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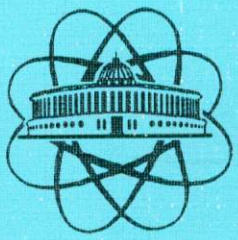

EDITORS:

M.A.IVANOV, A.B.ARBUZOV, E.A.KURAEV,
V.E.LYUBOVITSKIJ, A.G.RUSETSKY

TOP QUARK PRODUCTION AT THRESHOLD WITH $\mathcal{O}(\alpha_s^2)$ ACCURACY

KIRILL MELNIKOV[a] and ALEXANDER YELKHOVSKY[b]
[a] *Institut für Theoretische Teilchenphysik, Universität Karlsruhe, D–76128 Karlsruhe, Germany,*
[b] *Budker Institute for Nuclear Physics Novosibirsk, 630090, Russia*

ABSTRACT

We calculate the next-to-next-to-leading order correction to the cross section for top quark pair production in e^+e^- annihilation in the threshold region, resumming all $\mathcal{O}\left[(\alpha_s/\beta)^n \times (\alpha_s^2, \beta^2, \alpha_s\beta)\right]$ terms of perturbation series. We find that the magnitude of the NNLO correction is comparable to the size of the NLO corrections.

1. Introduction

The cross section of hadron production in e^+e^- annihilation belongs to the best known quantities in high energy physics. Far away from quark thresholds the cross section is well approximated by the results obtained in perturbative QCD (for a review see [1]). The situation is not so clear at quark thresholds which, however, are known to be of importance for a number of physical applications.

Among such applications, a special place is occupied by a threshold production of $t\bar{t}$ pairs at the Next Linear Collider. It was suggested in Ref. [2], that the large width of the top quark provides a natural cutoff for long–distance effects and, therefore, reliable predictions for the $t\bar{t}$ threshold production cross section are possible in perturbative QCD. Since then, the threshold production cross section of $t\bar{t}$ was studied in great detail [3, 4, 5, 6, 7]. The commonly accepted conclusion [6, 8] is that one can perform precision studies of various quantities of direct physical interest (top mass, top width, strong coupling constant, etc.), once accurate measurements in the threshold region are conducted. However, all these studies were performed using predictions for the top threshold production cross section valid up to $\mathcal{O}(\alpha_s)$ and, therefore, suffered from the ignorance of higher order QCD effects.

It is worth emphasizing, that calculation of radiative corrections to the threshold cross section differs from standard perturbative calculations, which are done for higher energies. The difference is because of the fact that close to the threshold, the conventional perturbation theory breaks down [9]. The physical origin of this phenomena is known from quantum mechanics: considering Coulomb potential as a perturbation, one gets series in α/β, where α is the strength of the potential and β is the particle velocity. When the velocity is small, this ratio becomes large and meaningful predictions can be only achieved once the series is resummed. It was demonstrated in [9], that if such resummation is performed, the threshold cross section becomes proportional to the square of the Coulomb wave function at the origin. In Ref. [2] this result was generalized to the situation, when

the produced particles are unstable. It was concluded, that the cross section for the quark pair production is proportional to the imaginary part of the non–relativistic Green function of the $\bar{Q}Q$ system, evaluated for complex energies.

Since then, it was also realized, that the $\mathcal{O}(\alpha_s)$ corrections can be easily incorporated, because contributions of soft and hard scales completely factorize with this accuracy. The absence of this factorization property, as well as the technical difficulties with explicit higher order calculations, were the stumbling blocks in achieving the $\mathcal{O}(\alpha_s^2)$ accuracy. It is remarkable, that new results, obtained in the last several years, permit a relatively easy determination of these corrections.

In what follows, we present the calculation of the threshold cross section for the $t\bar{t}$ pair production which is valid with $\mathcal{O}(\alpha_s^2, \alpha_s\beta, \beta^2)$ accuracy.

2. The framework of the calculation

We first discuss a framework of our calculations and introduce all relevant notations. The threshold region is characterized by a small value of the quark velocity β:

$$\beta = \sqrt{1 - \frac{4m^2}{s}} \ll 1. \tag{1}$$

To order $\mathcal{O}(\alpha_s^2, \alpha_s\beta, \beta^2)$, dynamics of slowly moving quark–antiquark pair is governed by a non–relativistic Hamiltonian*):

$$H = H_0 + V_1(r) + U(\boldsymbol{p}, \boldsymbol{r}), \tag{2}$$

$$H_0 = \frac{\boldsymbol{p}^2}{m} - \frac{C_F a_s}{r}, \tag{3}$$

$$V_1(r) = -\frac{C_F a_s^2}{4\pi r}\left\{2\beta_0 \ln(\mu' r) + a_1 + \frac{a_s}{4\pi}\left[\beta_0^2\left(4\ln^2(\mu' r) + \frac{\pi^2}{3}\right) + 2(\beta_1 + 2\beta_0 a_1)\ln(\mu' r) + a_2\right]\right\}, \tag{4}$$

$$U(\boldsymbol{p}, \boldsymbol{r}) = -\frac{\boldsymbol{p}^4}{4m^3} + \frac{\pi C_F a_s}{m^2}\delta^{(3)}(\boldsymbol{r}) - \frac{C_F a_s}{2m^2 r}\left(\boldsymbol{p}^2 + \frac{\boldsymbol{r}(\boldsymbol{r}\boldsymbol{p})\boldsymbol{p}}{r^2}\right) + \frac{C_F a_s}{4m^2 r^3}\boldsymbol{S}\boldsymbol{L} - \frac{C_F a_s}{2m^2}\left(\frac{\boldsymbol{S}^2}{r^3} - 3\frac{(\boldsymbol{S}\boldsymbol{r})^2}{r^5} - \frac{8\pi}{3}\delta^{(3)}(\boldsymbol{r})\right) - \frac{C_A C_F a_s^2}{2mr^2}. \tag{5}$$

In the above equations, the strong coupling constant is evaluated at the scale μ:

$$a_s \equiv \alpha_s(\mu). \tag{6}$$

The scale μ' equals to μe^{γ}, γ is the Euler constant.

The operator $U(\boldsymbol{p}, \boldsymbol{r})$ is the QCD generalization of the standard Breit potential [10]. The last term in Eq.(5) is the non–Abelian contribution, originating from a correction to the Coulomb gluon exchange, caused by a magnetic gluon [11]. The potential $V_1(r)$ represents a deviation of the static QCD potential from the Coulomb one. It was calculated

*) One can describe the $\bar{Q}Q$ system by means of the non–relativistic quantum mechanics to this order since the radiation of real gluons shows up only at $\mathcal{O}(\beta^3)$ order.

to order α_s^2 in [12] and to order α_s^3 in [13]. The coefficients there read explicitly:

$$\begin{aligned}
\beta_0 &= \frac{11}{3}C_A - \frac{4}{3}N_L T_R, \\
\beta_1 &= \frac{34}{3}C_A^2 - \frac{20}{3}C_A T_R N_L - 4C_F T_R N_L, \\
a_1 &= \frac{31}{9}C_A - \frac{20}{9}T_R N_L, \\
a_2 &= \left(\frac{4343}{162} + 6\pi^2 - \frac{\pi^4}{4} + \frac{22}{3}\zeta_3\right) C_A^2 - \\
&\quad \left(\frac{1798}{81} + \frac{56}{3}\zeta_3\right) C_A T_R N_L - \left(\frac{55}{3} - 16\zeta_3\right) C_F T_R N_L + \left(\frac{20}{9}T_R N_L\right)^2 .
\end{aligned} \tag{7}$$

For the $SU(3)$ color group, the color factors are $C_A = 3, C_F = 4/3, T_R = 1/2$. $N_L = 5$ is the number of quarks whose masses have been neglected.

Given the Hamiltonian H, one can find the Green function for the Schrödinger equation:

$$(H - E - i\delta)G(E; \boldsymbol{r}, \boldsymbol{r}') = \delta^{(3)}(\boldsymbol{r} - \boldsymbol{r}'). \tag{8}$$

Once the Green function is found, the cross section of the non-relativistic $Q\bar{Q}$ pair production in e^+e^- annihilation[†)] is obtained as:

$$\sigma(s) = \frac{4\pi\alpha^2}{3s}R(s), \tag{9}$$

where

$$R(s) = \lim_{r\to 0} \mathrm{Im}\left[N_c e_Q^2 \frac{24\pi}{s}\left(1 - \frac{\boldsymbol{p}^2}{3m^2}\right) G(E; \boldsymbol{r}, 0)\right], \qquad E = \sqrt{s} - 2m. \tag{10}$$

In Eq.(10), we have included the $\mathcal{O}(\beta^2)$ correction originating from the expansion of the vector current which produces and annihilates a heavy $Q\bar{Q}$ pair in the triplet S-state. The quantity $R(s)$ will be the central object for further discussion.

