# EQUATIONS OF MOTION OF SPINNING RELATIVISTIC PARTICLE IN EXTERNAL FIELDS

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The talk is based on the article gr-qc/9710098. We consider the motion of a spinning relativistic particle in external electromagnetic and gravitational fields, to first order in the external field, but to an arbitrary order in spin. The correct account for the spin influence on the particle trajectory is obtained with the noncovariant description of spin. Concrete calculations are performed up to second order in spin included. A simple derivation is presented for the gravitational spin-orbit and spin-spin interactions of a relativistic particle. We discuss the gravimagnetic moment (GM), a specific spin effect in general relativity. It is demonstrated that for the Kerr black hole the gravimagnetic ratio, i.e., the coefficient at the GM, equals to unity (as well as for the charged Kerr hole the gyromagnetic ratio equals to two). The equations of motion obtained for relativistic spinning particle in external gravitational field differ essentially from the Papapetrou equations.

# INSTRUCTIVE PROPERTIES OF QUANTIZED GRAVITATING DUST SHELL

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In this talk based on [1] we investigate quantum dynamics of self-gravitating spherical dust shell. The wave functions of discrete spectrum are not localized inside the Schwarzschild radius. We argue that such shells can transform into white holes (in another space). It is plausible that shells with bare masses larger than the Planck mass loose their mass emitting lighter shells.

Thin dust shell is one of the simplest models of collapsing gravitating bodies. The classical dynamics of this system was considered in [2–5]. This model was quantized in various nonequivalent ways with physically different results in [6–9]. The most natural in our opinion approach proposed in [8] reduces the problem to the usual $s$-wave Klein-Gordon equation in a Coulomb field. Curious effects arising here may turn instructive for more realistic situations.

Let us present the basic steps leading to the Klein-Gordon equation for this system. Two classical equations of motion for the shell radius $r$ have first integrals

$$\sqrt{1+\dot{r}^2-2km/r}+\sqrt{1+\dot{r}^2}=C_1; \tag{1}$$

$$\sqrt{1+\dot{r}^2-2km/r}-\sqrt{1+\dot{r}^2}=C_2-\frac{k\mu}{r}. \tag{2}$$

Here $\mu$ is the "bare" mass of the shell, i.e. the rest mass of each particle of the dust times their number, $k$ is the Newton gravitational constant, and $\dot{r}=dr/d\tau$ where $\tau$ is the proper time of the dust. Multiplying these two expressions we see that the two integrals are compatible only if $C_2=0$ and $C_1=2m/\mu$. Under these conditions the equations of motion are consistent and equivalent to

$$\frac{\ddot{r}}{\sqrt{1+\dot{r}^2}}=-\frac{k\mu}{2r^2}. \tag{3}$$

Eq. (3) has the following first integral

$$m=\mu\sqrt{1+\dot{r}^2}-\frac{k\mu^2}{2r}. \tag{4}$$

This expression will be taken as a classical Hamiltonian of the system (after going over from the velocity $\dot{r}$ to the canonical momentum $p$). Formally any function of a first integral can be chosen as a Hamiltonian. The choice is by no means unique, but eq. (4) is singled out because this Hamiltonian corresponds to the total energy of the shell, and this energy $m$ has an explicit and simple expression.

One can easily recognize in the rhs of eq. (4) the energy of a relativistic particle in a Coulomb-like field, $-k\mu^2/2r$, written in proper time $\tau$. It is convenient to go over from $\tau$ to the world time $t$ inside the shell:

$$d\tau = dt\sqrt{1-v^2},\ v = dr/dt.$$

We obtain now:

$$m = \frac{\mu}{\sqrt{1-v^2}} - \frac{k\mu^2}{2r}. \tag{5}$$

Clearly in this case the canonical momentum equals $p = \mu v/\sqrt{1-v^2}$ and the Hamiltonian has the well known form:

$$H = \sqrt{p^2+\mu^2} - \frac{k\mu^2}{2r}. \tag{6}$$

The quantum-mechanical wave equation corresponding to this Hamiltonian is derived by the standard procedure, taking the square of the root, i.e., rewriting (6) as:

$$(H + k\mu^2/2r)^2 = p^2 + \mu^2.$$

Thus one obtains the usual Klein-Gordon radial equation for $s$-wave:

$$\left(\partial_r^2 + \frac{2}{r}\,\partial_r + \frac{k\mu^2 m}{r} + \frac{k\mu^2}{4r^2} + m^2 - \mu^2\right)\psi = 0. \tag{7}$$

The discrete spectrum of this equation is well-known [10,11]:

$$m_n = \mu\left[1 + \frac{k^2\mu^4}{\left(2n+1+\sqrt{1-k^2\mu^4}\right)^2}\right]^{-1/2} \tag{8}$$

The radial quantum number $n$ is an integer and runs from 0 to infinity.

The spectrum has a singularity at $k\mu^2 = 1$. At larger values of $\mu$ the $r^{-2}$ potential becomes so strong that the "fall to the center" takes place, i.e., there are no stationary states. It looks natural that for heavy pressureless matter (with $\mu > m_{Pl} = 1/\sqrt{k}$) naive quantum mechanical effects cannot stop the collapse.

