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THE SCIENCE AND CULTURE SERIES — PHYSICS

UNIVERSALITY FEATURES IN MULTIHADRON PRODUCTION AND THE LEADING EFFECT

Proceedings of the 33rd Workshop of the INFN Eloisatron Project

Erice, Italy 19–25 October 1996

4755/ин

Editors
L. Cifarelli, A. Kaidalov and V. A. Khoze

Series Editor
A. Zichichi

World Scientific, 1998
Singapore • New Jersey • London • Hong Kong

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USE OF THE GRIBOV THEOREM ON SOFT EMISSION IN MASSLESS GAUGE THEORIES

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We present an exampe, which demonstrates, that the Gribov theorem about the region of applicability of the soft emission factorization can be used in massless gauge theories for exact calculations, inspite of the fact that the theorem cannot be literally applied in the case of massless charged particles. The example is closely connected with the programm of calculation of the next-to-leading corrections to the BFKL equation. We use the theorem for calculation of the gluon production amplitude in the multi-Regge kinematics with small transverse momentum of the produced gluon.

1 Introduction

We want to show how the theorem about the region of applicability of well-known formulas for accompanying bremstrahlung, which was proved by V.N. Gribov [1], can be used in massless gauge theories. Let us remind, that in 1967 V.N. Gribov proved, that for collision of two hadrons, A and B, with large c.m.s. energy $\sqrt{s} = \sqrt{(p_A + p_B)^2}$ this region is restricted by the inequalities

$$\frac{2p_A k}{s} \ll 1\,, \qquad \frac{2p_B k}{s} \ll 1\,,$$

$$\vec{k}_\perp^{\,2} \approx \frac{2(p_A k)\cdot 2(p_B k)}{s} \ll \mu^2\,, \tag{1}$$

where $\vec{k}_\perp$ is the projection of the momentum of the emitted photon on the plane orthogonal to the momenta of the colliding particles p_A and p_B, and μ is a typical hadron mass.

Notice that before the work of Ref. [1] it was generally accepted (see, for example, Ref. [2]) that for the applicability of the accompanying bremstrahlung formulas one has to have

$$2p_A k \ll \mu^2\,, \qquad 2p_B k \ll \mu^2\,. \tag{2}$$

Therefore, V.N. Gribov proved, that the region of applicability of these formulas is considerably extended at large energies. Indeed, the conditions (2) are much more stringent than the conditions (1), if

$$s \gg \mu^2\,. \tag{3}$$

Gribov proved[1] that in the region (1) the amplitude of the emission process is given only by those Feynman diagrams where the photon is attached to external charged particles. Furthermore, calculating the contributions of these diagrams one has to keep the non radiative part of the amplitude on the mass shell, i.e. to neglect virtualities of radiating particles. In the following we'll call the formulas obtained in such a way as "soft insertion formulas". Let us stress that these formulas are invariant under gauge transformation of the emitted photon. The possibility of using the factorized formulas with on mass-shell non radiative amplitude in the region (1) is quite non trivial and is connected with gauge invariance of the emission amplitude[1].

It is very attractive to make use of the Gribov theorem in more complicated cases, such as, for example, Quantum Chromodynamics (QCD). An evident obstacle for this is the masslessness of particles having colour charge. In other words, the typical mass μ in Eq. (1) is equal to zero for the case of QCD.

The main point in the proof of the Gribov theorem is the smallness of the transverse momentum $k_\perp$ of the emitted quantum of the gauge field (photon or gluon) in comparison with the essential transverse momenta of the other particles. In massive theories the latter momenta are of order (or larger than) μ. Contrary, in theories with massless particles, such as QCD, the essential transverse momenta of virtual particles can be arbitrary small (that appears as infrared and collinear divergences). Therefore, the Gribov theorem cannot be applied literally for these theories.

Nevertheless, the theorem can be used in such theories. It is necessary to say, that, in some sense, there are nothing new in this statement, because, in fact, the theorem was used already many times for QCD calculations in Double Logarithmic Approximation (DLA) (see, for example, [3]). We want to demonstrate here a validity of this statement in much more powerful sense: the theorem can be used for calculations not only in the DLA, but with an accuracy up to a constant. Below an example of such calculation is presented in framework of the programm of calculation of the next-to-leading corrections to the BFKL equation[4]. The theorem is used for calculation of the gluon production amplitude in the multi-Regge kinematics with small transverse momentum of the produced gluon.

