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Synchrotron radiation and spin light

G.N. Kulipanov[a], A.E. Bondar[a,*], V.A. Bordovitsyn[b,*], V.S. Gushchina[b]

[a]Budker Institute of Nuclear Physics, Novosibirsk, Russian Federation
[b]Tomsk State University, Tomsk, Russian Federation

Abstract

Spin light, or μ-radiation, is a new phenomenon of nature, which observation has become possible due to development of technology of the elementary particles acceleration. In this work the description of the Novosibirsk experiment, where the spin dependence of synchrotron radiation was observed for the first time, is given. It is shown, that the spin light is decomposed to different on principle μL- and μTh-radiation types. μL-radiation is conditioned by Larmor precision of the intrinsic magnetic moment of the electron (and its anomalous part), while μTh-radiation is connected with Thomas precession of the electron spin and has pure kinematic origin. In the relativistic quantum theory of radiation based on the Dirac equation, μL-radiation and μTh-radiation do not differ one from another and from the other quantum effects of electron radiation. Separate consideration is possible only in Jackson's relativistic semiclassical radiation theory used in the given work.

1. Semiclassical theory of mixed radiation

Spin dependence of synchrotron radiation (SR) in the quantum theory was first discovered in the work of Ternov et al. [1]. However, for a long time the physical interpretation of the corresponding quantum correction to the SR power has being remained vague. Only recently it was a success to brighten up this question in Ref. [2] with the help of Jackson's semiclassical theory [3].

According to the semiclassical theory, spin precession is defined by the Frenkel–BMT (Bargmann–Michel–Telegdi) equation

$$\frac{d\zeta}{dt} = \frac{e}{m_0 c\gamma}[\zeta \boldsymbol{H}_0^{\mathrm{eff}}] = [\omega\zeta]. \qquad (1)$$

The classical spin-vector $\zeta = (\sin\lambda\cos\nu, \sin\lambda\sin\nu, \cos\lambda)$ is calculated on the spin functions of the free electron as usually. The external magnetic field $\boldsymbol{H}_0^{\mathrm{eff}}$, we introduce in the form of decomposition [4]

$$\boldsymbol{H}_0^{\mathrm{eff}} = \boldsymbol{H}_0^{\mathrm{L}} + \boldsymbol{H}_0^{\mathrm{Th}}, \qquad (2)$$

where

$$\boldsymbol{H}_0^{\mathrm{L}} = \frac{g}{2}\boldsymbol{H}_0, \quad \boldsymbol{H}_0^{\mathrm{Th}} = \frac{\gamma}{\gamma+1}[\beta \boldsymbol{E}_0],$$

$\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ are fields in the electron rest system.

According to (1) and (2) the frequency of the spin precission also decomposes into two frequencies

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\mathrm{L}} + \boldsymbol{\Omega}_{\mathrm{Th}},$$
$$\boldsymbol{\Omega}_{\mathrm{L}} = -\frac{eg}{2m_0c}\frac{\boldsymbol{H}_0}{\gamma}, \quad \boldsymbol{\Omega}_{\mathrm{Th}} = -\frac{e}{m_0c}\frac{[\beta \boldsymbol{E}_0]}{\gamma+1}, \qquad (3)$$

which describe Larmor and Thomas precission of the spin correspondingly. We pay attention that the anomalous magnetic moment contains in Larmor precession completely. It follows that in the case of an electric neutral particle, whose magnetic moment is anomalous (neutron), Thomas precession disappears.

Let us consider the radiation fields replacing by

$$\boldsymbol{E} \to \tilde{\boldsymbol{E}}, \quad \boldsymbol{H} \to [\boldsymbol{n}\tilde{\boldsymbol{E}}], \quad \tilde{\boldsymbol{E}} \to -\mathrm{i}\frac{\tilde{\omega}}{c}\tilde{\boldsymbol{A}},$$

where $\tilde{\boldsymbol{A}}$ is the vector potential and $\tilde{\omega}$ is the frequency of the radiation field. According to Jackson's theory, the total Hamiltonian of interaction of the electron with the radiation field has the form

$$\tilde{H}^{\mathrm{int}} = \tilde{H}_e + \tilde{H}_\mu^{\mathrm{eff}} = \tilde{H}_e + \tilde{H}_\mu^{\mathrm{L}} + \tilde{H}_\mu^{\mathrm{Th}} = -e(\beta^{\mathrm{eff}}\tilde{\boldsymbol{A}}), \qquad (4)$$

