INTERACTIONS OF A MASSIVE SLOW MAGNETIC MONOPOLE WITH MATTER

I.V.Kolokolov[1], P.V.Vorob'ev[1], V.V.Ianovski

[1] **Budker Institute of Nuclear Physics (BINP), Novosibirsk, Russia**

A b s t r a c t

Interactions of a massive magnetic monopole with matter are considered. We discuss the possibility of creating the ferromagnetic detector to search for cosmic magnetic monopoles.

ВЗАИМОДЕЙСТВИЕ ТЯЖЕЛЫХ МЕДЛЕННЫХ МАГНИТНЫХ МОНОПОЛЕЙ С ВЕЩЕСТВОМ

П.В.Воробьев[1], И.В.Колоколов[1], В.В.Яновский

[1] **Институт ядерной физики им. Г.И.Будкера СО РАН, Новосибирск**

А н н о т а ц и я

Рассмотрено взаимодействие тяжелого магнитного монополя с ферромагнетиками, сверхпроводниками и другими средами. Обсуждается возможность создания ферромагнитного детектора для поиска магнитных монополей в космическом излучении с использованием современных технологий.

Introduction

A concept of a magnetic monopole has been introduced into modern physics in 1931 by Paul Dirac [1]. He postulated existence of an isolated magnetic charge g. Using general principles of quantum mechanics, he has related the electric and magnetic charge values: $gc = \frac{n}{2}\hbar c$, where e is the electron electric charge, $\hbar$ is the Plank constant, c is the speed of light, $n = \pm 1, 2...$ is an integer. Numerous but unsuccessful attempts of experimental search for this magnetic monopole on accelerators and in cosmic rays [2, 3] have been done since then. Among them, exotic installations like the steel furnace have been proposed for accumulation of monopoles for subsequent detection [4]. Attempts of search for Dirac's monopole by the ferromagnetic trap and the accelerators have been performed too [5].

The new interest to this problem has arised in 1974, when Polyakov [6] and 't Hooft [7] have shown that such objects exist as solutions in a wide class of models with spontaneously broken symmetry. Nature of their monopoles is absolutely different from nature of other elementary particles, since they represent a non-trivial topological construction of a finite size, which originates from non-Abel fields. So registration of Dirac monopoles or estimation of their flux limit could be an essential contribution to construction of the Grand Unified Theory, and as well it would give incentives for solutions of various problems in astrophysics.

The magnetic charge of the Polyakov—'t Hooft monopole is a multiple of the Dirac one $g = 2ne/\alpha$, and a value of its mass M_g lies in the range of 10^8 — 10^{16} GeV. It is assumed that a large fraction of the superheavy monopole flux at earth's surface can comprise the monopoles gravitationally attracted to the Sun. They have characteristic velocities of $v/c \sim 10^{-4}$. Such monopoles can pass significant thickness of material without any significant change of velocity.

At present the devices looking for the heavy slowly moving monopoles are superconducting induction detectors such as the Cabrera detector [8] or detectors, based on registration of proton decays induced by the monopole [9],[10]. Recently the group working with the Baikal lake Cherenkov detector [11] has set the following limit on the flux of heavy magnetic monopoles and the Q-balls, which are able to induce the proton decay: $\mathcal{F} < 3.9 \cdot 10^{-16} cm^{-2} s^{-1} sr^{-1}$.

The event, registered by the Cabrera detector, has originated a burst of experimental activity of searches for monopoles of a cosmological origin. However, the significant progress in sensitivity (and corresponding limits on the monopole flux) has been achieved only for ionising detectors [2, 12]. Sensitivity of these detectors comes close to the Parker limit [13]:

$$\mathcal{F} \leq 1 \cdot 10^{-16} (m/10^{17}\ GeV)\ cm^{-2} s^{-1}\ sr^{-1}.$$

Results, obtained recently by the induction experiments, are more modest. At the flux of $10^{-15} cm^{-2} sr^{-1} s^{-1}$ the observation of only one magnetic monopole per year would require effective area of a detector of about 1000 m^2. Modern superconducting inductive detectors with the superconducting quantum interference devices (SQUID) using magnetometer techniques to register the particle have an effective area only of the order 1 m^2.

