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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH
Section A

Coherent undulator radiation of an electron beam, microbunched for the FEL power outcoupling

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Abstract

The intensity of the coherent undulator radiation of an electron beam, preliminarily microbunched in the FEL master oscillator for the FEL power outcoupling, is approximately calculated by simple analytic considerations, taking into account the transverse emittance and the energy spread of the initial electron beam.

In our previous paper [1] we discussed the conservation of spatial-temporal correlations of longitudinal density of the electron beam passing through an achromatic magnetic system. This property provides the mutual coherence of radiation from two undulators separated by this achromatic bend, which we had observed earlier [2]. Thus, if the first undulator is a part of magnetic system of the oscillator FEL (see Fig. 1), at its exit the energy and density of electrons are modulated. Then, passing through the second undulator (radiator), this beam radiates coherently at the wavelength of its longitudinal modulation (the wavelength of the master oscillator FEL). Using an achromatic band between the undulators, we can deflect this coherent radiation from the axis and take it out of the FEL optical resonator. The theory of coherent undulator radiation was described in a number of papers (see, for example Refs. [3,4]). We will take into account the influence of the finite values of the energy modulation, the energy spread and the transverse emittances of the electron beam.

Consider an energy-modulated electron beam at the entrance of the dispersive section. Using the longitudinal coordinate Z as an independent variable we can write the distribution function of electrons in form:

$$f_1(x_0, y_0, x'_0, y'_0\, \delta_0, t_0) = f_0(x_0, y_0, x'_0, y'_0, \delta_0 - \Delta \sin(\omega t_0), t_0)\,, \quad (1)$$

where Δ and ω are the relative amplitude and the frequency of the modulation; $f_1(x_0, y_0, x'_0, y'_0, \delta_0, t_0)$ is the initial distribution function of transverse coordinates x_0, y_0, angles x'_0, y'_0, relative deviation of energy δ_0 and the time deviation t_0. Let us note, that f_0 is the density of particle flow (really independent on t_0) in a phase space with the normalization defined as

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_0(x_0, y_0, x'_0, y'_0\, \delta_0, t_0)\, dx_0\, dy_0\, dx'_0\, dy'_0\, d\delta_0 = 1. \quad (2)$$

After passing through a short magnetic system with longitudinal dispersion D, the distribution function is transformed as

$$f_2(x_0, y_0, x'_0, y'_0\, \delta_0, t_0) = f_1(x_0, y_0, x'_0, y'_0, \delta_0, t_0 - \delta_0 D/C) \quad (3)$$

$$= f_0(x_0, y_0, x'_0, y'_0, \delta_0 - \Delta \sin(\omega t_0 - kD\delta_0), t_0 - \delta_0 D/C)\,. \quad (4)$$

where $k = \omega/c$ is the longitudinal modulation wavenumber. Then, let such modulated electron beam pass through a planar undulator, in median plane of which the magnetic field H is parallel to axis Y and oscillates along the axis Z with amplitude H_0. In general case, when the undulator construction provides both the vertical and horizontal focusing, the vertical magnetic field near the undulator axis is described by the dependence

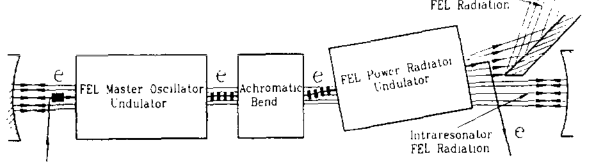

Fig. 1. The schematic diagram of the electron radiation outcoupling from the oscillator FEL.

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$$H_y(x, y, z) = H_0 \cosh(k_{ux}x)\cosh(k_{uy}y)\cos(k_u z), \tag{5}$$

where $k_{ux}^2 + k_{uy}^2 = k_u^2$; $k_u = 2\pi/\lambda_u$; λ_u is a period of undulator. Then the dependence of the electron longitudinal velocity along the undulator is of the form

$$v_z(z) \approx v\left\{1 - [(1-2\delta)\alpha_{u0}^2/2 + x_0^2/\beta_{ux}^2 + x_0'^2 + y_0^2/\beta_{uy}^2 + y_0'^2]/2 + \frac{\alpha_{u0}^2}{4}\cos[2k_u z]\right\}, \tag{6}$$

where $\alpha_{u0} = (eH_0)/(E_0 k_u)$ and E_0 are the mean deflection angle amplitude and the mean energy of the electrons in the undulator, correspondingly;

$$\beta_{ux} = \beta_u k_u/k_{ux}, \quad \beta_{uy} = \beta_u k_u/k_{uy}, \tag{7}$$