The outline of the talk is the following. In Section 2 we give a short review of the BFKL equation. In Section 3 the programm of calculation of the nex-to-leading correction to the BFKL kernel and its present status is presented. Section 4 is devoted to calculation of gluon production amplitude in the multi-Regge kinematics with small transverse momentum of the produced gluon. In Section 5 the Reggeon-Reggeon-Gluon vertex is obtained in the case of small gluon transverse momentum for arbitrary space-time dimension D.

2 BFKL equation

In the last years one can notice a wave of interest to the BFKL equation [4], connected with results of recent experiments on deep inelastic scattering of electrons on protons[5]. The equation permits to solve the problem of calculation of parton distributions in the region of small values of Bjorken variable x in the leading logarithmic approximation (LLA) of perturbation theory, which means summation of all terms of the type $[\alpha_s \ln(1/x)]^n$. It can be presented in the form of equation of evolution in variable $\ln(1/x)$:

$$\frac{\partial}{\partial ln(1/x)}\mathcal{F}(x,\vec{q}_1^{\,2}) = \int d^2q_2\mathcal{K}(\vec{q}_1,\vec{q}_2)\mathcal{F}(x,\vec{q}_2^{\,2}), \tag{4}$$

where function $\mathcal{F}(x,\vec{k}^{\,2})$ is so called unintegrated gluon density, connected with distribution of gluons, having squared transverse momenta up to Q^2, by equation

$$xg(x,Q^2) = \int_0^{Q^2} d\vec{k}^{\,2}\mathcal{F}(x,\vec{k}^{\,2}). \tag{5}$$

In the LLA the kernel of the equation has a form [4]

$$\mathcal{K}(\vec{q}_1,\vec{q}_2) = -\frac{g^2N}{(2\pi)^3}\vec{q}_1^{\,2}\int\frac{d^2k}{\vec{k}^2(\vec{q}_1-\vec{k})^2}\delta(\vec{q}_1-\vec{q}_2) + \frac{g^2N}{4\pi^3}\frac{1}{(\vec{q}_1-\vec{q}_2)^2}\,. \tag{6}$$

Here g is a gauge coupling constant ($\alpha_s = \frac{g^2}{4\pi}$), N is a number of colours ($N = 3$ for QCD), and the vector sign serves for denotation of components of 4-momenta of partons orthogonal to the plane of initial particle momenta p_A, p_B. Separate contributions to the kernel (6) leads to infrared singularities, which cancel each other in Eq.(4). In the LLA it is not difficult, performing an azimuthal integration in Eq.(4), to rewrite it in such a way that makes this cancellation evident[4]:

$$\frac{\partial}{\partial ln(1/x)}\mathcal{F}(x,\vec{q}_1^{\,2}) =$$

$$\frac{N\alpha_s}{\pi}\int d\vec{q}_2^{\,2}[\frac{\mathcal{F}(x,\vec{q}_2^{\,2})}{|\vec{q}_2^{\,2}-\vec{q}_1^{\,\text{œ[B2}}|} - \mathcal{F}(x,\vec{q}_1^{\,2})\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}(\frac{1}{|\vec{q}_2^{\,2}-\vec{q}_1^{\,2}|} - \frac{1}{\sqrt{(\vec{q}_1^{\,2})^2+4(\vec{q}_2^{\,2})^2}})]. \tag{7}$$

In the next-to-leading approximation, which will be discussed below, the cancellation of the infrared singularities is not so simple, therefore in the following we'll use dimensional regularization

$$\frac{d^2k}{(2\pi)^3} \rightarrow \frac{d^{D-2}k}{(2\pi)^{D-1}}, \tag{8}$$

where $D = 4 + 2\epsilon$ - space-time dimension, in order to use only well defined expressions at each step of calculations.