where

$$\beta^{\mathrm{eff}} = \beta + \beta^{\mathrm{L}} + \beta^{\mathrm{Th}},$$
$$\beta^{\mathrm{L}} = -\mathrm{i}\frac{\mu\tilde{\omega}}{ec}\left\{[\sigma(\boldsymbol{n}-\beta)] - \frac{\gamma}{\gamma+1}(\sigma\beta)[\beta\boldsymbol{n}]\right\}, \qquad (5)$$
$$\beta^{\mathrm{Th}} = -\mathrm{i}\frac{\hbar\tilde{\omega}}{2m_0c^2}\frac{\gamma}{\gamma+1}\{[\sigma\beta](1-(\boldsymbol{n}\beta)) + (\sigma[\beta\boldsymbol{n}])\}.$$

It should be noted that in this representation we do not take into account the recoil effects during radiation, although it is not difficult to do that (see Ref. [5]).

* Corresponding authors.

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The spectra-angular and polarization characteristics of radiation in the semiclassical theory are obtained on the base of the Fourier transform

$$E_{s\tilde{\omega}} = -\mathrm{i}\frac{e\tilde{\omega}}{Rc}\int_{-\infty}^{\infty}(\beta^{\mathrm{eff}}\boldsymbol{n}_s)\,\mathrm{e}^{\mathrm{i}\tilde{\omega}[t-(\boldsymbol{nr})/c]}\,\mathrm{d}t\,, \tag{6}$$

where $\boldsymbol{n}_s$ are the polarization unit vector ($s = \sigma$, π), R is the distance from the charge to the observation point.

Radiation of the intrinsic magnetic moment of the electron conditioned by Larmor or Thomas precession we shall call μL- and μTh-radiation correspondingly.

2. Mixed $e\mu$L- and $e\mu$Th-radiation

In the most visible form μL- and μTh-radiation are shown in the so-called mixed $e\mu$-synchrotron radiation. Now it is a well known fact that this radiation is a result of the interference of the charge and intrinsic magnetic moment radiation fields.

According to the relativistic semiclassical radiation theory we shall represent the all trajectory characteristics of the electron in the form of the decomposition in the small parameters γ^{-1}, ct/ρ and ψ, where ρ is the orbital radius of the electron in a homogeneous magnetic field and ψ is the angle between the plane of the orbit and the direction of radiation.

The quantum character of spin behavior during radiation is taken into account by the matrix element ($\zeta, \zeta' = \pm 1$)

$$\langle\zeta'|\sigma|\zeta\rangle = \frac{1-\zeta\zeta'}{2}(\boldsymbol{i}+\mathrm{i}\zeta\boldsymbol{j}) + \frac{1+\zeta\zeta'}{2}\zeta\boldsymbol{k} = \bar{\zeta}\,. \tag{7}$$

This formula can be interpreted as decomposition of the complex spin-vector $\bar{\zeta}$ to the parallel and perpendicular components

$$\bar{\zeta} = \bar{\zeta}_\perp + \bar{\zeta}_\parallel\,, \quad |\bar{\zeta}| = \frac{1+\zeta\zeta'}{2}\delta_{\zeta\zeta'} = \zeta_\parallel\,.$$

Further we shall consider that the quantum transitions take place along the direction of the magnetic field, although a more total consideration of this question is possible. Besides, it should be taken into account that $e = -e_0 < 0$ and the magnetic field has the direction along the Z axis.

Spectral-angular composition of the radiation power is determined by the expression

$$\frac{\mathrm{d}^2W_s}{\mathrm{d}\Omega\,\mathrm{d}\tilde{\omega}} = \frac{cR^2}{4\pi^2T}|\tilde{E}_{s\tilde{\omega}}|^2\,,$$

where $T = c/2\pi\rho$.