If a calculation of R will be attempted, one will find, that the Green function at the origin does not exist because there are terms in the Hamiltonian H, which behave as $1/r^n$, $n \geq 2$, for small values of r. The difficulty originates from the fact, that the region $r \to 0$ is not properly treated in the Hamiltonian. Indeed, small values of r correspond to a region in the momentum space, where a typical momentum transfer between $\bar{Q}$ and Q, is of the order of the quark masses and therefore quarks cannot be considered as non-relativistic. For this reason, the use of the Hamiltonian H in actual calculations leads to the divergencies, which appear for $r \to 0$.

The way to circumvent this difficulty is as follows. In order to perform a calculation, one introduces a cutoff λ, such that $\alpha_s m \ll \lambda \ll m$. The momenta region where $k \ll \lambda$ is a non-relativistic region and can be described using a Hamiltonian H. The momenta region with $k \gg \lambda$ is a relativistic one and the calculation in this region should be performed

[†)] In what follows, we consider only photon mediated process and do not take into account the Z-boson exchange. The axial-vector coupling of the Z-boson contributes $\mathcal{O}(\beta^2)$ relative correction to the threshold cross section. The vector Ze^+e^- coupling is also suppressed, but can be taken into account in the same way as the photon contribution, which we treat in this paper.

using the rules of quantum field theory. We note that this is rather standard procedure for calculations, related to bound state problems. It is also well known that its practical realization often requires substantial effort.

However, there is a possibility to use the result of the non-relativistic calculation with the cutoff in the following way: one takes the limit of the obtained result, considering kinematic region where $\alpha_s \ll \beta \ll 1$. In this particular region, the non-relativistic results are still valid; on the other hand as long as $\alpha_s/\beta \ll 1$, the resummation of the Coulomb effects is not necessary. Therefore, in this particular region, one can calculate the corrections applying the standard rules of the quantum field theory. In the framework of QCD, such calculations have been performed recently in Ref. [15]. One therefore can match the result of the non-relativistic calculation with the cutoff, directly to the result presented in [15] and in this way completely eliminate the cutoff dependence. This procedure was suggested in [14] and we will call it a direct matching procedure, in accordance with that reference.

In what follows, we pursue this program in QCD. We confine ourselves to a strictly perturbative approach and we do not attempt any discussion of non-perturbative effects. In order to accommodate the phenomenologically relevant case of the unstable top quark, we will consider the total energy E as the complex variable $E \to E + i\Gamma_t$, in the spirit of Ref. [2].

3. Matching and final result for R

We obtain the following final result for R at NNLO:

$$R_{\mathrm{NNLO}} = \frac{6\pi}{m^2}N_c e_Q^2 \left(1 + C_1 C_F\left(\frac{a_h}{\pi}\right) + C_2 C_F\left(\frac{a_h}{\pi}\right)^2\right) \mathrm{Im}\left\{\left(1 - \frac{5\beta^2}{6}\right) G(r_0, r_0)\right\}. \tag{11}$$

Here we have factored out all energy-independent corrections. They are parametrized by the constants C_1 and C_2, which are divergent in the limit $r_0 \to 0$. For this reason we use $a_h = \alpha_s(m)$ as the expansion parameter for these "hard" corrections. In (11), all non-singular vanishing in the limit $r_0 \to 0$ terms of the Green's function are omitted.

To get rid of the r_0-dependence we use the direct matching procedure, suggested in Ref. [14]. For this we consider $\sqrt{s} > 2m$, set the width of the top quark Γ_t to zero and equate our result (11) to its perturbative counterpart [15] in the kinematic region $\alpha_s \ll \beta \ll 1$, where both are supposed to be valid. We also set $\mu = m$, so that a_s coincides with a_h.

Let us note, that the direct matching procedure fixes the linear combination of C_2 and $\ln(mr_0)$

$$C_2 C_F\left(\frac{\alpha_s}{\pi}\right)^2 - 2\kappa\ln(mr_0). \tag{12}$$

If we were working strictly to NNLO, this last combination would be the only thing we need for the final result. However, because of the large difference in scales, which govern relativistic and non-relativistic physics, we would like to write Eq.(11) in a factorized form and include an exact dependence on r_0 into the non-relativistic Green function. For this reason, we have to set a factorization scale. We do this by choosing r_0 in such a way, that the correction to the Coulomb Green function due to the $1/r^2$ perturbation in the region $\alpha_s \ll \beta \ll 1$ is given by $\log(\beta)$, without additional constants. Any other choice of r_0 would correspond to other (also legitimate) value of the factorization scale.

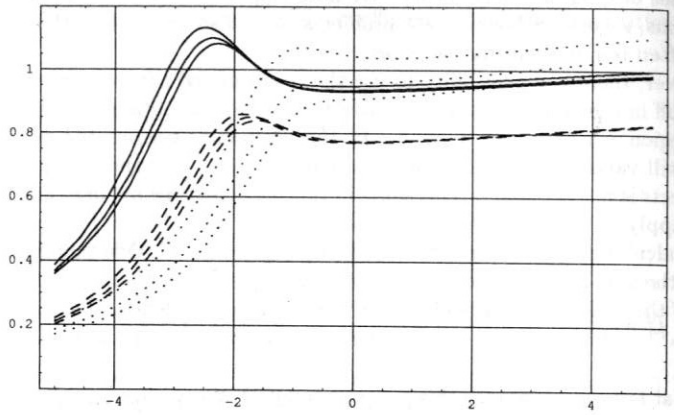

Figure 1: $R_{\rm LO}$ *(dotted lines),* $R_{\rm NLO}$ *(dashed lines),* $R_{\rm NNLO}$ *(solid lines) as a function of energy* $\sqrt{s}-2m$*, GeV. In all three cases, three curves correspond to different choices of the soft scale* $\mu = 50$ GeV *(upper curves),* $\mu = 75$ GeV *and* $\mu = 100$ GeV *(lower curves). We also use* $m = 175$ GeV, $\Gamma_t = 1.43$ GeV *and* $\alpha_s(M_Z) = 0.118$ *as the input parameters.*

A factorized form (11) of our final result makes sense only if a dependence on a choice of the factorization scale is weak. We have checked that changing the value of the cutoff between $r_0/2$ and $2r_0$ for r_0 given by Eq.(13), we obtain small ($\sim 1-3\%$) variation of the resulting values of R.

Therefore, accoring to our choice, we fix the value of the cutoff

$$r_0 = \frac{e^{2-\gamma}}{2m}, \tag{13}$$

and obtain finally:

$$C_1 = -4; \qquad C_2 = C_F C_2^A + C_A C_2^{NA} + T_R N_L C_2^L + T_R N_H C_2^H, \tag{14}$$

where

$$\begin{aligned} C_2^A &= \frac{39}{4} - \zeta_3 + \pi^2\left(\frac{4}{3}\ln 2 - \frac{35}{18}\right); \\ C_2^{NA} &= -\frac{151}{36} - \frac{13}{2}\zeta_3 + \pi^2\left(\frac{179}{72} - \frac{8}{3}\ln 2\right); \\ C_2^H &= \frac{44}{9} - \frac{4}{9}\pi^2; \\ C_2^L &= \frac{11}{9}. \end{aligned} \tag{15}$$

Eq.(11) with definitions provided by Eqs.(13 –15) is our final result for the top quark threshold cross section with the NNLO accuracy.

For numerical purposes, we have chosen $m = 175$ GeV and $\Gamma_t = 1.43$ GeV. As an input value for the strong coupling constant we used $\alpha_s(M_Z) = 0.118$. Fig.1 provides our final results for $R_{\rm NNLO}$ as a function of $\sqrt{s}-2m$ in comparison with LO and NLO results, for three values of the soft scale $\mu = 50,\ 75,\ 100$ GeV. One can see that the NNLO corrections are as large as the NLO ones.

There is also a moderate scale dependence of the NNLO corrections in the vicinity of the resonance peak. The position of the resonance peak appears to be sensitive to the variations in the scale μ on the level ~ 100 MeV. We note in this respect, that the shift of the ground–state energy due to the Breit perturbation is well known (see Ref. [10]) and its expected variation with μ is close to this value.

4. Conclusions

We have presented a calculation of the next-to-next-to-leading order corrections to the threshold cross section of the top quark pair production in QCD, summing all $\mathcal{O}\left[(\alpha_s/\beta)^n \times (\alpha_s^2, \beta^2, \alpha_s\beta)\right]$ terms of the perturbation series. We have found, that the NNLO effects are quite sizable.

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CAN α^2 CORRECTIONS BE HUGE ENOUGH TO RESOLVE THE POSITRONIUM DECAY PROBLEM?