The LLA leads to a sharp power growth of cross sections with c.m.s. energy $\sqrt{s}$. In terms of parton distributions this means a fast increase of the gluon density $g(x, Q^2)$ in small $x = \frac{Q^2}{s}$ region:

$$g(x, Q^2) \sim x^{-j_0}, \tag{9}$$

where j_0 is the LLA position of the singularity of the partial amplitude with vacuum quantum numbers in t-channel [4]:

$$j_0 = 1 + \frac{4\alpha_s}{\pi} N \ln 2, \tag{10}$$

with $N = 3$ for QCD. Such behaviour violates the Froissart bound $\sigma_{tot} < const(\ln s)^2$ and therefore the LLA can not be applied at asymptotically small x. Nevertheless, in the region of parameters accessible for modern experiments observed behaviour of the structure functions is consistent with LLA results [6], and we will not discuss here the unitarization problem, which appear at asymptotically large energies. From practical point of view it seems more important to determine the region of energies and momentum transfers where the LLA is applicable. For this purpose we have to calculate radiative corrections to the LLA. Importance of the radiative corrections is strengthened by the curcumstance that the dependence of the QCD running coupling α_s on virtuality is beyond of accuracy of the LLA. It diminishes a predictive power of the LLA, because numerical results of this approximation can be strongly modified by changing a scale of virtuality.

It is clear from the above discussion, that the problem of calculation of radiative corrections to the LLA is very important now.

3 Next-to-leading approximation

In the next-to-leading approximation, when all terms of the type $\alpha[\alpha_s \ln(1/x)]^n$ have to be summed, the equation for $\mathcal{F}(x, \vec{k}^{\,2})$ preserves its form [7], so that the problem is reduced to calculation of corrections to the kernel $\mathcal{K}(\vec{q}_1, \vec{q}_2)$. Up to the next-to-leading approximationis the kernel can be presented in the form:

$$\mathcal{K}(\vec{q}_1, \vec{q}_2) = 2\omega(t_1)\delta(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_{real}(\vec{q}_1, \vec{q}_2), \tag{11}$$

where $t_1 = -\vec{q}_1^{\,2}$, $\omega(t_1)$ - deviation of the gluon Regge trajectory $j(t_1) = 1 + \omega(t_1)$ from unity, and $\mathcal{K}_{real}(\vec{q}_1, \vec{q}_2)$ is determined by a probability of real particle

production in collision of two Reggeized gluons with momenta $q_1 = \beta p_A + \vec{q}_1$ and $-q_2 = \alpha p_B - \vec{q}_2$; $\alpha, \beta \ll 1$. More definitely,

$$\mathcal{K}_{real}(\vec{q}_1, \vec{q}_2) = \frac{1}{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \frac{1}{(N^2-1)} \sum_{i_1, i_2, f} \int d\kappa d\rho_f$$

$$\times \delta^{(D)}(q_1 - q_2 - \sum_{n \in \{f\}} k_n)[|\gamma^{\{f\}}_{i_1 i_2}(q_1, q_2)|^2 - |\gamma^{\{f\}}_{i_1 i_2}(q_1, q_2)|^2_{asympt}], \tag{12}$$

where the sum is taken over colours i_1, i_2 of the Reggeized gluons and over all discreet quantum numbers of the system $\{f\}$ of produced particles (including their number); $\kappa = (q_1 - q_2)^2$ -square of invariant mass of the two reggeons, k_n - momenta of produced particles; $d\rho_f$ - an element of their phase space,

$$d\rho_f = \prod_{n \in \{f\}} \frac{dk_n}{(2\pi)^{D-1} 2\omega_n}, \tag{13}$$

$\gamma^{\{f\}}_{i_1 i_2}(q_1, q_2)$ - effective vertices for production of the system $\{f\}$ in Reggeon-Reggeon collision and the subscript "*asympt*" means their values at asymptotically large κ. Substraction is performed in order to escape a double counting of the region of large κ, because in this region vertices $\gamma^{\{f\}}_{i_1 i_2}(q_1, q_2)$ with gluon quantum numbers in t-channels factorizes into products of more simple vertices and therefore the discussed region is taken into account by iterating of contributions of these vertices in the kernel.

