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ЭЛЕМЕНТАРНЫЕ ЧАСТИЦЫ И ПОЛЯ

$SO(8)$ COLOR AS POSSIBLE ORIGIN OF GENERATIONS

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A possible connection between the existence of three quark–lepton generations and the triality property of $SO(8)$ group (the equality between 8-dimensional vectors and spinors) is investigated.

1. INTRODUCTION

One of the most striking features of the quark–lepton spectrum is its cloning property: μ and τ families seem to be just heavy copies of electron family. Actually, we have two questions to be answered: what is an origin of family formation and how many generations do exist? Recent LEP data [1] strongly suggest three quark–lepton generations. Although Calabi–Yau compactifications of the heterotic string model can lead to three generations [2], there are many such Calabi–Yau manifolds, and additional assumptions are needed to argue why the number three is preferred [3].

There is another well-known example of particle cloning (doubling of states): the existence of antiparticles. Algebraically the charge conjugation operator defines an (outer) automorphism of underlying symmetry group [4, 5] and reflects the symmetry of the corresponding Dynkin diagram. We can think that the observed triplication of states can have the same origin.

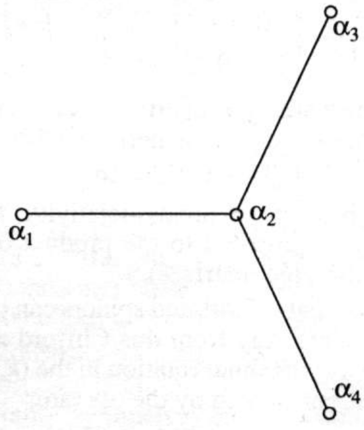
The most symmetric Dynkin diagram is associated with $SO(8)$ group. So it is the richest in automorphisms and, if $SO(8)$ plays some dynamical role, we can hope that its greatly symmetrical internal structure naturally leads to the desired multiplication of states in elementary particle spectrum. What follows is an elaboration of this idea.

Although the relevant mathematical properties of $SO(8)$ are known for a long time [6], they have not been discussed in the context of the generation problem, to my knowledge.

2. PECULIARITIES OF THE $SO(8)$ GROUP

It is well known [7, 8] that the structure of a simple Lie algebra is uniquely defined by the length and angle relations among simple roots. This information is compactly represented by the Dynkin diagram. On such a diagram each simple root is depicted by a small circle, which is made black, if the root is a short one. Each pair of vertices on the Dynkin diagram is connected by lines, the number of which equals $4\cos^2\varphi$, φ being the angle between the corresponding simple roots.

The main classification theorem for simple Lie algebras states that there exist only four infinite series and five exceptional algebras [7]. Among them D_4, the Lie algebra of the $SO(8)$ group, really has the most symmetric Dynkin diagram:



Actually, only the symmetry with regard to the cyclic permutations of the (α_1, α_3, α_4) simple roots (which we call triality symmetry) is new, because the symmetry with regard to the interchange $\alpha_3 \longleftrightarrow \alpha_4$ (last two simple roots) is shared by other D_n Lie algebras also.

Due to this triality symmetry, $8_v = (1000)$, $8_c = (0010)$ and $8_s = (0001)$ basic irreps (($a_1, a_2, \ldots, a_r$) being the highest weight in the Dynkin coordinates [8]) all have the same dimensionality 8 – the remarkable fact valid only for the D_4 Lie algebra. The corresponding highest weights are connected by the above mentioned triality symmetry. For other orthogonal groups (10 … 0) is a vector representation, (00 … 01) – a first kind spinor and (00 … 10) – a second kind spinor. So there is no intrinsic difference between (complex) vectors and spinors in the 8 - dimensional space [9], which object is vector and which ones are spinors depend simply on how we have enumerated symmetric simple roots and so is a mere convention.

It is tempting to use this peculiarity of the $SO(8)$ group to justify the observed triplication of the quark–lepton degrees of freedom. This possible connection between generations and $SO(8)$ can be formulated most naturally in terms of octonions.