β_u is the matched beta-function of electron beam in undulator:

$$\beta_u \approx \frac{E_0\sqrt{2}}{eH_0}. \tag{8}$$

The transverse components of the velocity vector are expressed as

$$v_x(z) \approx v[x'(z) - \alpha_{u0}\sin(k_u z)], \tag{9}$$

$$v_y(z) = vy'(z), \tag{10}$$

$$x'(z) = x_0'\cos(z/\beta_{ux}) - x_0\sin(z/\beta_{ux})/\beta_{ux}. \tag{11}$$

$$y'(z) = y_0'\cos(z/\beta_{uy}) - y_0\sin(z/\beta_{uy})/\beta_{uy}. \tag{12}$$

From Eq. (6), in particular, one can see that the electron longitudinal velocity v_z, averaged over the undulator period, conserves its initial value at passing along the undulator.

Let us describe the radiation field of a moving electron by the Fourier-harmonic of its vector-potential [5]

$$\boldsymbol{A}_\omega = \frac{e}{c}\int \frac{\boldsymbol{v}(t)}{R(t)}\exp\{i\omega[t + R(t)/c]\}\,dt, \tag{13}$$

where $R(t)$ is the distance from the electron to the observation point at the time t. In the far zone of the radiation field

$$\boldsymbol{R}(t) = \boldsymbol{R}_0 - \boldsymbol{r}(t), \quad |\boldsymbol{R}_0| \gg |\boldsymbol{r}(t)|, \tag{14}$$

where $\boldsymbol{R}_0$ is the radius-vector from the undulator to the observation point, $\boldsymbol{r}(t)$ is the radius-vector of an electron moving along the undulator; replacing the integration variable

$$t = t_0' + \int_0^z \frac{dx_1}{v_z(z_1)}, \tag{15}$$

where t_0' is the time of the electron coming to the undulator entrance, we get

$$\boldsymbol{A}_\omega = \frac{e}{cR_0}\exp(ikR_0)\int_0^L \frac{\boldsymbol{v}(z)}{v_z(z)} \times \exp\left\{i\left[\omega t_0' + \omega\int_0^z \frac{dz}{v_z(z_1)} - \boldsymbol{kr}(z)\right]\right\}dz, \tag{16}$$

where L is the length of the undulator; $\boldsymbol{k}$ is the radiation wave number vector ($|\boldsymbol{k}| = \omega/c$), directed to the observation point;

$$\boldsymbol{r}(z) = [x(z) + \alpha_{u0}\cos(k_u z)/k_u, y(z), z],$$

$$x(z) = x_0\cos(z/\beta_{ux}) + x_0'\beta_{ux}\sin(z/\beta_{ux}), \tag{17}$$

$$y(z) = y_0\cos(z/\beta_{uy}) + y_0'\beta_{uy}\sin(z/\beta_{uy}). \tag{18}$$

The total vector-potential of the radiation field of the electron beam with the distribution function $f_2(x_0, y_0, x_0', y_0', \delta_0', t_0')$ at the undulator entrance, taking into account the normalization (2), is expressed as

$$A(t) = 2\,\mathrm{Re}\left\{\frac{I\omega\exp(ikR_0 - i\omega t)}{2\pi cR_0}\int_0^L\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \times \int_{-\infty}^{+\infty}\int_{-\pi/\omega}^{+\pi/\omega}\frac{\boldsymbol{v}(z)}{v_z(z)}\exp\left\{i\left[\omega t_0' + \omega\int_0^z \frac{dz_1}{v_z(z_1)}\boldsymbol{kr}(z)\right]\right\} \times f_2(x_0, y_0, x_0', y_0', \delta_0', t_0') \times dt_0'\,dx_0\,dy_0\,dx_0'\,dy_0'\,d\delta_0'\,dz\right\}, \tag{19}$$

where I is total current of the electron beam.