In the LLA [4]

$$\omega^{LLA}(t_1) = \frac{g^2 t_1}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2}k}{k^2(\vec{q}_1 - \vec{k})^2}, \tag{14}$$

and only a one gluon can be produced, with the Reggeon-Reggeon-Gluon (RRG) vertex

$$\gamma^d_{i_1 i_2}(q_1, q_2) = g T^d_{i_2 i_1} e^*_\mu(k) C^\mu(q_2, q_1), \tag{15}$$

where $T^d_{i_2 i_1}$ - matrix elements of the $SU(N)$ group generators in the ajoint representation, $e(k)$ - polarisation vector of the produced gluon, k -its momentum, $k = q_1 - q_2$, and

$$C^\mu(q_2, q_1) = -q_1 - q_2 + p_A\left(\frac{q_1^2}{k_1 p_A} + 2\frac{k_1 p_B}{p_A p_B}\right) - p_B\left(\frac{q_2^2}{k_1 p_B} + 2\frac{k_1 p_A}{p_A p_B}\right). \tag{16}$$

Therefore

$$\mathcal{K}^{LLA}_{real}(\vec{q}_1, \vec{q}_2) = \frac{1}{2(2\pi)^{D-1}\frac{1}{(N^2-1)}\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \sum_{i_1, i_2, d, \lambda} |\gamma^d_{i_1 i_2}(q_1, q_2)|^2 =$$

$$\frac{g^2 N}{(2\pi)^{D-1}} \frac{2}{(\vec{q}_1 - \vec{q}_2)^2} \; . \tag{17}$$

Here λ is a helicity of the produced gluon. Evidently, substituting Eqs.(14),(17) in Eq.(11) and taking $D = 4$ we obtain Eq.(6).

The next-to-leading corrections to the kernel are expressed [7] in terms of the two-loop contribution $\omega^{(2)}(t)$ to the gluon Regge trajectory $\omega(t)$, one-loop correction to the RRG vertex $\gamma^{d}_{i_1 i_2}(q_1, q_2)$, and contrbutions from two-gluon and quark-antiquark production in the quasi-multi-Regge kinematics (QMRK), which, in turn, are expressed in terms of the Reggeon- Reggeon-Gluon-Gluon (RRGG) and Reggeon-Reggeon-Quark-anti-Quark (RR$Q\bar{Q}$) vertices.

Corrections to to the Reggeized gluon trajectory were calculated in Refs. [8] and [9]. The closed form for the correction $\omega^{(2)}(t)$ is presented in Ref.[9].

Investigation of the contributions from two-gluon and quark-antiquark production in the QMRK was started in Ref.[7] where the two-gluon production amplitude in the QMRK, and, correspondingly, the RRGG vertex, was found. The next step was done in Ref.[10], where the two-gluon and quark-antiquark production amplitudes in the QMRK were simplified using helicity representation, the corresponding next-to-leading contributions to the BFKL kernel were expressed in terms of the integrals from the squares of these helicity amplitudes over relative transverse and longitudinal momenta of produced particles, all infrared divergencies were extracted in an explicit form and cancellation of the divergences between real and virtual contributions of quark-antiquark pairs to the kernel was shown. In general case cancellation of infrared divergences was demonstrated in Ref.[11]. Finally, contribution of two-gluon production to the kernel was obtained in Ref.[12]. Resummation formulas for the quark-antiquark production were derived in Ref.[13].

The corrections to the RRG vertex were calculated in Refs. [14]. The calculations were performed in the space-time dimension $D = 4 + \epsilon$ for regularizing the infrared and collinear divergences, but terms vanishing at $\epsilon \to 0$ were omitted in the final expressions. Unfortunately, such terms can give non vanishing contributions to the total cross sections (and to corrections to the BFKL equation) because integration over transverse momenta of the produced gluon leads to divergences at $k_\perp = 0$ for the case $D = 4$. Therefore, in the region $k_\perp \to 0$ we need to know the production amplitude for arbitrary ϵ. As we shall see, the use of the Gribov theorem simplifies considerably the calculation of the RRG vertex in this region.