3. OCTONIONS AND TRIALITY

Eight-dimensional vectors and spinors can be realized through octonions [10, 11], which can be viewed as a generalization of the complex numbers: instead of one imaginary unit we have seven imaginary units $e_A^2 = -1$, $A = 1 - 7$, in the octonionic algebra. The multiplication table between them can be found in [11].

The octonion algebra is an alternative algebra (but not associative). This means that the associator $(x, y, z) = x(yz) - (xy)z$ is a skew symmetric function of the x, y, z octonions.

The conjugate octonion $\bar{q}$ and the scalar product of octonions are defined as

$$\bar{q} = q_0 - q_A e_A \quad (p, q) = \frac{1}{2}(p\bar{q} + q\bar{p}) = (\bar{p}, \bar{q}). \quad (1)$$

Let us consider eight linear operators Γ_m, $m = 0 - 7$, acting in the 16-dimensional bioctonionic space:

$$\Gamma_m \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} 0 & e_m \\ \bar{e}_m & 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} e_m q_2 \\ \bar{e}_m q_1 \end{pmatrix}. \quad (2)$$

Using the alternativity property of octonions, it can be tested that these operators generate a Clifford algebra

$$\Gamma_m \Gamma_n + \Gamma_n \Gamma_m = 2\delta_{mn}.$$

(Note, that, because of nonassociativity, the operator product is not equivalent to the product of the corresponding octonionic matrices).

The 8-dimensional vectors and spinors can be constructed in the standard way from this Clifford algebra [12]. Namely, the infinitesimal rotation in the (k, l)- plane by an angle θ is represented by the operator

$$R_{kl} = 1 + \frac{1}{2}\theta \Gamma_k \Gamma_l,$$

and the transformation law for the (bi)spinor $\Psi = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ is $\Psi' = R_{kl}\Psi$.

For Γ_m given by (2) the upper and lower octonionic components of Ψ transform independently under the 8-dimensional rotations

$$q'_1 = q_1 + \frac{1}{2}\theta e_k(\bar{e}_l q_1) \equiv q_1 + \theta F_{kl}(q_1),$$
$$q'_2 = q_2 + \frac{1}{2}\theta \bar{e}_k(e_l q_2) \equiv q_1 + \theta C_{kl}(q_2), \quad (3)$$

while the vector transformation law can be represented in the form

$$x' = x + \theta\{e_k(e_l, x) - e_l(e_k, x)\} \equiv x + G_{kl}(x)\theta. \quad (4)$$

One more manifestation of the equality between 8-dimensional vectors and spinors is the fact [9] that each spinor transformation from (3) can be represented as a sum of four vector rotations:

$$F_{0A} = \frac{1}{2}(G_{0A} + G_{A_1B_1} + G_{A_2B_2} + G_{A_3B_3}), \quad (5)$$

where A_i, B_i are defined through the condition $e_{A_i}e_{B_i} = e_A$, and

$$F_{A_1B_1} = \frac{1}{2}(G_{A_1B_1} + G_{0A} - G_{A_2B_2} - G_{A_3B_3}). \quad (6)$$

An algebraic expression of the equality between vectors and spinors in the 8-dimensional space is the following equation, valid for any two x, y octonions [11]:

$$S_{kl}(\overline{xy}) = G_{kl}(x)y + xC_{kl}(y), \quad (7)$$

where $S_{kl} = KF_{kl}K$, K being the (octonionic) conjugation operator $K(q) = \bar{q}$.

Equation (7) remains valid under any cyclic permutations of (S_{kl}, G_{kl}, C_{kl}). Note that

$$S_{kl} = \tau(G_{kl}), \quad C_{kl} = \tau(S_{kl}) = \tau^2(G_{kl}), \quad (8)$$

where τ is an automorphism of the D_4 Lie algebra. We can call it the triality automorphism, because it performs a cyclic interchange between vector and spinors: G_{kl} operators realize the (1000) vector representation, S_{kl} – a first kind spinor (0001), and C_{kl} – a second kind spinor (0010).