I.B. KHRIPLOVICH
Budker Institute of Nuclear Physics,
630090 Novosibirsk, Russia

ABSTRACT

Second-order corrections to the positronium decay rate, calculated up to now, are $35(\alpha/\pi)^2$ and $76(\alpha/\pi)^2$ for singlet and triplet states, respectively.

1. The strong disagreement between the experimental value of the orthopositronium decay rate [1]

$$\Gamma_{exp}^{o-Ps} = 7.048\,2(16)\ \mu s^{-1} \tag{1}$$

and its theoretical value which includes the order α and $\alpha^2\log(1/\alpha)$ corrections [2–6]

$$\Gamma_{th}^{o-Ps} = m\alpha^6\,\frac{2(\pi^2-9)}{9\pi}\left[1-10.2866(6)\frac{\alpha}{\pi}-\frac{1}{3}\alpha^2\log\frac{1}{\alpha}\right]$$

$$= 7.038\,236(10)\ \mu s^{-1} \tag{2}$$

is the most acute problem in the modern low-energy QED. For the disagreement to be resolved within the QED framework, the correction $\sim(\alpha/\pi)^2$, which has not been calculated completely up to now, should enter the theoretical result (2) with a numerical factor 250(40), which may look unreasonably large.

Though the result of a more recent experiment [7]

$$\Gamma_{exp}^{o-Ps} = 7.0398 \pm 0.0025(\text{stat}) \pm 0.0015(\text{syst})\ \mu s^{-1} \tag{3}$$

does not demand by itself such a large second-order correction, the problem of evaluating $\sim(\alpha/\pi)^2$ terms certainly exists.

One class of large second-order corrections arises as follows [8]. The large, ~ -10, factor at the α/π correction to the decay rate (see (2)) means that the factor at the α/π correction to the decay amplitude is roughly -5. Correspondingly, this correction squared contributes about $25(\alpha/\pi)^2$ to the decay rate. In fact, $1/4\cdot 10.28^2\,(\alpha/\pi)^2$ is the lower limit for this contribution and its true value, as obtained by numerical calculations, is [9, 10]

$$28.86\,(\alpha/\pi)^2. \tag{4}$$

There is one more class of potentially large contributions to the positronium decay rate. I mean relativistic corrections. A simple argument in their favour is that the corresponding parameter $(v/c)^2 \sim \alpha^2$ is not suppressed, as distinct from that of usual second-order radiative corrections, $(\alpha/\pi)^2$, by the small factor $1/\pi^2 \sim 1/10$. In this talk I present the results for the relativistic corrections to the positronium decay rate

obtained by A.I. Milstein and myself [11, 12]. Our technique is based on such simple tools as the tree-level QED, starting with the noncovariant perturbation theory, plus the Breit equation. The noncovariant perturbation theory, as distinct from the covariant, Feynman one, allows us to work in a natural way with bound particles, treating properly their binding energy (see, e.g., [13]).

The problem of relativistic corrections to the positronium decay rate was addressed previously in [14, 15]. We differ essentially from those authors in the approach to the problem and, which is more essential, in the conclusion made. At least one origin of the disagreement will be elucidated below. The result of the more recent paper [16] agrees with ours.

As to the relativistic correction to the parapositronium decay rate, also obtained in our article [11], its calculation was started by us as a warm-up exercise for the much more complicated orthopositronium problem. However, the correction in the singlet case also turns out large, quite close to the sensitivity of the recent experiment [17].

2. The central point when treating relativistic corrections to the positronium decay rate is as follows. When calculating the decay amplitude we have to integrate the annihilation kernel $M(\vec{p})$ over the distribution of the electron and positron three-momenta $\vec{p}$. In the present section we address the relativistic corrections to $M(\vec{p})$ only, i.e., we take as the ground-state wave function $\psi(p)$ the nonrelativistic one. Then the decay amplitude is

$$\int\frac{d\vec{p}}{(2\pi)^3}\,\psi(\vec{p})\,M(\vec{p}) = \int\frac{d\vec{p}}{(2\pi)^3}\,\frac{8\sqrt{\pi a^3}}{(p^2a^2+1)^2}\,M(\vec{p}), \tag{5}$$

where $a = 2/m\alpha$ is the positronium Bohr radius. To lowest order in v/c the kernel $M(0)$, both for para- and orthopositronium, is independent of those momenta, and we are left with

$$M(0)\int\frac{d\vec{p}}{(2\pi)^3}\,\frac{8\sqrt{\pi a^3}}{(p^2a^2+1)^2} = M(0)\,\psi(\vec{r}=0). \tag{6}$$

Thus, in the limit $p\to 0$ we obtain the common prescription: the positronium decay rate is proportional to $|\psi(r=0)|^2$.

However, already to first order in $(p/m)^2$ the momentum integral

$$\int d\vec{p}\,(p/m)^2\,\frac{8\sqrt{\pi a^3}}{(p^2a^2+1)^2} \tag{7}$$

linearly diverges at $p\to\infty$, which precludes the straightforward evaluation of these relativistic corrections.

The crucial observation is that the true relativistic expression for the annihilation kernel does not grow up at $p\to\infty$, as distinct from its expansion in p/m. So, the initial integral (5) in fact converges.

When treating relative corrections to the decay amplitude, it is convenient to single out from it the factor $\psi(r=0) = (\pi a^3)^{-1/2}$, and a trivial overall dimensional factor from $M(\vec{p})$. So, we investigate, instead of (5), the following expression:

$$\int\frac{d\vec{p}}{(2\pi)^3}\,\frac{8\pi a^3}{(p^2a^2+1)^2}\,M(\vec{p}) \tag{8}$$

with dimensionless $M(\vec{p})$.

Let us consider first an auxiliary integral

$$\int \frac{d\vec{p}}{(2\pi)^3}\,\frac{8\pi a^3}{p^4a^4}\,[M(\vec{p})-M(0)], \tag{9}$$

which converges both at low and high p. After the angular integration, the dimensionless kernel $M(\vec{p})$ depends on the ratio $y^2=(p/m)^2$ only, and the expression (9) reduces to

$$\frac{2\alpha}{\pi}\int_0^\infty \frac{dy}{y^2}\,[M(y^2)-M(0)]. \tag{10}$$

In other words, this auxiliary integral is of first order in α/π, and therefore of no interest for our problem. This is a first- order radiative correction absorbed already by $-10.286\,\alpha/\pi$ in (2) (for orthopositronium). In fact, we have neglected in this argument the kernel dependence on the positronium binding energy (this dependence certainly exists in the noncovariant perturbation theory). But corrections effectively neglected in this way, are of higher odd powers in α.

So, expression (9) can be used as a regulator, perfectly convergent and unique. In the now rapidly converging integral

$$\int \frac{d\vec{p}}{(2\pi)^3}\,8\pi a^3\left[\frac{1}{(p^2a^2+1)^2}-\frac{1}{p^4a^4}\right][M(\vec{p})-M(0)] \tag{11}$$

we can safely expand $M(\vec{p})$ up to $(p/m)^2$ included. In this way we obtain

$$\int \frac{d\vec{p}}{(2\pi)^3}\,8\pi a^3\left[\frac{1}{(p^2a^2+1)^2}-\frac{1}{p^4a^4}\right]\left(\frac{p}{m}\right)^2=-\frac{3}{4}\alpha^2. \tag{12}$$

It can be demonstrated in the analogous way that $(p/m)^4\to(5/16)\alpha^4$ (neglecting contributions $\sim\alpha^3$).

Our alternative derivation [11] of the result (12) demonstrates that α^2 correction is generated by the pole of the atomic wave function $8\sqrt{\pi a^3}/(p^2a^2+1)^2$ in the initial integral (5), and the contribution on the order of α is due to the square-root cuts in the annihilation amplitude $M(\vec{p})$.

3. We start calculations with a more simple case of parapositronium. Here the noncovariant annihilation amplitude can be written as

$$M=4\pi\alpha V^+(\vec{e}_2\vec{\alpha})\frac{\Lambda_+(\vec{p}-\vec{k}_1)-\Lambda_-(\vec{p}-\vec{k}_1)}{E-\omega-\epsilon(\vec{p}-\vec{k}_1)-\epsilon(p)}(\vec{e}_1\vec{\alpha})U+(1\leftrightarrow 2)\,,$$

$$V=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\left(1-\frac{\vec{\alpha}\vec{p}}{\epsilon(p)+m}\right)\begin{pmatrix}0\\ \chi\end{pmatrix},\ U=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\left(1+\frac{\vec{\alpha}\vec{p}}{\epsilon(p)+m}\right)\begin{pmatrix}\phi\\ 0\end{pmatrix} \tag{13}$$

In this expression χ and ϕ are nonrelativistic spinors; $E=2m-m\alpha^2/4$ is the positronium total energy; $\vec{e}_{1,2}$ and $\vec{k}_{1,2}$ are the polarizations and momenta of the photons; $\omega_1=\omega_2=\omega=E/2$ are their frequencies; $\epsilon(p)=\sqrt{m^2+p^2}$;

$$\Lambda_\pm(\vec{p})=\frac{1}{2}\left(1\pm\frac{\vec{\alpha}\vec{p}+\beta m}{\epsilon(p)}\right)$$

are the projectors onto the positive and negative energy states of a fermion with a momentum $\vec{p}$ correspondingly. The Coulomb interaction in the intermediate state can be neglected since the momentum of one particle in it is close to m.