The curious property of the states belonging to the discrete spectrum should be pointed out. Even in the most tightly bound ground state for $k\mu^2 = 1$ the wave function is not localized inside the gravitational radius of the shell. The probability to find the shell outside it is $3/e^2 \approx 0.4$. Here we naively use $r$ as the operator of coordinate. In the relativistic case a more refined definition of the coordinate should be used [8]. It is clear however that any reasonable definition of the coordinate operator would not change the localization considerably.

Let us consider now the continuous spectrum. Though we assume as above that $k\mu^2 < 1$ or in other words $\mu < m_{Pl}$, the total energy $m$ can be arbitrarily large. The wave function of such a state is a superposition of incoming and outgoing spherical waves with equal amplitudes. It is evidently non-localized. To consider the quantum analogue of the collapse of the classical shell we have to turn to wave packets. As was noted in ref. [8] the gravitational radius of this object will be as smeared as its energy. Let us assume that the initial radius of the maximum of the incoming spherical wave packet is much larger than its average gravitational radius $r_g$. From the point of view of a distant observer this packet moving towards the center freezes at $r = r_g$. However, in its proper time it reaches the center in a finite interval $\delta\tau$, bounces back and then after the same time interval returns to its initial position and form. Certainly, it returns not to "our" space, but to a quite different one. This is a possible realization of the white hole phenomenon [12,13].

The considered realization of a white hole based on quantum scattering differs essentially from the classical examples of white holes. In the classical case the very existence of the phenomenon depends crucially upon the presence of singularity at $r = 0$, while in our case the transformation of an incoming spherical wave into an outgoing one takes place even for nonsingular potentials.

Finally let us return to the case of a large bare mass, $\mu > m_{Pl}$. This problem formally coincides with that of a charged scalar particle in the field of a point-like nucleus with a supercritical electric charge, $Z\alpha > 1$. It is known that the vacuum around a supercharged nucleus is unstable and this nucleus discharges by emitting positively charged particles (see e.g. [14,15]). A similar scenario is plausible for our problem: the collapsing shell loses its bare mass $\mu$ by emitting light shell-lets till it reaches the subcritical mass. This phenomenon would resemble quantum evaporation of usual black holes. On the other hand, in the subcritical situation, $\mu < m_{Pl}$, the emission of shell-lets does not take place (even if the physical mass $m$ is larger than $m_{Pl}$). It can be considered as a hint that small black holes do not evaporate, though at $m > \mu$ they form white holes in another universe.

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# SUPERLUMINAL VELOCITIES OF PHOTONS IN GRAVITATIONAL BACKGROUND

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The talk is based on Ref. [1]. The influence of radiative corrections on the photon propagation in a gravitational background is investigated without the low-frequency assumption $\omega \ll m$. The conclusion is made in this way that the local velocity of light can exceed unity.

The question addressed in the talk was raised many years ago by Drummond and Hathrell [2]. They noted that tidal gravitational forces on the photons, induced by radiative corrections, alter in general the characteristics of propagation, and therefore, due to it photons may travel in some cases at local speeds greater than unity. To be more precise, in a local inertial frame the induced curvature terms in the Maxwell equations survive and modify the light cone in different way for different polarizations.

The approach of Ref. [2] consisted in expanding the contribution to the photon effective action from one-loop vacuum polarization to the lowest order in the inverse electron mass squared $1/m^2$. Therefore, their result by itself refers strictly speaking to low-frequency photons with $\omega \ll m$ only. Meanwhile, the velocity of the wave-front propagation in a dispersive medium is determined by the asymptotics of the refraction index $n(\omega)$ at $\omega \to \infty$ (see, e.g., Ref. [3]). It is argued however in Ref. [2] that due to the dispersion relation for the refraction index $n(\omega)$, its high-frequency asymptotics $n(\infty)$ is related to the low-frequency one $n(0)$ as follows:

$$n(\infty) = n(0) - \frac{2}{\pi}\int_0^\infty \frac{d\omega}{\omega}\,\mathrm{Im}\, n(\omega). \qquad (1)$$

Then, since $\mathrm{Im}\, n(\omega)$ is nonnegative,

$$n(\infty) \le n(0),$$

and this would guarantee the superluminal propagation of the wave front. The shortcoming of this argument, as pointed out in Ref. [4], is that the sign of $\mathrm{Im}\, n(\omega)$ in the problem of interest is not fixed, generally speaking. Indeed, the physical meaning of the condition

$$\mathrm{Im}\, n(\omega) \ge 0$$

is that in a homogenious medium without instabilities (i.e., without particle creation) the wave amplitude can decrease only, due to the loss of particles from the beam. However, in an inhomogenious medium (and this is the case of a gravitational background) the processes of the beam focusing and bunching are possible, leading to the increase of the wave amplitude, which corresponds to

$$\mathrm{Im}\, n(\omega) \le 0.$$

Even if the superluminal propagation takes place indeed in this way, it does not violate causality [2]. Still, the effect discussed is quite unexpected and interesting, and it is certainly worth efforts to find out whether the predicted phenomenon is a true one or just a result of an inadequate approximation. We address the problem without the low-frequency assumption $\omega \ll m$, and in this way arrive at the conclusion that in a gravitational background photons can propagate indeed with superluminal velocities.