4 Gluon production amplitude

Let us consider the process of emission of a gluon G with momentum $p_G \equiv k$ at scattering of the particles (quarks or gluons) A and B,

$$A + B \to A' + B' + G \; , \tag{18}$$

in the multi-Regge kinematics

$$s = (p_A + p_B)^2 \gg s_{1,2} \gg |t_{1,2}| \; ,$$
$$s_1 = (2p_{A'} + k)^2 \approx 2p_A k \; , \qquad s_2 = (2p_{B'} + k)^2 \approx 2p_B k \; ,$$
$$t_i = q_i^2 \approx -\vec{q}_{i\perp}^{\;2} \; , \qquad q_1 = p_A - p_{A'} \; , \qquad q_2 = p_{B'} - p_B \; , \tag{19}$$

for the case of the transverse momentum of the emitted gluon small compared with the transferred momenta:

$$|\vec{k}_\perp| \ll |\vec{q}_\perp| \; , \qquad q \equiv \frac{q_1 + q_2}{2} \; ,$$
$$|t_1 - t_2| \ll |t| \approx \vec{q}_\perp^{\;2} \; . \tag{20}$$

Let us start with the Born approximation. Obviously, the soft insertion œ[5 formula should be valid here in the region defined by Eqs. (19) and (20), because all transverse momenta are fixed and $k_\perp$ is the smallest one. The elastic scattering amplitude in the region of large s and fixed t in the Born approximation has the form

$$A_{AB}^{A'B'}(Born) = \Gamma^{(0)i}_{A'A} \frac{2s}{t} \Gamma^{(0)i}_{B'B} \; , \tag{21}$$

where $t = -\vec{q}_\perp^{\;2}$ and $\Gamma^{(0)i}_{A'A}$ are the particle-particle-Reggeon (PPR) vertices in the Born approximation[4]. In the helicity basis these vertices can be presented as

$$\Gamma^{(0)i}_{A'A} = g \langle A'|T^i|A\rangle \delta_{\lambda_{A'}, \lambda_A} \; , \tag{22}$$

where $\langle A'|T^i|A\rangle$ are the matrix elements of the colour group generators in the corresponding representation. It is easy to see that the soft insertion of a gluon with momentum $p_G \equiv k$, colour index c and polarization vector $e(k)$ gives us

$$A_{AB}^{A'GB'}(Born) = \Gamma^{(0)i_1}_{A'A} \frac{2s}{t} \Gamma^{(0)i_2}_{B'B} \, g T^c_{i_2 i_1} e^*_\mu(k) \left(\frac{p_A^\mu}{p_A k} - \frac{p_B^\mu}{p_B k} \right) . \tag{23}$$

Let us remind that in the kinematics defined by the relations (19) the gluon production amplitude in the Born approximation takes the form [4]

$$A_{AB}^{A'GB'}(Born) = 2s \Gamma^{(0)i_1}_{A'A} \frac{1}{t_1} \gamma^{c}_{i_1 i_2}(q_1, q_2) \frac{1}{t_2} \Gamma^{(0)i_2}_{B'B} \; , \tag{24}$$

where the effective production vertex $\gamma^{c}_{i_1 i_2}(q_1, q_2)$ is given by Eqs.(15),(16). Since in the region (20) of small $k_\perp$ we obtain that

$$C(q_2, q_1) \to t\left(\frac{p_A}{p_A k} - \frac{p_B}{p_B k}\right), \tag{25}$$

the expression (24) turns into the form (23). So, for the case of the Born approximation the soft insertion formula is valid, as it was reported.

Now let us consider the one-loop corrections to the production amplitude. Since in this case we need to integrate over the momenta of virtual particles, we cannot expect that the soft insertion gives a corrected answer here. However, analyzing the proof of the Gribov's theorem [1] one can conclude that the soft insertion should be valid for the contribution of the kinematical region where the transverse momenta of virtual particles are much larger than $k_\perp$. The idea is to use the soft insertion formula for this contribution and to add the contribution of the region of small virtual transverse momenta, which has to be calculated separately. From the first sight the idea appears doubtful, because for $D = 4$ the integrals over virtual transverse momenta have a logarithmic behaviour; therefore, it seems that the separation of two regions is not a simple problem. But for $D > 4$ the integrals are convergent, and we have two different scales where they can converge, $q_\perp$ and $k_\perp$, so that the separation is quite simple in this case. Evidently, the contribution of the integrals converging at $q_\perp$ can be obtained applying the soft insertion formula and the contribution of the integrals converging at $k_\perp$ has to be calculated.