Knowing, that the power of the first harmonic of electron undulator radiation is concentrated in the angular cone with the opening angle about $1/\gamma_*$ [6] (where $\gamma_* = \gamma/\sqrt{1 + K^2/2}$; γ is the electron relativistic factor; $K = \gamma\alpha_{u0}$ is the undulator parameter), we will derive an approximate expression for the coherent undulator radiation, valid when the electron angular spread $\sigma_{x',y'}$ in the beam

$$\sigma_{x',y'} \ll 1/\gamma_* \tag{20}$$

and the diffractive divergence of the coherent radiation of electron beam with r.m.s. transverse size $\sigma_{x,y}$

$$\frac{1}{k\sigma_{x,y}} \ll 1/\gamma_*, \quad \text{i.e.}$$

$$\sigma_{x,y} \gg \gamma_*/k. \tag{21}$$

With such an approximation, we get (see Ref. [7]), that at small observation angles with the undulator axis ($\theta_{x,y} \ll 1/\gamma_*$) the vector-potential of the first harmonic of the considered undulator radiation mainly contains only one component:

$$A_x(t) \approx \mathrm{Re}\left\{\frac{-iI}{cR_0}\exp(ikR_0 - i\omega t)\alpha_{u0}[J_0 - J_1]I_z\right\}, \tag{22}$$

where $J_0 = J_0(k\alpha_{u0}^2/8k_u)$ and $J_1 = J_1(k\alpha_{u0}^2/8k_u)$ are Bessel functions of zero and first orders. The quantity I_z is given for the initial beam (before the energy and density modulating) with the distribution function f_0, independent on time, as follows:

$$I_z = \int_0^L \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f_0(x_0, y_0, x_0', y_0', \delta_0) \times J_1(X_0 + kz\Delta/\gamma_*^2)\exp\{i[-\delta_0 X_0/\Delta + [k((1-2\delta_0)/\gamma_*^2 + \theta_x^2 + \theta_y^2 + x_0^2/\beta_{ux}^2 + x_0'^2 + y_0^2/\beta_{uy}^2 + y_0'^2)/2 - k_u]z - k\theta_x x(z) - k\theta_y y(z)]\} \times dx_0\, dy_0\, dx_0'\, dy_0'\, d\delta_0\, dz, \quad (23)$$

where $X_0 = -kD\Delta$ is the parameter of beam bunching at the undulator entrance. Then, expressing the radiation magnetic field as

$$H_{\omega y} \approx ikA_{\omega x}, \quad (24)$$

we write an expression for the angular distribution of the coherent radiation power:

$$\frac{dP}{d\Omega} \approx \frac{\overline{H_y^2}}{4\pi} cr_0^2 \approx \frac{\overline{A_x^2}}{4\pi} ck^2 R_0^2. \quad (25)$$

The total power of coherent radiation can be expressed as

$$P = I^2 Z_u, \quad (26)$$

where

$$Z_u = \frac{\{k\alpha_{u0}[J_0 - J_1]\}^2}{8\pi c} \int |J_z|^2\, d\Omega \quad (27)$$

is the effective impedance of the undulator. For negligible values of the transverse emittance, the energy spread and the energy modulation of the electron beam, the expression (23) takes the form

$$|J_{z0}| = J_1(X_0)L \left| \frac{\sin(kL\theta^2/4 - \chi)}{kL\theta^2/4 - \chi} \right|, \quad (28)$$

where $\theta^2 = \theta_x^2 + \theta_y^2$; $\chi = [k_u - (k/2\gamma_*^2)]L/2 = kL\theta_0^2/4$ is a resonance detune of the undulator; here θ_0 is the observation angle, which corresponds to the wavelength λ in angular dependence of the spontaneous undulator radiation spectrum:

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + K^2/2 + \gamma^2\theta_0^2).$$

From Eq. (27), after integration over the solid angle, supposing

$$k \sim \frac{4\pi\gamma^2}{\lambda_u(1 + K^2/2)}, \quad (29)$$

with $\chi \gg 1$ we get the maximal achievable impedance of the undulator

$$Z_{u0} \approx q \frac{2\{\pi K[J_0 - J_1]J_1(X_0)\}^2}{c(1 + K^2/2)}, \quad (30)$$

where $q = L/\lambda_u$ is the number of undulator periods. With $K \gg 1$ and $\max[J_1(X_0)] \approx J_1(1,84) \approx 0.582$ we have

$$Z_{u0} \approx q \times 196\,\Omega, \quad (31)$$

and the expression (27) can be rewritten in the form

$$Z_u \approx Z_{u0} \times 0.47 \frac{\int |I_z|^2\, d\Omega}{\lambda L}. \quad (32)$$

It is interesting to remark that for $\chi \gg 1$ the dependence of the total radiated power on the electron energy is very weak, and therefore we may obtain an electron efficiency much more than $1/q$, even without tapering.