In general, vectors and spinors transform differently under 8-dimensional rotations, because $G_{kl} \neq S_{kl} \neq C_{kl}$. But it follows from (6) that $G_{A_1B_1} - G_{A_2B_2}$ and $G_{A_1B_1} - G_{A_3B_3}$ are invariant with regard to the triality automorphism, and so, under such rotations, an 8-dimensional vector and both kinds of spinors transform in the same way. These transformations are automorphisms of the octonion algebra, because their generators act as derivations, as the principle of triality (7) shows. We can construct 14 linearly independent derivations of the octonion algebra, because the method described above gives two independent rotations per one imaginary octonionic unit $e_A = e_{A_i}e_{B_i}$. It is well known [10] that the derivations of the octonion algebra form G_2 exceptional Lie algebra. It was suggested [11, 13, 14] that the subgroup of this G_2, which leaves the seventh imaginary unit invariant, can be identified with the colour $SU(3)$ group. If we define the split octonionic units [11]

$$u_0 = \frac{1}{2}(e_0 + ie_7), \quad u_0^* = \frac{1}{2}(e_0 - ie_7),$$
$$u_k = \frac{1}{2}(e_k + ie_{k+3}), \quad u_k^* = \frac{1}{2}(e_k - ie_{k+3}), \quad (9)$$

where $k = 1 - 3$, then with regard to this $SU(3)$ u_k transforms as triplet, u_k^* – as antitriplet and u_0, u_0^* are singlets [11]. Therefore, all one-flavour quark–lepton degrees of freedom can be represented as one octonionic (super)field

$$q(x) = l(x)u_0 + q_k(x)u_k + q_k^C(x)u_k^* + l^C(x)u_0^*, \quad (10)$$

here, $l(x)$, $q_k(x)$ are lepton and (three coloured) quark fields and $l^C(x)$, q_k^C – their charge conjugates.

Note that it does not matter what an octonion, first kind spinor, second kind spinor or vector we have in (10), because they all transform identically under $SU(3)$.

So $SO(8)$ can be considered as a natural one-flavour quark–lepton unification group. We can call it also a generalized colour group in the Pati–Salam sense, remembering their idea about the lepton number as the fourth colour [15]. Then the triality property of the $SO(8)$ gives a natural reason why the number of flavours should be triplicated.

4. FAMILY FORMATION AND $SO(10)$

Unfortunately, $SO(8)$ is not large enough to be used as a grand unification group: there is no room for weak interactions in it. This is not surprising, because weak interactions connect two different flavours and we are considering $SO(8)$ as a one-flavour unification group.

The following observation points out the way how $SO(8)$ can be extended to include the weak interactions. Because $C_{AB} = F_{AB}$ and $C_{A0} = -F_{A0}$ for $A, B = 1$ - 7, the $SO(8)$ (Hermitian) generators for the (bi)spinor transformation (3) can be represented as $M_{AB} = -iF_{AB}$ and $M_{A0} = -M_{0A} = -i\sigma_3 F_{A0}$.

The last equation suggests to consider $M_{A,7+k} = -i\sigma_k F_{A0}$ generators, where $k = 1$ - 3 and summation to the modulus 10 is assumed, i.e., 7 + 3 = 0. So we have two new operators $-i\sigma_1 F_{A0}$ and $-i\sigma_2 F_{A0}$ which mix the upper and lower (bi)spinor octonionic components. Besides, if we consider these operators as rotations, then we have to add two extra dimensions and it is expected that $SO(8)$ will be enlarged to $SO(10)$ in this way and two different $SO(8)$ spinors (two different flavours) will join in one $SO(10)$ spinor (family formation). Indeed, the following generators

$$M_{AB} = -iF_{AB}, \quad M_{7+i,7+j} = \frac{1}{2}\varepsilon_{ijk}\sigma_k,$$
$$M_{A,7+k} = -i\sigma_k F_{A0}, \quad M_{7+k,A} = -M_{A,7+k}, \tag{11}$$

where $A, B = 1$ - 7 and $i, j, k = 1$ - 3, really satisfy the $SO(10)$ commutation relations

$$[M_{\mu\nu}, M_{\tau\rho}] = -i(\delta_{\nu\tau}M_{\mu\rho} + \delta_{\mu\rho}M_{\nu\tau} - \delta_{\mu\tau}M_{\nu\rho} - \delta_{\nu\rho}M_{\mu\tau}). \tag{12}$$