The advanced accelerator experiments [14] have looked for production of monopoles by searching for the high ionization tracks that would be produced by monopoles created at high energy collisions. The first study of such a kind was made by the L3 experiment at the LEP e^+e^- collider by searching for the $Z \to \gamma\gamma\gamma$ decay going through the monopole loop [15]. It resulted in the 95% confidence level for lower mass of 510 GeV for pointlike spin 1/2 monopoles. Further, the D0 collaboration has searched for central production of a pair of photons with high transverse energies in $p\bar{p}$ collisions at $\sqrt{s} = 1.8$ TeV using 70 pb^{-1} of data collected with the D0 detector at the Fermilab Tevatron. They observe no excess of events above background, and set 95% C.L. lower limits of 610, 870, or 1580 GeV/c^2 on the mass of a spin 0, 1/2, or 1 Dirac monopole respectively [16].

In this paper we discuss the interaction of slow-moving magnetic monopole with ferromagnetic and other media and a possibility to create the detector for magnetic monopole searches.

1 Ionizing and Cherenkov energy losses

Ionization and Cherenkov losses of the monopole in matter were considered by many authors [17]-[19].

The charged particle energy loss due to Cherenkov radiation per unit length is:

$$\frac{dW}{dl} = \frac{e^2}{c^2} \int\limits_{v>c_{med}} \left(1 - \frac{c_{med}^2}{v^2}\right) \omega d\omega \,, \tag{1}$$

where e is the particle charge, c_{med} is the electro-magnetic wave velocity in a matter.

For $v >> c_{med}$ Eq.(1) gives

$$\frac{dW}{dl} = \frac{e^2}{c^2}\omega^2 = \frac{e^2}{\lambda^2} \,, \tag{2}$$

here λ is the length of a Cherenkov radiation wave in vacuum. The Cherenkov radiation of electrical charge in isotropic ferromagnet (ferrite) is given by

$$\frac{dW}{dl} = \frac{e^2}{c^2} \int\limits_{v>c_{med}} \mu \left(1 - \frac{1}{\beta^2 \epsilon \mu}\right) \omega d\omega \,. \tag{3}$$

What should be expected in case of monopole movement in a magnetic medium? In case if

$$\nabla \cdot \mathbf{B} = 4\pi\mu\rho_g \,, \tag{4}$$

then for the magnetic charge radiation we obtain the expression [17, 19]

$$\frac{dW}{dl} = \frac{g^2\mu\epsilon}{c^2} \int\limits_{v>c_{med}} \mu \left(1 - \frac{1}{\beta^2\mu\epsilon}\right) \omega d\omega = \frac{g^2}{c_{med}^2} \int\limits_{v>c_{med}} \mu \left(1 - \frac{c_{med}^2}{v^2}\right) \omega d\omega \ . \tag{5}$$

Hence, for the ratio of the magnetic charge radiation to electrical we have

$$\frac{W_g}{W_e} = \frac{g}{e} \frac{c^2}{c_{med}^2} \simeq 4700 \cdot \frac{c^2}{c_{med}^2} \ . \tag{6}$$

Such an approach meets the idea of duality

$$E \leftrightarrow B$$

and we shall adhere just such an agreement herein.

A heavy slow monopole can not emit usual Cherenkov radiation in a ferromagnet. It is due to high value of the phase velocity of electro-magnetic waves in ferromagnetic, about $c/10$, which is always much more than the monopole velocity.

For the fast monopole, the intensity of losses is appreciably ($2n/\alpha$, i.e. about in 4700 times!) exceeds the loss of mip's. However, practically for all magnito-dielectrics, the Cherenkov radiation threshold velocity appears to be about 0.1 c, that is of no interest for the search of cosmic slow heavy monopoles, the speeds of which do not exceed 10^{-3} c.

The monopole ionization loss is given by [20]:

$$\left(\frac{dE}{dR}\right)_g = \left(\frac{dE}{dR}\right)_{Ze} \left(\frac{g}{Ze}\right)^2 \left(\frac{v}{c}\right)^2 . \tag{7}$$

As was already mentioned, the energy loss of ultra-relativistic monopole is 4700 times higher, than of a particle with $Z = 1$ in the ionization minimum (it is about $2\frac{MeV}{g \cdot cm^2}$). However, because monopole is a very heavy particle, it is practically always non-relativistic.