The expansion of the amplitude in p/m is straightforward. Averaging over the directions of $\vec{p}$ (an S-state is under discussion) and using relation (12) we obtain

$$M+\delta M=\left[1+\alpha^2\left(\frac{1}{2}+\frac{\sqrt{2}}{8}\right)\right]M, \tag{14}$$

where M is the lowest order annihilation amplitude. The corresponding relative correction to the decay rate is

$$\frac{\delta\Gamma}{\Gamma}=\alpha^2\left(1+\frac{\sqrt{2}}{4}\right)=1.35\alpha^2. \tag{15}$$

4. The calculation of relativistic corrections for the triplet state (decaying into three photons) is much more tedious problem. We believe that have managed to simplify it considerably, but still it is too lengthy to be presented in detail here. So, only its brief outline is given below.

The construction of noncovariant perturbation theory amplitude is straightforward. Then we rewrite the initial energy E in the perturbative denominators as

$$E=2\epsilon(p)+E-2\epsilon(p)$$

and expand the amplitude in

$$E-2\epsilon(p)=\frac{m\alpha^2}{2}$$

(here we use already the recipe (12) for p^2/m^2).

Zeroth term of this expansion transforms into usual covariant Feynman amplitude for electron and positron with 4-momenta $(\epsilon(p),\pm\vec{p})$. Now we expand this covariant amplitude in p/m, average the terms of second order in p/m over the directions of $\vec{p}$ and make the substitution (12). We obtain in this way the following expression:

$$\begin{aligned}M_c=\frac{5}{8}\alpha^2M_0-\frac{(4\pi\alpha)^{3/2}}{2m^2}\frac{\alpha^2}{4}\chi^\dagger\{&[(\vec{e}_2\vec{e}_3)(\vec{n}_1\vec{n}_3+\vec{n}_1\vec{n}_2-\vec{n}_2\vec{n}_3)\\ &-(\vec{n}_2\vec{n}_3)(\vec{h}_2\vec{h}_3)](\vec{\sigma}\vec{e}_1)+[(\vec{e}_2\vec{h}_3)(\vec{n}_1\vec{n}_3)+(\vec{e}_3\vec{h}_2)(\vec{n}_1\vec{n}_2)](\vec{\sigma}\vec{h}_1)\\ &+\frac{4m^2}{\omega_2\omega_3}(\vec{e}_3\vec{e}_3)(\vec{\sigma}\vec{e}_1)\\ &+\frac{2m}{\omega_1}[(\vec{e}_2\vec{e}_3+\vec{h}_2\vec{h}_3)(\vec{\sigma}\vec{e}_1)+(\vec{e}_1\vec{n}_3)([\vec{e}_2\vec{h}_3]\vec{\sigma})+(\vec{e}_1\vec{n}_2)([\vec{e}_3\vec{h}_2]\vec{\sigma})]\\ &+(1\leftrightarrow 2)+(1\leftrightarrow 3)\}\phi.\end{aligned} \tag{16}$$

Here $\vec{n}_i=\vec{k}_i/\omega_i$; $\vec{h}_i=[\vec{n}_i\vec{e}_i]$;

$$\begin{aligned}M_0=-\frac{(4\pi\alpha)^{3/2}}{2m^2}\chi^\dagger[&(\vec{e}_2\vec{e}_3-\vec{h}_2\vec{h}_3)(\vec{\sigma}\vec{e}_1)+(\vec{e}_2\vec{h}_3+\vec{e}_3\vec{h}_2)(\vec{\sigma}\vec{h}_1)\\ &+(1\leftrightarrow 2)+(1\leftrightarrow 3)]\phi\end{aligned} \tag{17}$$

is the amplitude of zeroth order in v/c.

The interference of this α^2-correction with the lowest order amplitude after the summation over the polarizations of the photons and integration over the final phase space, generates the following correction to the decay rate:

$$\frac{\delta\Gamma_c}{\Gamma} = \alpha^2 \frac{31\pi^2 - 240}{16(\pi^2 - 9)}. \tag{18}$$

In fact, before starting the tedious algebra which leads to the last formula, we tried a simplified estimate of this correction. Both M_c and M_0 were taken at the center of the Dalitz plot, and then the product of those expressions, after summation over the polarizations, was multiplied by the three-photon phase space. The result turned out huge (in good agreement with formula (18) obtained afterwards). Then the accuracy of the trick was checked by applying it to the estimate of the zeroth order probability generated by M_0. This last estimate coincided with the exact expression within few percent: the exact factor $\pi^2 - 9 \simeq 0.87$ (see (2)) was approximated by $27/32 \simeq 0.84$*). Only then, with good reasons to expect that the correction is large indeed, did we resort to the lengthy calculations.

The correction (18) is conveniently combined with that for the phase space. The shift of the total energy from $2m$ to $E = 2m - m\alpha^2/4$ changes the phase space and therefore the decay rate by

$$\frac{\delta\Gamma_p}{\Gamma} = -\frac{1}{4}\alpha^2. \tag{19}$$

In this way we come to the following total "covariant" correction to the decay rate

$$\frac{\delta\Gamma_c + \delta\Gamma_p}{\Gamma} = \alpha^2 \frac{27\pi^2 - 204}{16(\pi^2 - 9)}. \tag{20}$$

The prescription $v^2 \to -\alpha^2/4$ used in [15] is obviously correct for the phase space, but it differs from our formula (12) for the expansion of the annihilation kernel. We believe that the above simple and transparent derivation of formula (12) makes obvious which of the two results for v^2 in the expansion of $M(\vec{p})$ is correct. Contrary to some assertions, this discrepancy cannot be ascribed to the fact that neither of the two approaches results in complete α^2 correction. The disagreement refers to a contribution which is gauge-invariant, well- defined, unique.

Let us consider now the "noncovariant" correction to the annihilation amplitude (i.e., terms proportional to $E - 2\epsilon(p) = m\alpha^2/2$ in the expansion of the exact amplitude):

$$M_n = \frac{(4\pi\alpha)^{3/2}\alpha^2}{8m}\chi^\dagger \Big\{ [(\vec{e}_1\vec{e}_3)(\vec{\sigma}\vec{e}_2) + (\vec{e}_1\vec{e}_2)(\vec{\sigma}\vec{e}_3) - (\vec{e}_2\vec{e}_3)(\vec{\sigma}\vec{e}_1)]$$

$$\times \left[\frac{(m + \omega_2 - \epsilon_2)(m + \omega_3 - \epsilon_3)}{2\epsilon_2\epsilon_3(\epsilon_2 + \epsilon_3 - \omega_1)} + \frac{\epsilon_2 - \omega_2}{m\epsilon_2} + \frac{\epsilon_3 - \omega_3}{m\epsilon_3} \right]$$

*) An assertion is widely spread that the problem with the orthopositronium decay rate is not that the higher-order corrections to it are large, but that the lowest order probability is accidentally small. It is based on the observation that this probability is proportional to a small difference of two large numbers: $\pi^2 - 9 \sim 1$. Our estimate demonstrates that the assertion is wrong: $\pi^2 - 9$ is nothing but an educated presentation of a number naturally close to unity.

$$+[(\vec{e}_1\vec{h}_3)(\vec{\sigma}\vec{h}_2) + (\vec{e}_1\vec{h}_2)(\vec{\sigma}\vec{h}_3) - (\vec{h}_2\vec{h}_3)(\vec{\sigma}\vec{e}_1)] \tag{21}$$

$$\times \left[\frac{(m - \omega_2 + \epsilon_2)(m - \omega_3 + \epsilon_3)}{2\epsilon_2\epsilon_3(\epsilon_2 + \epsilon_3 - \omega_1)} + \frac{m + \epsilon_2 - \omega_2}{\epsilon_2(m - \epsilon_2 - \omega_2)} + \frac{m + \epsilon_3 - \omega_3}{\epsilon_3(m - \epsilon_3 - \omega_3} \right]$$

$$+(1 \leftrightarrow 2) + (1 \leftrightarrow 3) \Big\} \phi.$$

Here $\epsilon_i = \sqrt{\omega_i^2 + m^2}$. This correction to the decay rate demands numerical calculations which give

$$\frac{\delta\Gamma_n}{\Gamma} = 0.807\alpha^2. \tag{22}$$

Let us note here that the weird term with $\sqrt{2}$ in the correction to the singlet decay rate (see eqs. (14), (15)) is of the same "noncovariant" origin.