Further we shall not consider synchrotron radiation of the electron for it is studied very well. Main attention will be paid only to mixed $c\mu$-radiation, which is included in the interference terms

$$\tilde{E}^{e}_{s\tilde{\omega}}\tilde{E}^{\mu\mathrm{L}}_{s\tilde{\omega}}\,, \quad \tilde{E}^{e}_{s\tilde{\omega}}\tilde{E}^{\mu\mathrm{Th}}_{s\tilde{\omega}}\,.$$

After the integration over the angles and spectrum we will get:

$$\begin{aligned} W^{e\mu\mathrm{L}}_{\sigma} &= \frac{1+\zeta\zeta'}{2}W_{\mathrm{SR}}\frac{1}{6}\xi\zeta = W^{e\mu\mathrm{L}}_{\pi}\,,\\ W^{e\mu\mathrm{Th}}_{\sigma} &= -\frac{1+\zeta\zeta'}{2}W_{\mathrm{SR}}\frac{7}{6}\xi\zeta\,,\\ W^{e\mu\mathrm{Th}}_{\pi} &= -\frac{1+\zeta\zeta'}{2}W_{\mathrm{SR}}\frac{1}{6}\xi\zeta\,, \end{aligned} \tag{8}$$

where

$$W_{\mathrm{SR}} = \frac{2}{3}\frac{e_0^2c}{\rho^2}\gamma^4 = \frac{2}{3}\frac{e_0^2H^2}{m_0^2c^3}$$

is the total power of synchrotron radiation of the electron charge and ξ represents a well-known invariant parameter (sometimes χ is used instead of ξ)

$$\tfrac{2}{3}\xi = \frac{\hbar}{m_0c^3}\sqrt{w_\mu w^\mu} = \frac{\hbar\gamma^2}{m_0c\rho} = \gamma\frac{H}{H^*} = \chi\,,$$

where w^μ is a four-dimensional acceleration and H^* is the Schwinger magnetic field.

The fact attracts attention that mixed $e\mu$-radiation occurs due to transitions without spin flip exceptionally.

Summing $e\mu$L- and $e\mu$Th-radiation we will get the well-known in the synchrotron radiation theory result [1,6]:

$$\begin{aligned} W^{e\mu}_{\sigma} &= W^{e\mu\mathrm{L}}_{\sigma} + W^{e\mu\mathrm{Th}}_{\sigma} = -W_{\mathrm{SR}}\xi\zeta\,,\\ W^{e\mu}_{\pi} &= W^{e\mu\mathrm{L}}_{\pi} + W^{e\mu\mathrm{Th}}_{\pi} = 0\,. \end{aligned}$$

Thus in π-component $e\mu$L- and $e\mu$Th-radiation completely make up for each other when $g = 2$.

3. Correspondence principle in μ-radiation of electron

Relativistic classical theory of μ-radiation (to be more precise as μL-radiation) has been worked out for a long time. At first the potentials of the radiation fields were got by Frenkel [7]. Then the fields and integral characteristics of radiation were found in the work of Bhabha and Corben [8]. Later results were repeated by many authors (Horvath [9], Bialas [10], Kolsrud and Leer [11], Cohn and Wiebe [12]), but nobody considered or made up one's mind to consider the concrete applications. For the first time such attempt was undertaken in the series of our works [13–17]. As a result of this research it was set that for neutral Dirac particles having only anomalous magnetic moment (neutron), the precise coincidence of the classical and quantum theory of μL-radiation in the uniform and homogeneous external fields of various configurations with respect to $\boldsymbol{E}$ and $\boldsymbol{H}$ takes place.

However, such agreement was not got for the electron even for uniform magnetic field. Only in the case of the electron moving uniformly and rectilinearly in the fields of Veen filter type, when Thomas precession is absent, classical and quantum theory of μ-radiation gives the same result [18]. Thus the relativistic classical theory of μ-radiation is a theory of μL-radiation.

Now let us show that the classical theory of μL-radiation developed earlier in Refs. [8–12] and in our works [13–17] completely coincides with the semiclassical (and quantum) theory.

At the beginning we prove this assertion in the general form. According to Ref. [5], the tensor of the electromagnetic radiation field of a charged particle with intrinsic magnetic moment can be represented in the form

$$\tilde{H}^{\alpha\beta} = \frac{1}{c}\frac{\partial}{\partial\tilde{t}}\tilde{A}^{[\alpha}n^{\beta]} - \frac{1}{c^2}\frac{\partial^2}{\partial\tilde{t}^2}\tilde{Q}^{[\alpha\rho}n_\rho n^{\beta]}, \tag{9}$$

where

$$\tilde{A}^\alpha = -e\frac{v^\alpha}{R_\rho v^\rho},\quad \tilde{Q}^{\alpha\rho} = -c\mu\frac{\Pi^{\alpha\rho}}{R_\rho v^\rho}.$$