It is clear from (12), that $M_{\alpha\beta}$ ($\alpha, \beta = 0, 7, 8, 9$) and M_{mn} ($m, n = 1$ - 6) subsets of generators are closed under commutation and commute to each other. They correspond to $SU_L(2) \otimes SU_R(2)$ and $SU(4)$ subgroups of $SO(10)$. The generators of the $SU_L(2) \otimes SU_R(2)$ can be represented as

$$T_L^i = \frac{1}{2}\sigma_i u_0, \quad T_R^i = \frac{1}{2}\sigma_i u_0^* . \tag{13}$$

So multiplication by u_0 or u_0^* split octonion units plays the role of projection operator on the left and right weak isospin, respectively.

The $SU(4)$ generators can be also expressed via split octonionic units:

$$E_{ij} = -u_i(u_j^* , \quad E_{0i} = -u_j(u_k, \quad E_{i0} = u_j^*(u_k^* . \tag{14}$$

In the last two equations (i, j, k) is a cyclic permutation of (1, 2, 3) and it is assumed that, for example, $E_{ij}(q) = -u_i(u_j^* q)$.

Under $SU(4)$ u_α ($\alpha = 0$ - 3) transforms as a $\underline{4}$ fundamental representation and u_α^* – as its conjugate $\underline{4^*}$. So $SU(4)$ unifies u_0 colour singlet and u_k colour triplet in one single object, and therefore plays the role of the Pati–Salam group [15].

Note that all one-family (left-handed) quark–lepton degrees of freedom are unified in one bioctonionic (super)field (16-dimensional $SO(10)$ spinor) [16]

$$\Psi_L = \begin{pmatrix} \nu(x) \\ l(x) \end{pmatrix}_L u_0 + \begin{pmatrix} q_i^u(x) \\ q_i^d(x) \end{pmatrix}_L u_i + \begin{pmatrix} l^C(x) \\ \nu^C(x) \end{pmatrix}_L u_0^* + \begin{pmatrix} q_i^{dC}(x) \\ q_i^{uC}(x) \end{pmatrix}_L u_i^* . \tag{15}$$

The fact that we should take the Weyl (left-handed) spinors instead of Dirac (that the weak interactions are flavour chiral) indicates close interplay between space-time (space inversion) and internal symmetries [17].

Thus our construction leads to $SO(10)$ as a natural one-family unification group. But doing so, we have broken the triality symmetry: only the spinoric octonions take part in family formation and the vectoric octonion is singled out. Can we in some way restore equivalence between vector and spinor octonions?

First of all, we need to realize vector octonion in terms of the $SO(10)$ representation and this can be done by means of 2×2 octonionic Hermitian matrices, which together with the symmetric product $X \circ Y = \frac{1}{2}(XY + YX)$ form the M_2^8 Jordan algebra [18]. $SO(10)$ appears as a (reduced) structure group of this Jordan algebra [18] and 10-dimensional complex vector space generated by the M_2^8 basic elements (the complexification of M_2^8) gives (10000) irreducible representation of its D_5 Lie algebra.

Thus, now we have at hand the realization of spinoric octonions as a 16-dimensional $SO(10)$ spinor $\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ and vectoric octonion as a 10-dimensional $SO(10)$ vector $\begin{bmatrix} \alpha & q \\ \bar{q} & \beta \end{bmatrix}$. How to unify them? The familiar unitary symmetry example how to unify an isodublet and an isotriplet in the 3×3 complex Hermitian matrix can give a hint and so let us consider 3×3 octonion Hermitian matrices.