5. Let us consider at last the effects originating from relativistic corrections to the wave function $\psi(\vec{r})$ itself. We will use here the Breit equation in the way it was done in [8]. Para- and orthopositronium can be treated thus in parallel.

The part of the Breit Hamiltonian (BH) that corresponds to the relativistic corrections to the dispersion law of the particles and to their Coulomb interaction,

$$V_c = -\frac{p^4}{4m^3} + \frac{\pi\alpha}{m^2}\delta(\vec{r}), \tag{23}$$

can be easily transformed to

$$V_c = \frac{\alpha^3}{8r}. \tag{24}$$

We omit constant, independent of r terms in the perturbations (obviously, they do not lead to observable effects) and substitute $-m\alpha/2$ for ∂_r acting on the ground state positronium wave function.

The next spin-independent term in the BH

$$V_m = -\frac{\alpha}{2m^2 r}\left(p^2 + \frac{1}{r^2}\vec{r}(\vec{r}\vec{p})\vec{p}\right), \tag{25}$$

describes the magnetic electron-positron interaction due to the orbital motion. For the ground state it transforms into

$$V_m = \frac{\alpha^3}{4r} - \frac{\alpha^2}{2mr^2}. \tag{26}$$

The last term in BH of interest for our problem is the contact spin-spin interaction

$$V_{ss} = \frac{\pi\alpha}{m^2}A\delta(\vec{r}); \quad A = \frac{7}{3}S(S+1) - 2. \tag{27}$$

It is conveniently rewritten as

$$V_{ss} = A\frac{1}{4m}\left[H, \frac{\alpha}{r}\right] + A\frac{\alpha^2}{4mr^2}; \quad H = \frac{p^2}{m} - \frac{\alpha}{r}. \tag{28}$$

Straightforward calculation leads now to the following correction due to the relativistic modification of the ψ-function:

$$\frac{\delta\Gamma_\psi}{\Gamma} = \alpha^2 \begin{cases} 31/8 - 2C - 2\log(mr_0), & S=0 \\ -19/24 + 1/3C + 1/3\log(mr_0), & S=1 \end{cases} . \tag{29}$$

Here $C = 0.577$ is the Euler constant. The effective short-distance cut-off $r_0 \sim 1/m$ is provided by the range at which the annihilation takes place. We have omitted in (29) the logarithmically enhanced part of this correction

$$\alpha^2 \log\left(\frac{1}{\alpha}\right) \begin{cases} 2, & S=0 \\ -1/3, & S=1 \end{cases} , \tag{30}$$

which has been calculated previously for the triplet (see formula (2)) and singlet cases in [4] and [8] respectively. We believe that ± 1 is a reasonable estimate for the scatter of possible values of $\log(mr_0)$ introduced by the uncertainty in the short- distance cut-off r_0.

To complete the calculation of the α^2 corrections and to get rid of this uncertainty, one should proceed in the same way as it is being done in the Lamb shift problem. The correction calculated here for the atomic region of momenta with the full account for the bound-state effects, should be combined with the correction originating from higher momenta where the atomic effects can be neglected. In practice, it demands calculating the whole set of the corresponding Feynman diagrams.

Such calculations are usually performed in the momentum representation. Therefore, it is useful to rewrite formula (29) in terms of the cut-off at high momenta, instead of short distances. It can be easily done by going over to the momentum representation for the positronium wave function and introducing the maximum momentum p_0. The result differs from formula (29) by the substitutions $m_0 \to 1/p_0$, $C \to 1$. It is

$$\frac{\delta\Gamma_\psi}{\Gamma} = \alpha^2 \begin{cases} 15/8 + 2\log(p_0/m), & S=0 \\ -11/24 - 1/3\log(p_0/m), & S=1 \end{cases} . \tag{31}$$

Our prediction for the atomic relativistic correction in orthopositronium, $-11/24\alpha^2$, differs from that given in [15]. We cannot explain the disagreement, since the authors of [15] present only their numerical result for this correction, $1.16\alpha^2$, without any details.

6. To summarize, the total relativistic corrections in para- and orthopositronium constitute, respectively,

$$\frac{\delta\Gamma_r}{\Gamma} = 3.229\alpha^2 = 31.86\left(\frac{\alpha}{\pi}\right)^2, \quad S=0; \tag{32}$$

$$\frac{\delta\Gamma_r}{\Gamma} = 4.839\alpha^2 = 47.76\left(\frac{\alpha}{\pi}\right)^2, \quad S=1 \tag{33}$$

(it is instructive perhaps to present these relativistic corrections in the usual "radiative" units (α/π) as well).

The known corrections to the orthopositronium decay rate comprise, in line with (33), the already mentioned square of the first-order correction (see (4)) and the result of the polarization operator insertion into the first-order diagrams [18, 19]

$$0.965\left(\frac{\alpha}{\pi}\right)^2 . \tag{34}$$

Taken together, corrections (4), (33), and (34) constitute

$$\frac{\delta\Gamma}{\Gamma} = 77.59\left(\frac{\alpha}{\pi}\right)^2 . \tag{35}$$

Since the parapositronium decay is described by a single amplitude, the analogue of the contribution (4) is derived immediately from the well-known result [20] for the first-order correction. This second-order correction is $(1/4)(5 - \pi^2/4)(\alpha/\pi)^2 = 1.60\,(\alpha/\pi)^2$. The polarization operator insertion into the first-order correction gives here [18, 19] $0.45\,(\alpha/\pi)^2$. Taken together with (15 and (32), these corrections result in

$$\frac{\delta\Gamma}{\Gamma} = 33.91\left(\frac{\alpha}{\pi}\right)^2 . \tag{36}$$

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ANALYSIS OF SOME SECOND ORDER RADIATIVE CORRECTIONS TO THE ORTHOPOSITRONIUM DECAY WIDTH.

V. ANTONELLI, V. IVANCHENKO, E. KURAEV, V. LALIENA
Talk given by Vito Antonelli
Institute for Theoretical Physics - University of Bern,
Sidlerstrasse 5 CH-3012 Bern Switzerland

ABSTRACT

The problem of the evaluation of the orthopositronium decay width, including second order radiative corrections, is discussed. After a review of the results available in literature, some $O(\alpha^2)$ annihilation-type contributions, recently computed in [11], are analyzed.

1. Introduction

Positronium is essentially a pure QED system; strong and weak interactions can be neglected at the level of accuracy at which we are interested. Its study is an interesting test of QED calculations and of the formalism used to describe bound states in quantum field theory. It can exist in two different spin states: a singlet $S = 0$, called parapositronium, that decays mainly into a couple of photons, and a triplet $S = 1$, called orthopositronium, decaying into an odd number of photons greater than one. At the 10 ppm level of accuracy, one can restrict the analysis for orthopositronium to the decay mode with three photons in the final state.

We will focus our attention to the analysis of orthopositronium (O_{ps}) decay width. With the present results one cannot exclude a possible discrepancy between theory and experiment. This could eventually indicate also a problem in the formalism used to study this bound state system.

The more precise experimental determinations of O_{ps} decay rate are [1, 2]:

$$\lambda^{exp}_{Ops} = 7.0514 \pm 0.0014\ \mu s^{-1}\ ;\ \lambda^{exp}_{Ops} = 7.0482 \pm 0.0016\ \mu s^{-1}\ . \qquad (1)$$

A more recent experiment [3] found a different value: $\lambda^{exp}_{Ops} = 7.0398 \pm 0.0025 \pm 0.0015\ \mu s^{-1}$, where the first error is statistical and the second systematic.

Considering only the decay into three photons, the theoretical value is [†]:

$$\lambda^{th}_{Ops} = \lambda_0 \left[1 + A\frac{\alpha}{\pi} - \frac{1}{3}\alpha^2 \ln\frac{1}{\alpha} + B\left(\frac{\alpha}{\pi}\right)^2 - \frac{3}{2}\frac{\alpha^3}{\pi}\ln^2\frac{1}{\alpha} + O((\alpha/\pi)^3) \right], \qquad (2)$$

where the lowest order decay rate λ_0, first obtained by Ore and Powell [4], is given by $\lambda_0 = 7.211169 \pm 0.000004 \mu s^{-1}$. The coefficient A has been computed by many authors [5, 6, 7, 8] and its most accurate determination $A = -10.286606 \pm 0.000010$ has been found in [8]. The constants $\frac{-1}{3}$ and $\frac{-3}{2\pi}$ of the logarithmic terms have been found respectively in [6, 7, 9] and [10]. Using these results, one gets: $\lambda^{th}_{Ops} = 7.038204 \pm 0.000010 \mu s^{-1} + \lambda_0 \left[B\left(\frac{\alpha}{\pi}\right)^2 + O\left(\frac{\alpha}{\pi}\right)^3 \right]$.