For the maximum of monopole velocity v it is natural to take the velocity of the Sun relatively to the relict radiation

$$\frac{v}{c} \simeq 10^{-3} \ . \tag{8}$$

We shall remind, that the "virial" velocity for our Galaxy does not exceed 10^{-3} too. Now from the Eq.(7, 8) it is possible find the ionization energy loss of monopole

$$\left(\frac{dW}{dl}\right)_g \simeq 2 \left[\frac{MeV}{g \cdot cm^2}\right] \cdot 4700 \cdot \left(\frac{v}{c}\right)^2 \leq 9.4 \cdot 10^{-3} \left[\frac{MeV}{g \cdot cm^2}\right] \ . \tag{9}$$

Thus, for the monopole with velocity 10^{-3} c we have

$$(dW/dl) = 10^4 \frac{eV}{g \cdot cm^2} \ .$$

And for the monopole with velocity 10^{-4} c the loss is already

$$(dW/dl) = 10^2 \frac{eV}{g \cdot cm^2} .$$

This is certainly very small loss, and registration of heavy slow magnetic monopole by ionization in matter is practically impossible.

2 The Drell effect and the Penning effect

Some expansion of the ionizing detector sensitivity for slow monopoles is reached due to the Drell effect [21]. The essence of the Drell effect is in following. Let assume, that monopole fly near the helium atom. In a strong magnetic field of monopole by the Zeeman effect occurs a crossing of main and excited levels. Under the action of a magnetic field, which is variable in time, and due to of some non-adiabatic of process it is possible the electron transition in a excited level. After monopole passage some of helium atoms in state with energy an order of 20 eV are excited. An dope of a gas with the ionization potential smaller, than the excitation energy of helium atom, results in this gas (quenching gases—CH_4, CO_2 or $n - pentane$) ionization by collision with excited helium (the Penning effect). Therefore an ionization can be detected as by direct methods, as by the radiative recombination. The gas counters for cosmic monopole registration on the Drell effect base were created and used in a number of experiments [3]. In detail the ionization and Drell effect are discussed in[22].There the review of experiments on search of the massive monopole is addused too.

We shall discuss some new mechanisms of the energy losses of slow monopole now. And on this base we shall discuss than an idea of a new type detector construction of slow monopole as well.

3 Excitation of spin wave Cherenkov radiation by the heavy magnetic monopole

As well known, the slowly moving heavy monopole cannot emit usual Cherenkov radiation in ferromagnetic media, because the phase speed of electromagnetic waves is of the order $c/10$ and always much faster than the monopole speed.

We shall consider the slow monopole passage an ordered magnetic matter [23]. In such case a main mechanism of kinetic energy loss is the Cherenkov radiation of magnons. This is because the magnon phase velocity reaches zero and the coupling of monopole to magnons is linear and large.

For definiteness we shall consider a ferromagnet, but the evaluations below are of more general character.

Magnon's Hamiltonian in presence of magnetic field of a moving monopole can be written in the form

$$H = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} + \sum_{\mathbf{k}} \left(f_{\mathbf{k}} e^{-i\Omega_{\mathbf{k}} t} a_{\mathbf{k}}^{\dagger} + c.c \right) , \tag{10}$$

where $a_{\mathbf{k}}^{\dagger}$ — operator of a magnon creation with a wave vector $\mathbf{k}$, $\omega_{\mathbf{k}}$—his dispersion law, $\Omega_{\mathbf{k}} = \mathbf{kv}$, $\mathbf{v}$—vector of a monopole speed and $f_{\mathbf{k}}$ — coupling factor of a monopole magnetic field $\mathbf{B} = g\nabla\frac{1}{r}$ with magnon.

The magnon energy, radiated in a unit of time, is

$$\epsilon = \frac{2\pi}{\hbar} \sum_{\mathbf{k}} \omega_{\mathbf{k}} |f_{\mathbf{k}}|^2 \delta(\Omega_{\mathbf{k}} - \omega_{\mathbf{k}}) . \tag{11}$$

Let the monopole velocity $\mathbf{v}$ be directed along the direction of the spontaneous magnetization, along Z-axis. The general case is investigated absolutely similarly and the result differs *only by a factor close to 1.* Then

$$F_{\mathbf{k}} = \frac{4\pi g \mu_B}{a^{3/2}\sqrt{V}} \sqrt{\frac{S}{2}} \frac{k_x - ik_y}{k^2} , \tag{12}$$

here a is the lattice constant, V is the sample volume, S is the spin size on the node and μ_B is the Bohr magneton.