We derive first of all the general structure of the photon-graviton vertex $\gamma\gamma g$ at any frequencies and momentum transfers $q^2$:

$$-\frac{1}{2}h_{\mu\nu}\,[F'_{\mu\lambda}F_{\nu\lambda}+F'_{\nu\lambda}F_{\mu\lambda}-\frac{1}{2}\delta_{\mu\nu}F'_{\kappa\lambda}F_{\kappa\lambda}]f_1(q^2)$$

$$+R_{\mu\nu\kappa\lambda}F'_{\mu\nu}F_{\kappa\lambda}f_2(q^2)+RF'_{\kappa\lambda}F_{\kappa\lambda}f_3(q^2). \tag{2}$$

Here $h_{\mu\nu}$ is the deviation of the metric from the flat one; $R_{\mu\nu\kappa\lambda}$ and $R$ are the Riemann tensor and the scalar curvature, respectively; $F'_{\mu\lambda}$ and $F_{\nu\lambda}$ are field strengths of the outgoing and incoming photons. The lowest order QED contribution to the form factors $f_i(q^2)$ was calculated in Refs. [5,6]. The first nontrivial terms of the form factors expansions in $q^2$ are (see [2])

$$f_1=1+\frac{11\alpha}{720\pi}\,\frac{q^2}{m^2}\,;\;\; f_2=-\frac{\alpha}{360\pi m^2}\,;\;\; f_3=-\frac{\alpha}{144\pi m^2}\,. \tag{3}$$

Let us emphasize that the form factors in the amplitude (2) depend on the momentum transfer $q^2$ only, but not on the photon frequency $\omega$ itself. Of course, this property is in no way confined to the lowest order loop calculated in Refs. [5,6], but refers to a general vertex with two on-mass-shell particles. Moreover, when light propagates in a gravitational field of a macroscopic length scale $L$, the typical impact parameters $\sim L$ are large as compared to the Compton wave length $m^{-1}$ (or any other dimensional parameter possibly involved in the radiative corretions), and therefore one can confine to the values of the form factors $f_i$ at $q^2=0$.

The lowest order correction discussed modifies the Maxwell equations in the region where $R_{\mu\nu}=R=0$, and at $\omega L \gg 1$ as follows [2]:

$$D_\mu F^{\mu\nu}+\xi R^{\mu\nu}_{\kappa\lambda}D_\mu F^{\kappa\lambda}=0\,;\;\; \xi=\frac{\alpha}{90\pi m^2}\,. \tag{4}$$

The structure $\xi R^{\mu\nu}_{\kappa\lambda}$ in this expression can be considered obviously as an anisotropic contribution to a refraction index, which in general leads to a superluminal photon velocity.

However, the photon interaction with a strong gravitational field, induced by radiative corrections, contains terms of higher order in curvature. How will they influence the photon propagation?

As distinct from the $\gamma\gamma g$ vertex discussed, the diagrams generating the terms nonlinear in $R$ (from now on $R$ is a generic notation for $R_{\mu\nu\kappa\lambda}$, $R_{\mu\nu}$, $R$) have more external lines and therefore certainly depend on the photon frequency. But by dimensional reasons it is quite natural to expect that it is $R/\omega^2$ which serves as a parameter for the high-frequency expansion of the photon interaction with background. Such a behaviour in the high-frequency limit is much more natural than the expansion in $R/m^2$, with mass singularities in the asymptotic region. In this sense the $\gamma\gamma g$ vertex considered above is an exception: being $\omega$-independent (kind of a subtraction constant in the dispersion relation), it has no choice at $q^2/m^2$ but generate linear terms of the type $R/m^2$. Thus, the terms nonlinear in $R$ die out as $\omega \to \infty$ and do not influence the wave-front propagation. On the other hand, there is a case when those nonlinear terms are certainly inessential at any frequency: that of a weak gravitational background.

The low-frequency limit being now abandoned, we get rid also of the following difficulty pointed out in Ref. [2]. If the curvature length scale is $L$ ($R \sim L^{-2}$), then according to Eq. (4) the velocity shift caused by the radiative correction is

$$\delta v \sim \frac{\alpha}{m^2 L^2}\,. \tag{5}$$

The time of the signal propagation is also $\sim L$. So, the corresponding position shift for the signal is

$$\delta s \sim L\delta v \sim \frac{\alpha}{m^2 L} \ll \frac{1}{m}\,. \tag{6}$$

It is not exactly clear how such a distance can be resolved with frequencies $\omega \ll m$. Of course, going beyond the low-frequency approximation in principle removes this difficulty.

The presented arguments give strong reasons to believe that the effect of superluminal propagation of photons in a gravitational background does exist.

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