Putting $B = 0$ in the previous equation, we obtain a value at $O(\alpha)$, with the inclusion of logarithmic terms up to $O(\alpha^3 \ln^2 \alpha)$, which differs by the experimental results of [2, 1] by 6.2σ and 9.4σ. To reproduce these results one should have an unnaturally big value of the coefficient $B \simeq 250$. On the other hand, $B \simeq 40$ would be enough to reproduce the result of [3]. Hence, up to now, one cannot discriminate whether the so called "O_{ps} decay width puzzle" is a theoretical or an experimental problem. It is clear, in any case, that a complete $O(\alpha^2)$ calculation is needed. Some second order contributions to the decay rate still need to be evaluated.

Here we focus our attention on some second order annihilation type radiative corrections that have recently been computed in [11].

The first simplification in the study of a bound state like positronium [12] is to consider it as a two body system. The development of a consistent relativistic two body formalism for bound state calculations is mainly due to the works of Schwinger [13] and Bethe and Salpeter [14]. We have to look for the poles of a four point Green function, that must obey the well known Bethe-Salpeter (BS) equation. In this equation a fundamental role is played by the sum of all the two particle irreducible graphs, usually denoted as the BS kernel. We can introduce the "BS wave functions", obtained by considering the projections on the two particle (antiparticle) states. Using them, one obtains an equation for the residua of the poles that has non trivial solutions for specific values of P, [††] corresponding to the energy momenta of the bound states coupled to the two particle-antiparticle states. One usually looks for perturbative solutions of this equation, by writing the kernel K as the sum of a piece K_0, whose solution is known, and a remainder ΔK. Using the fact that in a QED bound state, like positronium, the relative momentum p of the components is small, ($p \simeq \alpha\ m$), we can make a non relativistic approximation and choose the lowest order kernel of BS equation in such a way to recover the well known Schrödinger equation with a coulombic potential.

2. The orthopositronium decay rate

To obtain the lowest order O_{ps} decay width, one has to consider only one graph. At the next order of perturbation theory, we must, instead, take into account different classes of diagrams, like self-energy and vertex corrections and the radiative corrections given by the two graphs of fig. 1, usually denoted as annihilation and binding diagrams. One can easily show that the matrix element $\mathcal{M}_B$ for the second graph can be written in the form $\mathcal{M}_B = \mathcal{M}_0\left(1 - 3\frac{\alpha}{\pi}\right)$, where $\mathcal{M}_0$ is the lowest order matrix element. So it seems that this

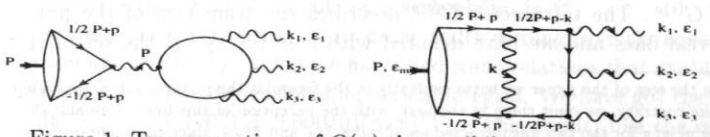

Figure 1: Two corrections of $O(\alpha)$:the annihilation graph and the binding diagram.

[†] We adopt the convention to collect an explicit power of $(\frac{\alpha}{\pi})^n$ in front of the contribution of order n, to be consistent with what usually done in literature. We would like, however, to stress that this convention can be misleading. In fact, in this way, the coefficients of the higher order radiative corrections appear unnaturally big.

[††] P is the sum of the tetramomenta of the two particles ($P = p_1 + p_2 = p_3 + p_4$) and m is the electron mass.

graph contributes not only at order α, but also at order zero. This is due to the fact that we are considering also the possibility that the additional binding photon inserted in the original lowest order diagram is a coulombic one, but in the determination of the O_{ps} wave function one has already taken into account the exchange of any number of coulombic photons between the e^+ and e^- lines. Hence, we must subtract the part corresponding to the exchange of a Coulomb photon, that is the term of order zero appearing in the original expression for $\mathcal{M}_B$. In this way, we get for the matrix element $(\alpha/\pi)\mathcal{M}'_B$ of the "subtracted binding diagram" [‡]: $(\alpha/\pi)\mathcal{M}'_B = -3\ (\alpha/\pi)\ \mathcal{M}_0$. The binding diagram is particularly important also because it gives a contribution to the decay rate, which is bigger than the 90% of the total $O(\alpha)$ radiative corrections [7].

As already said, we must consider also the second order radiative corrections, some of which have been already computed. A first relevant contribution of this order is given [15, 8] by the sum of the squares of all the first order amplitudes. They give a contribution equal to 28.860 ± 0.002 to the coefficient B of eq. (2). A second contribution [16] to B, equal to 9.0074 ± 0.0009, comes from the radiative corrections to the light-light scattering block. The inclusion of the vacuum polarization corrections to the first order graphs [17] enhances the value of B of 0.964960 ± 0.000004. Considering the decay channel into five photons, one gets [18] an additional contribution to B equal to 0.187 ± 0.011. The very important second order relativistic corrections have been studied by different authors. Khriplovich et al.[19] have found a big contribution to the coefficient B, equal to 46 ± 3, in agreement with the result of Faustov et al.[20]. Quite a different result have been found with a different approach, in the second paper of [10], by Labelle et al., that, using the so called "NRQED" have got for this contribution the value 24.6.

Generally speaking, we can write the matrix element for the sum of all the diagrams contributing up to $O(\alpha^2)$ in the following way:
$\mathcal{M} = \mathcal{M}_0 + \frac{\alpha}{\pi}\left(\mathcal{M}'_B + \mathcal{M}_A + \mathcal{M}_1\right) + \left(\frac{\alpha}{\pi}\right)^2\left(\mathcal{M}'_{AB} + \mathcal{M}_{AR} + \mathcal{M}_2\right) + O(\alpha^3)$, where M_1 is the sum of all the first order amplitudes with the exceptions of the first order annihilation diagram, M_A, and the subtracted binding amplitude, M'_B. The second order annihilation type corrections are given by the subtracted binding diagram, M'_{AB} (fig. 2(A)) and the radiative corrections to the light–light scattering block, M_{AR} (an example is given in fig. 2(B)); M_2 denotes the remaining (non-annihilation type) second order amplitudes. For what already said, it seems reasonable to look at second order corrections obtained by graphs containing some additional binding photon.

In a recent paper [11], we examined the contribution to the decay width coming from the interference between the graph of fig. 2(A) and the zero order diagram. We also considered the square of the first order annihilation amplitude $\mathcal{M}_A$, contributing at $O(\alpha^2)$ to the decay rate, and we verified the existence of a logarithmic enhanced contribution arising from the radiative correction to the light-light scattering block depicted in fig. 2(B). Let's recall briefly how one can evaluate these contributions (for more details look at [11]). The matrix element of the annihilation binding diagram can be written as $\mathcal{M}^{(m,\varepsilon)}_{AB} = -\frac{i}{4m^2} T^{(m)}_\rho\, G^{(\varepsilon)\rho}$. The tetravector $G^{(\varepsilon)\rho}$ describes the transition of the heavy photon to the three real ones and we have denoted with $\varepsilon \equiv (\varepsilon_1, \varepsilon_2, \varepsilon_3)$ the set of the

[‡]Notice that here and in the rest of the paper we write explicitly in the formulas the powers of α/π appearing in all the amplitudes; on the contrary, we omit them in the text, with the exception of this line. Note also that we will not write any power of α/π for the two unsubtracted amplitudes M_B and M_{AB}, since they contain terms of different order in α/π.

