. In particular, the inclusion of the contact vertex (39) into the XZ Compton scattering amplitude makes the result of the calculation independent of the term $k_\nu k_\sigma/M^2$ in the propagator of the Z-boson. Simple calculations give now the following result for this contribution to the EDM d of fermion 1:

$$\frac{d}{e} = -\frac{\alpha\,\beta\,Q_2 a_1 v_2}{3\pi^2 m}\log\frac{\Lambda^2}{M_>^2}. \tag{40}$$

This formula refers to the general case in which fermion 2 propagating in the loop differs from the fermion 1 propagating in the lower line. In particular, a_1 is the axial weak charge of the first fermion, v_2 is the vector weak charge of the second fermion, and Q_2 is its electric charge in the units of e. The logarithmic dependence on the cut-off parameter Λ is the result of the nonrenormalizable coupling of the X-boson to the vertex with derivatives $i\gamma_5\sigma_{\mu\rho}k_\rho$. Although the result (40) is presented with logarithmic accuracy, in all our numerical estimates we will conservatively assume $\log\Lambda^2/M_>^2$ to be of the order of unity ($M_>$ is the largest of the masses μ and M). Let us emphasize again the gauge invariance of this result with respect to the Z-field. As to its gauge invariance with respect to the electromagnetic field, it is self-evident from expression (38).

The contribution of diagram 1b

$$\frac{d_b}{e} = \frac{\alpha\,\beta\,Q_1 a_2 v_1}{36\pi^2 m}\log^2\frac{\Lambda^2}{M_>^2} \tag{41}$$

is much smaller numerically and can be neglected.

In the case of identical fermions there is also the contribution to the EDM of diagram 1c, but we will neglect it in our estimates with the expectation that the result will not be grossly affected. One might expect here that in the local limit, $\mu \rightarrow \infty$, the effect for identical fermions 1 and 2 should vanish, which would correspond to exact cancellation of diagrams 1a and 1c. However, even for $\mu \gg M$ we can, to logarithmic accuracy, restrict the integration over k to $k \gg \mu$, where the TOPE interaction of identical fermions is in no way a local one and therefore no cancellation takes place.

A consistent, gauge-independent calculation of the W-boson exchange contribution to the induced EDM is again much more model-dependent and we will not discuss it. The same arguments as in the previous section lead us to expect that the Z-boson contribution alone serves as a conservative estimate for the induced EDM.

We will start the application of the general result (40) from the case of the electron-electron TOPE interaction. Substituting into formula (40) the numerical values (32) for a_e, v_e, as well as $Q_e = -1$, we get

$$\frac{d_e}{e} \sim \beta_{ee} \cdot 10^{-19} \text{ cm}. \tag{42}$$

The experimental upper limit on the electron EDM[62,63]

$$\frac{d_e}{e} < 10^{-26} \text{ cm}. \tag{43}$$

leads to the following result for the constant β_{ee} of the electron-electron TOPE interaction:

$$\beta_{ee} < 10^{-7}. \tag{44}$$

In the same way we can get very strict upper limits on the electron-nucleon and nucleon-nucleon TOPE interactions. The axial charge of a fermion is always (up to a sign) 1/2 and for any quark, independently of its sort, the product Qv is numerically close to 1/9. Then, using the experimental upper limit (25) on the neutron EDM and assuming for dimensional reasons that the neutron dipole moment induced by the quark EDM is of about the same magnitude as the latter, we get for β_{qe}, the TOPE quark-electron interaction constant with derivatives in the quark vertex, the limit

$$\beta_{qe} < 10^{-6}. \tag{45}$$

For another electron-quark constant β_{eq} (with the derivative in the electron vertex) the constraint (43) on the electron EDM gives the upper limit

$$\beta_{eq} < 3 \cdot 10^{-8}. \tag{46}$$

For all quark-quark constants β_{qq} the limit (25) on the neutron EDM gives

$$\beta_{qq} < 3 \cdot 10^{-7}. \tag{47}$$

The latter refers as well to the "coloured" TOPE interaction, with the $SU(3)$ generators t^a in each vertex. In this case the external field on diagram 1 should not be electromagnetic, but rather a gluon one. Again, for dimensional reasons, the neutron EDM induced in this way should be of the same order of magnitude as the chromoelectric dipole moment described by this diagram.