Taking into consideration Eq.(12) the equation for ϵ can be written as

$$\epsilon = \frac{2g^2\mu_B^2 S}{a^3\hbar} \int d^3\mathbf{k}\,\omega_{\mathbf{k}} \frac{k_x^2 + k_y^2}{k^4} \delta(k_z v - \omega_{\mathbf{k}}) . \tag{13}$$

The integration in Eq.(13) is performed on the first Brillouin zone.

If $v \geq u$, where u — magnon velocity near a border of Brillouin zone, then the magnons with large $\mathbf{k}$ are essential. Then

$$\epsilon \simeq \frac{\bar{\omega} g^2 \omega_M}{v} , \tag{14}$$

where the frequency $\omega_M = \frac{4\pi\mu_B^2 S}{\hbar a^3}$ characterizes magnetization of media [24],

$$\bar{\omega} = \frac{1}{2\pi} \int \frac{d^2\mathbf{k}_{\perp}}{\mathbf{k}_{\perp}^2} \omega_{k_{\perp}} , \tag{15}$$

here $\mathbf{k}_{\perp} = (\mathbf{k_x}, \mathbf{k_y})$, and $\bar{\omega}$ has the value about maximal frequency of magnons.

For $g^2 \simeq 4700 \cdot e^2$ we obtain

$$\epsilon \simeq 10^3 \cdot Ry \cdot \omega_M(\bar{\omega}\tau) , \tag{16}$$

where $\tau = a/v$ is the characteristic time of interaction.

The typical values for magneto-ordered dielectrics are such: $\bar{\omega} \simeq 10^{-13}s^{-1}$, $\omega_M \simeq 10^{-11}s^{-1}$ and for $v/c \simeq 10^{-4}$ — $\epsilon \simeq 10^{14}$eV/s, that corresponds to losses per unit of length:

$$\frac{dE}{dl} \simeq 10^8 \ eV/cm$$

From Eq.(14) it is clear, that the losses ϵ and dE/dl grow with slowing down of monopole. When the speed v becomes $v < u$, the main contribution to losses contribute the magnons from "the bottom" of the spectrum.

For them, $\omega_{\mathbf{k}} = \omega_{ex}(ak)^2$, where ω_{ex} is the frequency, characterizing the exchange interaction [24, 25] , and the expressions for losses acquire the shape:

$$\epsilon = g^2 \frac{\omega_M v}{4\omega_{ex} a^2} ; \tag{17}$$

$$\frac{dE}{dl} = \frac{\epsilon}{v} = g^2 \frac{\omega_M}{4\omega_{ex} a^2} . \tag{18}$$

As one can see, the energy losses per unit of a length became a constant with decrease of the monopole speed. The characteristic speed values will be: $\omega_M/\omega_{ex} \simeq 10^{-2}$, $a \simeq 10^{-8}\ cm$ and for $v/c \simeq 10^{-4}$ have

$$\frac{dE}{dl} \simeq 10^8 \ eV/cm .$$

We'd like to stress, that the square-law for a magnon dispersion leads to a non-trivial spatial structure of the Cherenkov radiation field of the spin waves. The structure of the radiation field is similar to a shock wave and moving in front of the charge radiation. The radiation field for the square-law dispersion advances charge and is not equally to zero before charge. It is due to that for the square-law dispersion the group velocity of a wave is more then phase velocity and more than velocity of a charge movement.

From these evaluations it is clear, that a level of energy losses of the slow magnetic monopole in the magneto-ordered matter can be comparable to the ionization losses of a fast monopole. This opens new opportunities for construction of detectors of the monopoles in the range of velocities $v/c < 10^{-4}$. Note, that conversion of the spin waves to electromagnetic ones [25] permits to detect a monopole passing through a magnetic layer by traditional techniques.