An operation at high χ of the undulator-radiator makes other schemes of the electron FEL outcoupling possible (see Fig. 2a). An advantage of this configuration is the absence of the achromatic bend before the radiator, moreover, which may be installed between two undulators of the optical klystron (OK) master oscillator and serves also as the OK dispersive section (see Fig. 2b).

The real effective impedance Z_u of the undulator-radiator can be much lower than Z_{u0}, due to the emittance and the energy spread of electron beam, the influence of whose we will study further. Let us study the beam with the Gaussian function of the electron distribution

$$f_0(x_0, y_0, x_0', y_0', \delta_0) = \frac{1}{(2\pi)^{5/2}\varepsilon_x\varepsilon_y\sigma_e} \times \exp\left(-\frac{\delta_0^2}{2\sigma_e} - \frac{\beta_x x_0'^2 + 2\alpha_x x_0' x_0 + \gamma_x x_0^2}{2\varepsilon_x} - \frac{\beta_y y_0'^2 + 2\alpha_y y_0' y_0 + \gamma_y y_0^2}{2\varepsilon_y} \right). \quad (33)$$

where σ_e is the relative dispersion of the electron energy spread; $\varepsilon_{x,y}$ are the transverse emittances of the electron beam; $\alpha_{x,y}$, $\beta_{x,y}$, $\gamma_{x,y}$ are the Twiss parameters for transverse phase ellipses of the electron beam at the undulator

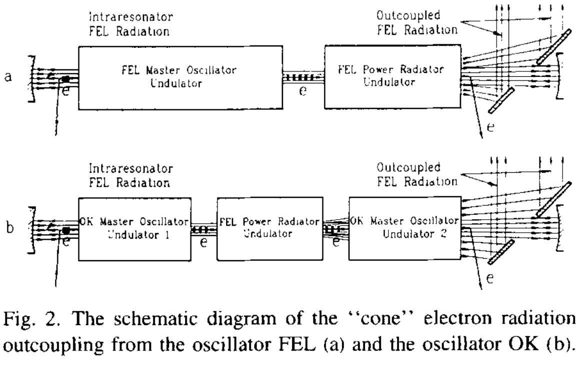

Fig. 2. The schematic diagram of the "cone" electron radiation outcoupling from the oscillator FEL (a) and the oscillator OK (b).

entrance. Having omitted the details of the integration (23) over the energy spread and the transverse emittances of electron beam, we write its final result:

$$I_z = \int_0^L F_x(z)F_y(z)F_e(z) \times \exp\{iz[k(1/\gamma_*^2 + \theta_x^2 + \theta_y^2)/2 - k_u]\}\,dz\,, \tag{34}$$

where

$$\begin{aligned} F_x(z) &= F(\theta_x, \varepsilon_x, \alpha_x, \beta_x, \gamma_x, z) \\ &= \exp\left\{ -\frac{k^2\theta_x^2\varepsilon_x}{2[\gamma_x - ikz\varepsilon_x/\beta_{ux}^2]} \left[\cos^2(z/\beta_{ux}) + \frac{[(\gamma\beta_{ux} - ikz\varepsilon_x/\beta_{ux})\sin(z/\beta_{ux}) - \alpha_x\cos(z/\beta_{ux})]^2}{1 - ikz\varepsilon_x(\beta/\beta_{ux}^2 + \gamma_x) - (kz\varepsilon_x/\beta_{ux})^2} \right] \right\} \\ &\quad /\sqrt{1 - ikz\varepsilon_x(\beta/\beta_{ux}^2 + \gamma_x) - (kz\varepsilon_x/\beta_{ux})^2}\,; \end{aligned} \tag{35}$$

$$\begin{aligned} F_y(z) &= F(\theta_y, \varepsilon_y, \alpha_y, \beta_y, \gamma_y, z); \\ F_e(z) &= J_1(X_0 + kz\Delta/\gamma_*^2)\exp\{-[(X_0 + kz\Delta/\gamma_*^2)\sigma_e/\Delta]^2/2\}\,. \end{aligned} \tag{36}$$