5. E_6, TRIALITY AND FAMILY TRIPLICATION

Together with the symmetric product, 3×3 octonion Hermitian matrices form the M_3^8 exceptional Jordan algebra [10]. A general element from it has a form

$$X = \begin{pmatrix} \alpha & x_3 & \bar{x}_2 \\ \bar{x}_3 & \beta & x_1 \\ x_2 & \bar{x}_1 & \gamma \end{pmatrix}$$

and can be uniquely represented as $X = \alpha E_1 + \beta E_2 + \gamma E_3 + F_1^{x_1} + F_2^{x_2} + F_3^{x_3}$. This is the Peirce decomposition [18] of M_3^8 relative to the mutually orthogonal idempotents E_i.

A reduced structure group of M_3^8 is E_6 exceptional Lie group [19]. Its Lie algebra consists of the following transformations:

1) 24 linearly independent $\{a_1, a_2, a_3\}$ generators, which are defined as $\{a_1, a_2, a_3\}X = [A, X]$, where

$$A = \begin{pmatrix} 0 & a_3 & \bar{a}_2 \\ -\bar{a}_3 & 0 & a_1 \\ -\bar{a}_2 & -\bar{a}_1 & 0 \end{pmatrix}$$

is a 3×3 octonion anti-Hermitian matrix with zero diagonal elements.

2) $\{\Delta_1, \Delta_2, \Delta_3\}$ triality triplets (7), (8), which annihilate E_i idempotents and in the F_i Peirce components act according to

$$\{\Delta_1, \Delta_2, \Delta_3\} F_i^a = F_i^{\Delta_i(a)}.$$

Because a triality triplet is uniquely defined by its first element: $\Delta_2 = \tau(\Delta_1)$ and $\Delta_3 = \tau(\Delta_2)$, this gives extra 28 linearly independent generators. Together with $\{a_1, a_2, a_3\}$ type operators, they form 52-dimensional F_4 exceptional Lie algebra [10, 20].

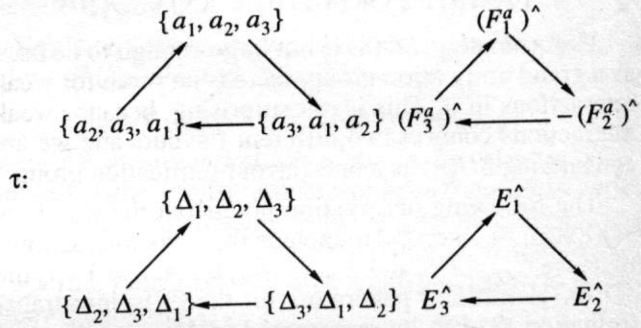
3) $T^{\wedge}$ linear transformations of M_3^8, defined as $T^{\wedge}X = T \circ X$, where T is any element from M_3^8 with zero trace.

The way how E_6 exceptional Lie algebra was constructed shows the close relationship between D_5 and E_6: the latter is connected to the exceptional Jordan algebra M_3^8 and the former – to the M_2^8 Jordan algebra [21]. But M_3^8 has three M_2^8 (Jordan) subalgebras, consisting correspondingly from elements:

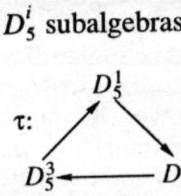
$$\begin{pmatrix} \alpha & a & 0 \\ \bar{a} & \beta & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} \alpha & 0 & \bar{a} \\ 0 & 0 & 0 \\ a & 0 & \beta \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & a \\ 0 & \bar{a} & \beta \end{pmatrix},$$

therefore E_6 has three equivalent D_5 subalgebras. Let D_5^i be that D_5 subalgebra of E_6 which acts in the M_2^8 Jordan algebra, formed from the F_i^a, E_j, E_k elements. It consists from $\{\Delta_1, \Delta_2, \Delta_3\}$, $(F_i^a)^{\wedge}$, $(E_j - E_k)^{\wedge}$, $\{\delta_{i1}a_1, \delta_{i2}a_2, \delta_{i3}a_3\}$ operators and their (complex) linear combinations. Therefore the intersection of these D_5^i subalgebras is D_4 formed from the $\{\Delta_1, \Delta_2, \Delta_3\}$ triality triplets, and their unification gives the whole E_6 algebra.