The constants β introduced here, are related as follows to the constants q used above:

$$\frac{4\pi\beta}{\mu^2} = \frac{G}{\sqrt{2}} q, \tag{48}$$

or

$$q = 4\pi\beta\sqrt{2}\left(\frac{m_p}{\mu}\right)^2 \cdot 10^5 = 1.8 \cdot 10^6 \left(\frac{m_p}{\mu}\right)^2 \beta. \tag{49}$$

Thus, the upper limits corresponding to (45), (46) and (47) are

$$q_{qe} < 2\left(\frac{m_p}{\mu}\right)^2, \quad q_{eq} < 0.05\left(\frac{m_p}{\mu}\right)^2, \quad q_{qq} < 0.5\left(\frac{m_p}{\mu}\right)^2, \tag{50}$$

respectively.

The upper limits on the constants q in the interval $q < 10^{-2} - 1$, were derived in the previous section under the assumption $\mu \geq M \sim 100\, m_p$. Under the same assumption, the limits we obtain here are much better:

$$q_{qe} < 10^{-4}, \quad q_{eq} < 10^{-5}, \quad q_{qq} < 10^{-4}. \tag{51}$$

Let us come back to the explanation of this gain. In the transition from the effective four-fermion T- and P-odd operators obtained in the previous section to the neutron EDM, we used the usual hadronic scale of 1 GeV. But here the transition takes place on a much higher scale of 100 GeV.

3.4 *Electron-Nucleon and Nucleon-Nucleon Interactions. Conclusions*

Now, having obtained the above limits on the TOPE electron-quark and quark-quark interactions, what can we say about the corresponding electron-nucleon and nucleon-nucleon interactions?

The answer for the electron-nucleon interaction is quite straightforward one. Simple dimensional arguments lead to the following estimates for the nucleon expectation values of the relevant quark operators (the second one has been already mentioned):

$$\langle N|\,\bar{q}\gamma_\mu\gamma_5 q\,|N\rangle \sim \bar{N}\gamma_\mu\gamma_5 N, \quad \langle N|\,\bar{q}\gamma_5\sigma_{\mu\nu}q\,|N > \sim \bar{N}\gamma_5\sigma_{\mu\nu}N. \tag{52}$$

Therefore, the limits (51) for $q_{qe,eq}$ are readily translated into those for the constants of TOPE electron-nucleon interactions:

$$q_{Ne} < 10^{-4}, \quad q_{eN} < 10^{-5}. \tag{53}$$

Let us address now the nucleon-nucleon interactions. Note first of all that in contrast to T- and P-odd nuclear forces, TOPE ones cannot be mediated by π^0-meson exchange [64]. Indeed, looking at the classification of the particle-antiparticle states in the annihilation channel presented in Section 2.3, we see that at $j = 0$ the state 2 just does not exist.

The absence of this exchange can be attributed also to vanishing of a TOPE $\pi^0 NN$ vertex. As to the TOPE $\pi^\pm NN$ coupling, being hermitian, it should be written as

$$\bar{p}\gamma_5 n\,\pi^+ - \bar{n}\gamma_5 p\,\pi^-. \tag{54}$$

This coupling does not lead to TOPE NN scattering amplitude in the one-boson exchange approximation since after the interchange of this vertex and of the strong one, the corresponding diagrams cancel. TOPE one-boson exchange starts therefore with vector and pseudovector bosons. Being mediated by heavier particles, the effective NN interaction is further suppressed as compared to simple estimates.

On the other hand, it follows already from general formulae (23) that a TOPE nucleon-nucleon scattering amplitude contains an extra power of p/m as compared to the usual P-odd weak interaction. This means an extra suppression of roughly by an order of magnitude as compared with the mentioned naive estimate $G\,m_\pi^2\,q$.

Thus, even taking into account all the uncertainties of our estimates, one can state that the relative strength of the TOPE nuclear forces does not exceed $10^{-4} G m_\pi^2$, or

$$\alpha_T < 10^{-11}. \tag{55}$$

Various objections to this conclusion have died away, being withdrawn implicitly or explicitly. The only one still worth mentioning is the possibility that the contributions of various particles to the fermion loop in diagram 1a cancel out. However, this possibility emphasized in [53] refers obviously to any estimates (including those of [53]) made in the absense of a reliable theory. As to the analogy with the well-known GIM mechanism mentioned in [53], it does not seem relevant here. The reasons for the GIM cancellation in the standard model are well-known, but they also seem irrelevant to the issue of nonrenormalizable TOPE interactions. On the other hand, too strong cancellation here looks especially unlikely due to the large mass of the t-quark. Moreover, a cancellation at the level of 10^{-6}, which is required to change the upper limit 10^{-11}, set in [59], to 10^{-5}, as discussed in [53], seems quite improbable.

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