Let us consider the particular case of negligible transverse emittances, when $F_{x,y}(z) \approx 1$ is carried out along the whole undulator ($0 < z < L$). Then we can approximately rewrite the integral (34) in the form

$$I_{ze} \approx \int_0^L F_e(z)\exp\{ikz(\theta^2 - \theta_0^2)/2\}\,dz\,. \tag{37}$$

Using Eq. (36), after the integral over the solid angle in Eq. (27), for the infinite long undulator at high detune ($\chi \approx 1$), with $X_0 = 0$, and taking into account, that

$$\max\left[\int_0^\infty J_1^2(\xi\sqrt{2}\Delta/\sigma_e)\exp(-\xi^2)\,d\xi\right] \approx 0.148 \quad \text{with} \quad \Delta/\sigma_e \approx 1.80\,, \tag{38}$$

we get the expression for the undulator maximal achievable impedance at the energy spread of the electron beam:

$$Z_{ue} \approx 6.83\ \Omega/\sigma_e\,. \tag{39}$$

Let us note, that when the magnetic system, used for the electron beam bunching before the undulator, has the positive value of longitudinal dispersion (sign opposite to the undulator dispersion value), the maximal impedance of the infinite undulator is reached at $-X_0 \gg 1$ and is twice as much as the value (39).

Now let us study the particular case of negligible electron energy spread, when $F_e(z) \approx \max[J_1(X_0)] \approx J_1(1.84) \approx 0.582$ is carried out along the whole undulator ($0 < z < L$). Then the integral (34) can be approximately rewritten as

$$I_{zp} \approx 0.582\int_0^L F_x(z)F_y(z)\exp\{ikz(\theta^2 - \theta_0^2/2\}\,dz\,. \tag{40}$$

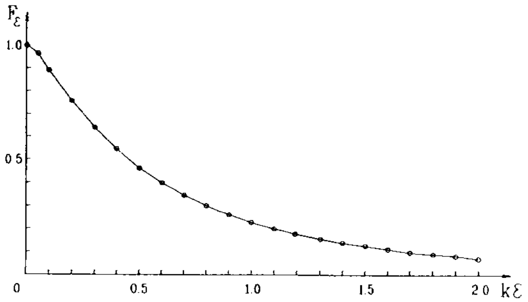

Fig. 3. The computed function $F_\varepsilon(k\varepsilon)$.

Let us consider the long undulator with equal horizontal and vertical focusing, with matched beta-functions of the electron beam at the undulator entrance:

$$\alpha_{x,y} = 0; \quad \beta_{x,y} = \beta_u\sqrt{2}; \quad \gamma_{x,y} = 1/\beta_{x,y}\,. \tag{41}$$

For the infinite long undulator, using Eq. (35), supposing $\varepsilon_x = \varepsilon_y = \varepsilon$, from Eq. (27) we get the expression for the undulator maximal achievable impedance at the transverse emittance of the electron beam:

$$Z_{u\varepsilon} = \frac{\beta_u}{\lambda_u}\frac{F_\varepsilon(k\varepsilon)}{k\varepsilon} \times 109\ \Omega \approx \frac{\beta_u}{\lambda_u k\varepsilon} \times 109\ \Omega \quad \text{with} \quad k\varepsilon \ll 1\,, \tag{42}$$

where we have defined

$$F_\varepsilon(k\varepsilon) = \max[F_\varepsilon(k\varepsilon, \theta_0^2)] \quad \text{over} \quad \theta_0^2\,, \tag{43}$$

$$F_\varepsilon(k\varepsilon, \theta_0^2) = \frac{2}{\pi^2}\int_0^\infty |I_{z\varepsilon}(k\epsilon, \theta_0^2, \theta^2)|^2\,d\theta^2\,, \tag{44}$$

$$I_{z\varepsilon}(k\varepsilon, \theta_0^2, \theta^2) = 0.582\int_0^\infty \exp\left[-\frac{k^2\varepsilon^2\theta^2}{(1 - iz)} + iz(\theta^2 - \theta_0^2)\right]/(1 - iz)^2\,dz\,. \tag{45}$$

The computed function $F_\varepsilon(k\varepsilon)$ is plotted in Fig. 3.

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