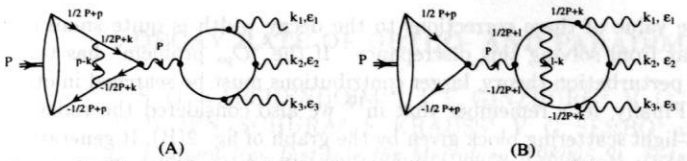

Figure 2: Two different kinds of corrections to the annihilation graph. (A) is the vertex correction, (B) represents the insertion of a photon into the light–light scattering block.

three polarizations of these photons ($\varepsilon_i = \pm 1$). The symbol $T^{(m)}_\rho$ represents the $O(\alpha)$ correction to the annihilation current 4-vector of the positronium in the polarization state ε_m. It contains a double integral, over p and k, a trace of γ matrices, including also the O_{ps} wave function $\Psi^{(m)}(p)$, and one photon propagator, that we can write as $-\frac{i\ \Delta_{\mu,\nu}(k-p)}{(k-p)^2}$. The $\Delta_{\mu\nu}$ tensor depends on the gauge we use. The choice of the gauge is subtle when dealing with bound state problems (as discussed for instance in the last paper of [6]). We have computed $T^{(m)}_\rho$ both in the Fried–Yennie (FY) gauge [21] and in the Coulomb gauge. As expected, the result is the same in both cases, and no gauge correction term must be added when using the FY gauge. Let's report the basic steps of the calculation in the FY gauge, for the analogous computation in the Coulomb gauge we refer the interested reader to [11]. In the FY gauge we have $\Delta_{\mu\nu} = g_{\mu\nu} + 2\frac{k_\mu k_\nu}{k^2}$. To perform the computation, we have splitted the trace entering $T^{(m)}_\rho$ into two pieces, one remaining non-singular at $k = 0$ and another one containing the contribution of the coulombic photon.
Formally we have used the equality: $\mathrm{Tr}_{\ \mu\nu\rho}(k) = \mathrm{Tr}_{\ \mu\nu\rho}(0) + (\mathrm{Tr}_{\ \mu\nu\rho}(k) - \mathrm{Tr}_{\ \mu\nu\rho}(0))$.
The first term gives a contribution to the matrix element proportional to the $O(\alpha)$ annihilation amplitude $(\alpha/\pi)\,\mathcal{M}_A$: $\mathcal{M}_{AB,1} = (\alpha/\pi)\,\mathcal{M}_A\,(1 - 3\,\alpha/\pi)$. The second one is infrared finite and we can safely put $p = 0$ in the loop integral, introducing an error of order $O(\alpha^2)$. The ultraviolet divergence can be regulated either with dimensional regularization or with the use of a cut-off [11]. We must add the contribution of the "annihilation vertex" counterterm, that cancels the divergence, as explicitly proved in [11]. The sum of the contributions of the second term in which we splitted $\mathrm{Tr}_{\ \mu\nu\rho}(k)$ and of the counterterm gives: $\mathcal{M}_{AB,2} = +\,(\alpha/\pi)\,(\alpha/\pi)\,\mathcal{M}_A$.
Hence we have found the following expression for the unsubtracted annihilation binding diagram in the FY gauge: $\mathcal{M}_{AB} = [1 - 2(\alpha/\pi)]\,(\alpha/\pi)\mathcal{M}_A$.

We must subtract the lowest order contribution $(\alpha/\pi)\,\mathcal{M}_A$ corresponding to the exchange of a coulombic binding photon and consider the interference of this subtracted annihilation binding diagram with the order zero graph. In this way we have got the following $O(\alpha^2)$ contribution to the O_{ps} decay width:
$\Gamma'_{AB} = -2\,(\alpha/\pi)\,\Gamma_A = 1.6281\,(\alpha/\pi)^2\,\Gamma_0$, where we have used the numerically improved value of the lowest order annihilation width Γ_A, that can be found in the first paper of [17]. This result is in very good agreement with the estimate, based on factorization arguments, of this correction, that was made by Karshenböim in [22].
To compute the contribution to the decay width coming from the square of the $O(\alpha)$ annihilation amplitude (fig. 1(A)) we have used some relations that enabled us to reduce this calculation to a one dimensional numerical integral. We have got the following numerical result: $\Gamma_{A^2} = (0.17021 \pm 0.00010)(\alpha/\pi)^2\,\Gamma_0$. This result, like the one for annihilation-binding contribution (and differently from the lowest order annihilation contribution of $O(\alpha)$) has the right sign to reduce the discrepancy between theory and experiment. Nev-

ertheless, the absolute value of these corrections to the decay width is quite small and they are manifestly far from solving this discrepancy. If the "O_{ps} problem" has to be solved by this kind of perturbation theory, larger contributions must be searched in other classes of diagrams. Finally, let's remember that in[11] we also considered the radiative correction to the light-light scattering block given by the graph of fig. 2(B). It generates a logarithmically enhanced term that gives a contribution to the decay width proportional to $\alpha^2 \ln(\alpha)\Gamma_0$. Our result is in agreement with the ones of [16], where the all set of these radiative corrections has been computed, and with those of [9] and of the first paper of [6].

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DECAY RATE OF ORTHO- AND PARADIMUONIUM

S. G. KARSHENBOIM[1•], I. F. GINZBURG[2], V. G. IVANOV[3], U. D. JENTSCHURA[4], F. KRAUSS[4], V. G. SERBO[5] and G. SOFF[3]

[1] *D. I. Mendeleev Institute for Metrology, 198005 St. Petersburg, Russia,*
[2] *Institute for Mathematics, 630090 Novosibirsk, Russia,*
[3] *Pulkovo Observatory, 196140 St. Petersburg, Russia,*
[4] *Technische Universität Dresden, 01062 Dresden, Germany,*
[5] *Novosibirsk State University, 630090 Novosibirsk, Russia.*

ABSTRACT

We present our recent results on the spectrum, the decay and the production of the dimuonium bound system.

This talk is devoted to a presentation of our recent results on the spectrum, the decay rate and the production of the $\mu^+\mu^-$ bound system (dimuonium). The main feature of dimuonium which leads to a difference with respect to positronium is that the electron is the lightest charged particle. As a result, orthodimuonium, which consists of the heavier leptons, decays into electron-positron pair through annihilation over a single virtual photon. In calculating the leading order corrections one has to take into consideration the Uehling potential effects which contribute in dimuonium in the relative order α.

We start our investigation with the **dimuonium hyperfine splitting** which is quite similar to the positronium *hfs*. The result

$$E_{hfs}(1s) = E_F \left\{ 1 + \frac{\alpha}{\pi}\left[\frac{2}{7}\ln\left(\frac{m_\mu}{m_e}\right) - 0.040(9)\right] + \left(\frac{\alpha}{\pi}\right)^2 \left[\frac{5\pi^2}{14}\ln\left(\frac{1}{\alpha}\right) + \frac{4}{21}\ln^2\left(\frac{m_\mu}{m_e}\right)\right]\right\},$$

where $E_F = 7/12\,\alpha^5 m_\mu$, includes all order α corrections (Fig. 1) [1, 2] and the leading logarithmic terms in relative order α^2 [3]. Details of the calculations in one-loop order can be found in Table 1. The higher order leading logarithmic terms with $\ln(\alpha)$ are similar to those for positronium, and the double logarithm can be easily obtained within the effective charge approach.

diagram	$C(1S)$	$C(2S)$	diagram	$C(1S)$	$C(2S)$
(g-2)-T	0.571	0.571	VPC-T	0.605	0.523
Rec	-0.857	-0.857	VPT	0.345	0.355
Vert-A	-1.714	-1.714	VPC-A	0.454	0.393
VP-μ-A	-0.381	-0.381	VP-e-A	1.483	1.483
2A	0.263	0.263	VP-h-A	-0.080(9)	-0.080(9)

•) E-mail: sgk@onti.vniim.sbp.su

NUCLEON POLARIZABILITY CONTRIBUTION TO THE HYDROGEN LAMB SHIFT AND HYDROGEN – DEUTERIUM ISOTOPE SHIFT

I.B. KHRIPLOVICH and R.A. SEN'KOV
Budker Institute of Nuclear Physics,
630090 Novosibirsk, Russia,
and Novosibirsk University

ABSTRACT

The correction to the hydrogen Lamb shift due to the proton electric and magnetic polarizabilities is expressed analytically through their static values, which are known from experiment. The numerical value of the correction is $-71 \pm 11 \pm 7$ Hz. Correction to the H-D 1S-2S – isotope shift due to the proton and neutron polarizabilities is estimated as $61 \pm 10 \pm 6$ Hz.

1. High experimental precision attained in the hydrogen and deuterium spectroscopy (see, e.g., [1, 2]) stimulates considerable theoretical activity in this field. In particular, the deuteron polarizability contribution to the Lamb shift in deuterium was calculated in [3–11]. An estimate of the proton polarizability contribution to the Lamb shift in hydrogen was made in [4]. A special feature of these corrections is that they contain logarithm of the ratio of a typical nuclear excitation energy to the electron mass, $\ln \bar{E}/m_e$.

In the present note we consider the problem of the proton polarizability correction to the Lamb shift in hydrogen. The typical excitation energy for the proton $\bar{E}_p \sim 300$ MeV is large as compared to other nuclei (to say nothing of the deuteron). So, $\ln \bar{E}_p/m_e$ is not just a mere theoretical parameter, it is truly large, about $6-7$, which makes the logarithmic approximation quite meaningful quantitatively.