Fortunately, a simple inspection of the Feynman diagrams shows that only those ones of Fig. 1 lead to the integrals of the second kind. This statement is valid for all possible choices of colliding particles: they can be gluons (in this case all lines in the diagrams of Fig. 1 are gluon lines) or quarks (in this case the upper and lower lines in the diagrams are quark lines) and so on.

Let us split the production amplitude as the sum of the factorizable and non factorizable contributions:

$$\mathcal{A}^{A'GB'}_{AB} = \mathcal{A}^{A'GB'}_{AB}(f) + \mathcal{A}^{A'GB'}_{AB}(nf). \tag{26}$$

The first term in Eq.(26) comes from the soft insertion while the second one is represented in the one-loop approximation by the diagrams of Fig. 1.

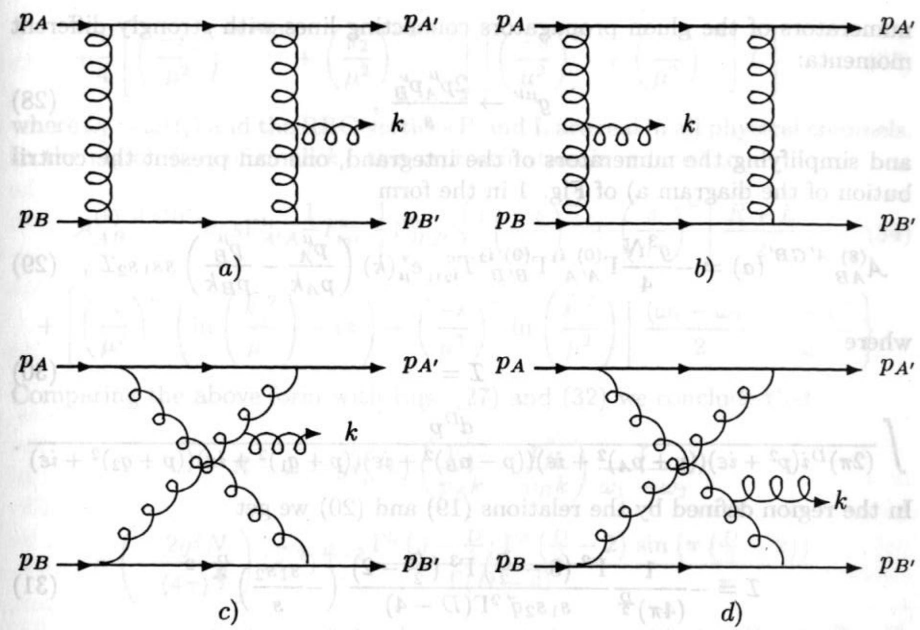

Fig. 1: *Feynman diagrams giving a non factorizable contribution to the gluon emission amplitude.*

Contrary to the Born case, in higher orders the colour structure of the production amplitude is not so simple. For definiteness, let us consider the part of the amplitude with the gluon quantum numbers in t_1 and t_2 channels. This part is the most important one because it determines the RRG vertex. The factorizable contribution to this part has a form similar to the expression (23):

$$\mathcal{A}^{(8)\,A'GB'}_{AB}(f) =$$

$$\Gamma^{i_1}_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^{i_2}_{B'B}\, g T^{c}_{i_2 i_1} e^{*}_{\mu}(k)\left(\frac{p^{\mu}_{A}}{p_A k} - \frac{p^{\mu}_{B}}{p_B k}\right). \tag{27}$$

Here $j(t) = 1 + \omega(t)$ is the gluon trajectory [4,9] and $\Gamma^{i}_{A'A}$ are the PPR vertices. The one-loop corrections to the LLA vertices (22) are calculated in Refs. [14,15]