Here $\Pi^{\alpha\beta}$ is a dimensionless antisymmetric spin tensor, $\tilde{t}$ is the radiation fixation time in the observation point, $R^\rho = (R, \boldsymbol{R})$, $R = c(\tilde{t} - t)$, $n^\rho = R^\rho/R$, $v^\rho = c\gamma(1, \boldsymbol{\beta})$. The square brackets signify the anticommutation on the corresponding indexes. Yielding in (9) the polarization components of the electric field and taking into account that $(n, n_s) = 0$, we will have

$$\tilde{E}_{s\tilde{\omega}} = -\mathrm{i}\frac{e\tilde{\omega}}{Rc}\int_{-\infty}^{\infty}(\boldsymbol{\beta}^{\mathrm{eff}}_{\mathrm{Cl}}\boldsymbol{n}_s)\,\mathrm{e}^{\mathrm{i}\tilde{\omega}[t-(\boldsymbol{nr})/c]}\,\mathrm{d}t,$$
$$\boldsymbol{\beta}^{\mathrm{eff}}_{\mathrm{Cl}} = \boldsymbol{\beta} - \mathrm{i}\frac{\mu\tilde{\omega}}{ec}\left\{[\boldsymbol{\zeta}(\boldsymbol{n}-\boldsymbol{\beta})] - \frac{\gamma}{\gamma+1}(\boldsymbol{\zeta\beta})[\boldsymbol{\beta n}]\right\} = \boldsymbol{\beta} + \boldsymbol{\beta}^{\mathrm{L}}. \tag{10}$$

We see that the latter expressions are different from (5) and (6) only so that $\sigma \to \zeta$ and by the absence of the term β^{Th}. Thus for μL-radiation the correspondence principle works irreproachably.

4. Classical theory of mixed $e\mu$L-synchrotron radiation

The possibility of classical theory can be demonstrated especially obviously on the example of mixed $e\mu$-synchrotron radiation considered above. Actually, according to the general classical theory of μ-radiation, loss of energy due to radiation for $e\mu$-radiation is defined by the expression (see Ref. [14], formula (6.17))

$$\left(\frac{\mathrm{d}P^\alpha}{\mathrm{d}\tau}\right) = \frac{2}{3}\frac{e\mu}{c^4} \times \left(\overset{\circ\circ}{\Pi}{}^{\alpha\beta}w_\beta - \frac{2}{c^2}v^\alpha\overset{\circ}{w}_\rho\Pi^{\rho\sigma}w_\sigma - \frac{1}{c^2}\Pi^{\alpha\beta}w_\beta w_\rho w^\rho\right). \tag{11}$$

Here P^α is the four-dimensional radiation momentum and the circle signifies the differentiation on the own time τ. The radiation power is got from the zero component

$$W = \frac{c}{\gamma}\left(\frac{\mathrm{d}P^0}{\mathrm{d}\tau}\right)^{e\mu}.$$

Substituting here the equations for the charge motion and the Frenkel–BMT equation for the spin and averaging over the period of motion and spin precession in the case of $g = 2$ we get [15]

$$\bar{W}^{e\mu} = W_{\mathrm{SR}}(1 + \tfrac{1}{3}\,\xi\pi_{1z}),$$

where π_{1z} is the initial value of the spin projection. As π_z is the motion integral and $\pi_{1z} = \zeta_\parallel$, then

$$\bar{W}^{e\mu} = W_{\mathrm{SR}}(1 + \tfrac{1}{3}\xi\zeta_\parallel) = \bar{W}^{e\mu\mathrm{L}}, \tag{12}$$

where $\bar{W}^{e\mu\mathrm{L}}$ is a sum of σ- and π-components of $e\mu$L-radiation defined in (8) (multiplier $(1 + \zeta\zeta')/2 = \delta_{\zeta\zeta'}$ is of no essential importance in this case).

It can be shown that the spectral-angular and polarization characteristics of $e\mu$L-radiation also coincide. We note that it is not necessary to do special calculations for this purpose, as one can use known in the semiclassical theory of mixed $e\mu$-synchrotron radiation results for the anomalous magnetic moment of the electron $\mu_a = \mu_0(g-2)/2 = \mu_0 a$, which, as we know, is not subjected the Thomas precession.

The question about the possibility of the inclusion of μTh-radiation in the more general classical theory of the magnetic moment radiation remains open. It is possible to suggest that after some modification of the energy–momentum tensor of the electromagnetic field this question will be solved positively.

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