In our calculation we follow closely the approach of [9]. In particular, we use the gauge $A_0 = 0$ for virtual photons, so that the only nonvanishing components of the photon propagator are $D_{im} = d_{im}/k^2$, $d_{im} = \delta_{im} - k_i k_m/\omega^2$ $(i, m = 1, 2, 3)$. The electron-proton forward scattering amplitude, we are interested in, is

$$T = 4\pi i\alpha \int \frac{d^4k}{(2\pi)^4}\, D_{im} D_{jn} \frac{\gamma_i(\hat{l} - \hat{k} + m_e)\gamma_j}{k^2 - 2lk}\, M_{mn}. \tag{1}$$

Here $l_\mu = (m_e, 0, 0, 0)$ is the electron momentum. The nuclear-spin independent Compton forward scattering amplitude, which is of interest to as, can be written as

$$M = \bar{\alpha}(\omega^2, \mathbf{k}^2)\mathbf{E}^*\mathbf{E} + \bar{\beta}(\omega^2, \mathbf{k}^2)\mathbf{B}^*\mathbf{B} = M_{mn} e_m e_n{}^*, \tag{2}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the nuclear electric and magnetic polarizabilities, respectively. The structure $\gamma_i(\hat{l} - \hat{k} + m_e)\gamma_j$ in (1) reduces to $-\omega\delta_{ij}$. Perhaps, the most convenient succession of integrating expression (1) is as follows: the Wick rotation; transforming the integral over

the Euclidean ω to the interval $(0, \infty)$; the substitution $\mathbf{k} \to \mathbf{k}\omega$. Then the integration over ω is easily performed with the logarithmic accuracy:

$$\int_0^\infty \frac{d\omega^2}{\omega^2 + 4m_e{}^2/(1+\mathbf{k}^2)^2} \left[(3 + 2\mathbf{k}^2 + \mathbf{k}^4)\bar{\alpha}(-\omega^2, -\omega^2\mathbf{k}^2) - 2\mathbf{k}^2\bar{\beta}(-\omega^2, -\omega^2\mathbf{k}^2)\right] \tag{3}$$
$$= \left[(3 + 2\mathbf{k}^2 + \mathbf{k}^4)\bar{\alpha}(0) - 2\mathbf{k}^2\bar{\beta}(0)\right] \ln \frac{\bar{E}^2}{m_e^2}.$$

The crucial point is that within the logarithmic approximation both polarizabilities $\bar{\alpha}$ and $\bar{\beta}$ in the lhs can be taken at $\omega = 0$, $\mathbf{k}^2 = 0$. The final integration over $d^3\mathbf{k}$ is trivial.

The resulting effective operator of the electron-proton interaction (equal to $-T$) can be written in the coordinate representation as

$$V = -\alpha m_e \left[5\bar{\alpha}(0) - \bar{\beta}(0)\right] \ln \frac{\bar{E}}{m_e}\, \delta(\mathbf{r}). \tag{4}$$

This expression applies within the logarithmic accuracy for arbitrary nuclei. It should be mentioned that a similar relation for hydrogen was obtained in [3], our numerical result agrees with theirs. On the other hand, the formula derived in [12] for an arbitrary nucleus differs from ours (4) by the absence of the magnetic polarizability $\bar{\beta}(0)$ only.

The experimental data on the proton polarizabilities, which follow from the Compton scattering, can be summarized as follows [13]:

$$\bar{\alpha}_p(0) + \bar{\beta}_p(0) = (14.2 \pm 0.5) \times 10^{-4}\ \text{fm}^3;$$
$$\bar{\alpha}_p(0) - \bar{\beta}_p(0) = (10.0 \pm 1.8) \times 10^{-4}\ \text{fm}^3. \tag{5}$$

Now,

$$5\,\bar{\alpha}_p(0) - \bar{\beta}_p(0) = 2\left[\bar{\alpha}_p(0) + \bar{\beta}_p(0)\right] + 3\left[\bar{\alpha}_p(0) - \bar{\beta}_p(0)\right] = (58.4 \pm 5.3) \times 10^{-4}\ \text{fm}^3. \tag{6}$$

The errors are added in quadratures.

Finally, at $\bar{E}_p \sim 300$ MeV the proton polarizability correction to the hydrogen $1S$ state is

$$-71 \pm 11 \pm 7\ \text{Hz}. \tag{7}$$

Here the first error is that of the logarithmic approximation, which we estimate as 15%. The second one originates from the values of the polarizabilities.

The corresponding estimate presented in [4] differs from our result by the factor at $\bar{\alpha}_p(0)$ (2 instead of 5) and by the absence of $\bar{\beta}_p(0)$. Besides, the authors of [4] (and of [3]) choose the inverse nucleon radius, instead of the excitation energy, for the logarithmic cut-off in the corresponding formulae.

2. Though being calculated rather accurately from the theoretical point of view, the correction (7) to the hydrogen Lamb shift is too small to be observed. However, the corresponding effect in the hydrogen–deuterium isotope shift is comparable both with the experimental accuracy (150 Hz) attained for it [2], and with the theoretical precision (70 Hz) for the contribution of the deuteron polarizability due to relative motion of the proton and neutron [11].

The deuteron is a weakly bound system. Then it is natural to assume that deuteron polarizability is the sum of the polarizability due to relative motion of the nucleons and the internal polarizabilities of the nucleons. Simple physical arguments, supported by model estimates, demonstrate that nucleon polarizabilities in deuteron coincide with polarizabilities for free nucleons, well within the accuracy of our logarithmic approximation. Therefore, in the corresponding effect in the H-D isotope shift the proton contributions cancel, and we are left with that of a neutron (with opposite sign)

$$\delta V_{H-D} = \alpha m_e \left(5\alpha_n(0) - \beta_n(0) \right) \ln \frac{\bar{E}_n}{m_e} \delta(\mathbf{r}). \tag{8}$$

The neutron electric polarizability is [14]

$$\alpha_n(0) = (9.8 \pm 1.3 \pm 0.7) \times 10^{-4} \text{ fm}^3;$$

Its magnetic polarizability β_n is not known. Assuming that it does not change the result considerably, the discussed contribution to the isotope shift between hydrogen and deuterium due to the internal polarizabilities of the nucleons can be estimated as

$$61 \pm 10 \pm 6\text{Hz}. \tag{9}$$

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ON THE POSSIBLE EXPERIMENTS ON COHERENT PRODUCTION OF RELATIVISTIC POSITRONIUM ATOMS IN A CRYSTAL BY HIGH-ENERGY PHOTONS AND ELECTRONS

Yu.L.Pivovarov and Yu.P.Kunashenko
Nuclear Physics Institute,
Tomsk Polytechnic University, 634050 Tomsk, P.O.Box 25, Russia
I.Endo, T.Isshiki and Y.Okazaki
Faculty of Science,
Hiroshima University, Higashi-Hiroshima 739, Japan

ABSTRACT

Production of relativistic singlet positronium atoms by high-energy photons and electrons in aligned crystal targets is considered. The detailed numerical calculations show that due to coherence effect in a crystal, the significant enhancement of positronium yield, at certain positronium energies and emission angles, can be achieved.

1. Introduction

The production of relativistic positronium (Ps) atom by high-energy photon in collision with an atom has been calculated first by H.Olsen in [1] and V.L.Lyuboshits in [2], and production of Ps by relativistic electrons - by G.V.Meledin, V.G.Serbo and A.K.Slivkov in [3] and A.A. Akhundov, D.Yu. Bardin and L.L. Nemenov in [4]; more detailed theory of Ps electroproduction was developed by E.Holvik and by H.Olsen in [5]. When created, the Ps can be broken during their passage through the target [1]. The Ps break-up cross-sections in relativistic collisions with an atom have been calculated very precisely by St.Mrowchynski in [6]. Later, the Ps production in a crystal has been studied [7, 8, 9] and it was shown that due to coherent production effect, the considerable enhancement occurs at certain emission angles and incident photon energies. Recently, similar coherent effects have been predicted for the creation of relativistic fermionium by relativistic electrons in a crystal [10]. The aim of this paper is to present the detailed numerical studies of coherent production of beams of relativistic singlet Ps in aligned crystals, in relevance to possible experiments at REFER facility of Hiroshima University.

2. Coherent photoproduction of relativistic singlet Ps in a crystal

The differential cross-section for production of singlet Ps by a high-energy photon has the form [1, 2] (here, $\hbar = c = 1$):

$$\frac{d\sigma_1}{d\Omega} = \frac{Z^2\alpha^6}{n^3} \times \frac{[1 - F(q)]^2}{q^4} \times \frac{p_p^3 m_p^2 \sin^2\theta}{2E_p(E_p - p_p\cos\theta)^2}. \tag{1}$$

where $F(q)$ is the atomic formfactor, q is the momentum transfered, $m_p = 2m_e - \varepsilon$, ε is the Ps binding energy, m_e is electron mass and n is the principal quantum number (below, $n = 1$). Here, E_p and p_p are the energy and momentum of Ps travelling as