Now let us pass to the non factorizable contribution. Evidently, the diagrams of Fig. 1 are connected each other by crossing, therefore it is sufficient to calculate the contribution of the diagram a). Performing usual tricks with the numerators of the gluon propagators connecting lines with strongly different momenta:

$$g^{\mu\nu} \to \frac{2p_A^\mu p_B^\nu}{s} , \tag{28}$$

and simplifying the numerators of the integrand, one can present the contribution of the diagram a) of Fig. 1 in the form

$$\mathcal{A}_{AB}^{(8)\ A'GB'}(a) = -\frac{g^3 N}{4}\Gamma^{(0)\ i_1}_{A'A}\Gamma^{(0)\ i_2}_{B'B}T^c_{i_2 i_1}e^*_\mu(k)\left(\frac{p_A^\mu}{p_A k}-\frac{p_B^\mu}{p_B k}\right) s s_1 s_2 \mathcal{I} , \tag{29}$$

where

$$\mathcal{I} = \int \frac{d^D p}{(2\pi)^D i(p^2+i\varepsilon)((p+p_A)^2+i\varepsilon)((p-p_B)^2+i\varepsilon)((p+q_1)^2+i\varepsilon)((p+q_2)^2+i\varepsilon)} . \tag{30}$$

In the region defined by the relations (19) and (20) we get

$$\mathcal{I} = \frac{1}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{s_1 s_2 \vec{q}^{\,2}\Gamma(D-4)}\left(-\frac{s_1 s_2}{s}\right)^{\frac{D}{2}-2} . \tag{31}$$

Consequently, using a simple colour algebra and the crossing relations we obtain

$$\mathcal{A}_{AB}^{(8)\ A'GB'}(nf) = \Gamma^{(0)\ i_1}_{A'A}\frac{2s}{t}\Gamma^{(0)\ i_2}_{B'B}\, g T^c_{i_2 i_1}e^*_\mu(k)\left(\frac{p_A^\mu}{p_A k}-\frac{p_B^\mu}{p_B k}\right)\left(-\frac{g^2 N}{8(4\pi)^{\frac{D}{2}}}\right)(\vec{k}_\perp^{\,2})^{\frac{D}{2}-2}$$
$$\times\left[3exp\left(-i\pi\left(\frac{D}{2}-2\right)\right)+exp\left(i\pi\left(\frac{D}{2}-2\right)\right)\right]\frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{\Gamma(D-4)} \tag{32}$$

The total amplitude is given by the sum of Eqs.(27) and (32).

5 Reggeon-Reggeon-Gluon vertex

Assuming the Regge behaviour of the amplitude in the sub-channels s_1 and s_2, from general requirements of analiticity, unitarity and crossing symmetry one has (see Refs. [7,16])

$$\mathcal{A}_{AB}^{(8)\ A'GB'} = s\Gamma^{i_1}_{A'A}\frac{1}{t_1}T^c_{i_2 i_1}\frac{1}{t_2}\Gamma^{i_2}_{B'B}$$
$$\times\left\{\frac{1}{4}\left[\left(\frac{-s_1}{\mu^2}\right)^{\omega_1-\omega_2}+\left(\frac{s_1}{\mu^2}\right)^{\omega_1-\omega_2}\right]\left[\left(\frac{-s}{\mu^2}\right)^{\omega_2}+\left(\frac{s}{\mu^2}\right)^{\omega_2}\right]R\right.$$

$$\left.+\frac{1}{4}\left[\left(\frac{-s_2}{\mu^2}\right)^{\omega_2-\omega_1}+\left(\frac{s_2}{\mu^2}\right)^{\omega_2-\omega_1}\right]\left[\left(\frac{-s}{\mu^2}\right)^{\omega_1}+\left(\frac{s}{\mu^2}\right)^{\omega_1}\right]L\right\} , \tag{33}$$

where $\omega_i = \omega(t_i)$ and the RRG vertices R and L are real in all physical channels. In the region (20) of small $k_\perp$ this representation is reduced to

$$\mathcal{A}_{AB}^{(8)\ A'GB'} = s\Gamma^{i_1}_{A'A}\frac{1}{t}T^c_{i_2 i_1}\frac{1}{t}\Gamma^{i_2}_{B'B}\left\{\left[\left(\frac{-s}{\mu^2}\right)^{\omega}+\left(\frac{s}{\mu^2}\right)^{\omega}\right]\frac{R+L}{2}\right. \tag{34}$$
$$\left.+\left[\left(\frac{s}{\mu^2}\right)^{\omega}\left(\ln\left(\frac{\vec{k}_\perp^{\,2}}{\mu^2}\right)-i\pi\right)+\left(\frac{-s}{\mu^2}\right)^{\omega}\ln\left(\frac{\vec{k}_\perp^{\,2}}{\mu^2}\right)\right]\frac{(\omega_1-\omega_2)}{2}\frac{(R-L)}{2}\right\} .$$