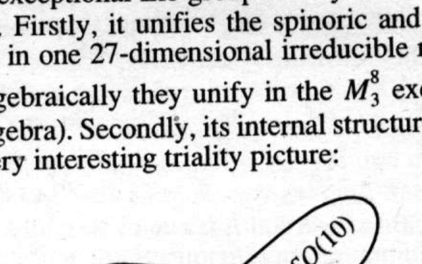
The triality automorphism for D_4, can be continued on E_6:



It can be verified [22] that (15) actually gives an E_6 automorphism. This τ automorphism causes a cyclic permutation of the D_5^i subalgebras:

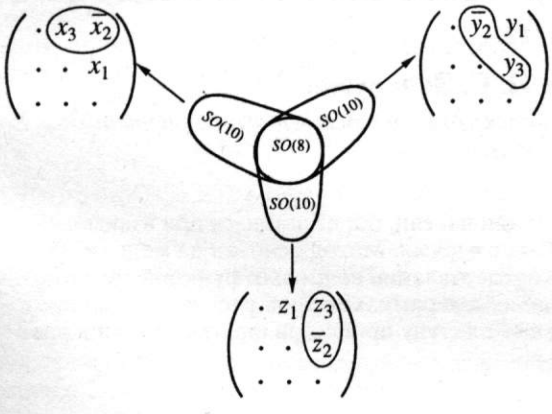


So E_6 exceptional Lie group is very closely related to triality. Firstly, it unifies the spinoric and vectoric octonions in one 27-dimensional irreducible representation (algebraically they unify in the M_3^8 exceptional Jordan algebra). Secondly, its internal structure also reveals a very interesting triality picture:



The equality between $SO(8)$ spinors and vectors now results in the equality of three $SO(10)$ subgroups (in the existence of the triality automorphism τ, which interchanges these subgroups).

To form a quark–lepton family, we have to select one of these $SO(10)$ subgroups. But *a priori* there is no reason to prefer any of them. The simplest possibility to have family formation which respects this equality between various $SO(10)$ subgroups (E_6 triality symmetry is to take three copies of M_3^8 and arrange matters in such a way that in the first M_3^8 the first $SO(10)$ subgroup acts as a family formating group, in the second M_3^8 – the second $SO(10)$ and in the third one – the third $SO(10)$:



More formally, we have $\underline{27} + \underline{27} + \underline{27}$ reducible representation of E_6, such that when we go from one irreducible subspace to another, the representation matrices are rotated by the triality automorphism τ.

6. CONCLUSION

If we take seriously that octonions play some underlying dynamical role in particle physics and $SO(8)$ appears as a one-flavour unification group, then the triality property of $SO(8)$ gives a natural reason for the existence of three quark–lepton generations. Family formation from two flavours due to weak interactions can be connected naturally enough to $SO(10)$ group, but with the triality symmetry violated. An attempt to restore this symmetry leads to the exceptional group E_6 and three quark–lepton families.

Note added. $SO(8)$ triality plays a crucial role in separating $(V - A)$ and $(V + A)$ families in the $SO(18)$ grand unification models, as is explained in Wilczek F., Zee A., Phys. Rev. D, 1982, vol. 25, p. 553. I am grateful to A. Zee for drawing my attention to this work.

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$SO(8)$-ЦВЕТ КАК ВОЗМОЖНЫЙ ИСТОЧНИК ПОКОЛЕНИЙ

З. К. Силагадзе

Исследуется возможная связь между существованием трех кварк-лептонных поколений и свойством триальности для группы $SO(8)$ (эквивалентность восьмимерных векторов и спиноров). $SO(8)$ выступает как естественная группа объединения в рамках одного аромата. Формирование кварк-лептонного семейства из-за слабых взаимодействий связывается с группой $SO(10)$, но с нарушением триальной симметрии. Попытки восстановить эту симметрию приводят к исключительной группе E_6 и трем кварк-лептонным поколениям.