Comparing the above form with Eqs. (27) and (32) we conclude that

$$R-L = ge^*_\mu(k)\left(\frac{p_A^\mu}{p_A k}-\frac{p_B^\mu}{p_B k}\right)\frac{t}{\omega_1-\omega_2}$$
$$\times\left(-\frac{2g^2 N}{(4\pi)^{\frac{D}{2}}}(\vec{k}_\perp^{\,2})^{\frac{D}{2}-2}\frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{\Gamma(D-4)}\frac{\sin\left(\pi\left(\frac{D}{2}-2\right)\right)}{\pi}\right)$$

$$R+L = 2ge^*_\mu(k)\left(\frac{p_A^\mu}{p_A k}-\frac{p_B^\mu}{p_B k}\right)t\left\{1-\omega(t)\ln\left(\frac{-t}{\mu^2}\right)-\frac{\Gamma^2\left(3-\frac{D}{2}\right)\Gamma^3\left(\frac{D}{2}-2\right)}{2\Gamma(D-4)}\right.$$
$$\left.\times\frac{g^2 N}{(4\pi)^{\frac{D}{2}}}(\vec{k}_\perp^{\,2})^{\frac{D}{2}-2}\left[\cos\left(\pi\left(\frac{D}{2}-2\right)\right)-\frac{\sin\left(\pi\left(\frac{D}{2}-2\right)\right)}{\pi}\ln\left(\frac{\vec{k}_\perp^{\,2}}{\mu^2}\right)\right]\right\} . \tag{35}$$

At $D\to 4$ the above expressions for the vertices coincide, taking into account the charge renormalization, with the small $k_\perp$ limit of the corresponding expressions of Ref. [14] (see Eq.(86) there). Independently we have performed the straightforward calculation of the RRG vertices at small $k_\perp$ for arbitrary D and have obtained the result (35.

6 Conclusion

We used the Gribov theorem about the region of applicability of the soft insertion formulas in framework of the programm of calculation of the next-to-leading corrections to the BFKL equation in order to find the Reggeon-Reggeon-Gluon vertex in QCD in the region of small transverse momentum of the gluon for the case of arbitrary space-time dimension D.

This example demonstrates, that the theorem can be used in massless gauge theories not only for calculations with double logarithmic accuracy, as it was done before, but for calculations with accuracy up to a constant. The crucial point for applications of the theorem in both cases is that the soft insertion is valid for (and only for) contributions of kinematical regions, where transverse momenta of virtual particles are much smaller than transverse momentum of considered gauge particle (gluon in QCD). Since in theories with massless particles the essential transverse momenta of virtual particles can be arbitrary small, the theorem can not be applied for them literally. In the double logarithmic approximation the idea, which permits to use the theorem, is based on the fact, that in this approximation all transverse momenta can be considered as strongly ordered; therefore, one can always find a particle with transverse momentum, which is much smaller than others, and apply the theorem to this particle. Evidently, this idea can not work for calculation with accuracy up to a constant.

Here the idea is that for $D > 4$ integrals over virtual transverse momenta are convergent in the infrared region, and scales of their convergency are determined by transverse momenta of external particles. In the case, when transverse momentum for considered gluon is much smaller than for other particles, only for small part of Feynman diagramms the scale of the convergency of integrals over virtual transverse momenta is determined by this smallest external transverse momentum. The contributions of such diagramms have to be calculated. But for the most part of Feynman diagrams the scale of the convergency is much larger than the transverse momentum of the considered gluon, and their contributions can be obtained, according to the Gribov theorem, applying soft insertion formulas.

Acknowledgements

I would like to thank the Organizing Committee of the Workshop for the invitation, for wonderful and stimulating atmosphere in Erice and for the financial support of my participation in the Wirkshop.
This talk is based on the work which was partially supported by the Russian Fund for Basic Researches and INTAS.

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