4 Excitation of Cherenkov acoustic (phonon) radiation by the magnetic monopole

4.1 Radiation due to the polarization of matter

For estimation of energy losses by radiation of sound waves (excitation of phonons) by the monopole moving through isotropic matter, we shall write the Hamiltonian of an elastic system in an external field as follows

$$H = \sum_n \frac{\mathbf{p}_n^2}{2M} + \frac{a}{2}\sum_{n,\Delta}(\mathbf{x}_n - \mathbf{x}_{n+\Delta})^2 + \sum_n \mathbf{F}_n(t)\mathbf{x}_n \ . \tag{19}$$

Here $n+\Delta$ are the numbers the closest neighbors to the node n,

$$\mathbf{F}_n(t) = \mathbf{F}(\mathbf{r}_n - \mathbf{v}t) \ . \tag{20}$$

We shall estimate the strength of the force $\mathbf{F}(\mathbf{r}_n)$, acting from the monopole to the given node, as follows. First, we shall assume that this force is located on one node (it's short-range nature allows this):

$$\mathbf{F}(\mathbf{r_n}) = \mathbf{F}\delta_{n0} \ . \tag{21}$$

Secondly, at rest this force causes deformation, and the affected node is shifted by

$$\delta a \sim \frac{F}{A} \ , \tag{22}$$

and, assuming the deformation energy to be $\epsilon_{def} \sim A\delta a^2$, we have the force as

$$\mathbf{F} \sim A\sqrt{\frac{\epsilon_{def}}{A}} \sim \sqrt{\epsilon_{def}A} \ . \tag{23}$$

The Hamiltonian Eq.(19) can be expressed similar to Eq.(10)

$$H = \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \sum_{\mathbf{k}} \left(f_{\mathbf{k}} e^{-i\Omega_{\mathbf{k}}t} a_{\mathbf{k}}^\dagger + c.c \right) \ , \tag{24}$$

but now: $A_{\mathbf{k}}^\dagger$ is the operator of phonon creation with the wave vector $\mathbf{k}$, $\omega_{\mathbf{k}}$ is it's dispersion, $\Omega_{\mathbf{k}} = \mathbf{kv}$, $\mathbf{v}$ is the vector of monopole speed, and $f_{\mathbf{k}}$ is the coupling factor of the magnetic monopole field $\mathbf{B} = g\nabla\frac{1}{r}$ with the phonon. We shall write the following expressions for them:

$$F_{\mathbf{k}} = \frac{1}{2i} h^{1/2} \frac{1}{\sqrt{N}} \frac{F}{(D_{\mathbf{k}}M)^{1/4}} \ , \tag{25}$$

$$\omega_{\mathbf{k}} = \sqrt{\frac{2D_{\mathbf{k}}}{M}} \tag{26}$$

$$D_{\mathbf{k}} = \frac{A}{2}\sum_{\Delta}\left|1 - e^{i\mathbf{k}\Delta}\right| . \tag{27}$$

Accordingly, the energy of phonons, radiated in a unit of time, is equal to

$$\epsilon = \frac{a^3}{4(2\pi)^2}\int d^3\mathbf{k} \frac{F^2}{(D_k M)^{1/2}} \omega_{\mathbf{k}} \delta(k_z v - \omega_{\mathbf{k}}) \ , \tag{28}$$

where a is the constant of the lattice $a^3 = V/N$, V is the sample volume.

For the essentially supersound monopoles, integration over dk_z gives the factor $1/v$, and the integral Eq.(28) results to:

$$\epsilon = \frac{a^3}{4(2\pi)^2}\frac{1}{v}\int d^2\mathbf{k}_\perp \frac{F^2}{(D_{\mathbf{k}_\perp} M)^{1/2}}\omega_{\mathbf{k}_\perp} = \frac{a}{2\sqrt{2}v}\frac{F^2}{M} . \tag{29}$$

Using the evaluation from Eq.(23), for F we shall obtain:

$$\epsilon \simeq \epsilon_{def}\frac{a}{v}\frac{A}{M} . \tag{30}$$

Now from decomposition Eq.(27) at small $\mathbf{k}$ and using Eq.(26) it is possible to express A/M through speed of sound. As a result Eq.(30) acquires the shape

$$\epsilon \simeq \epsilon_{def}\frac{1}{Z}\frac{c_s}{v}\frac{c_s}{a} \simeq \epsilon_{def}\frac{c_s}{v}\bar{\omega}_a , \tag{31}$$

where $\bar{\omega}_a$ is the frequency limit for phonons, Z is the number of nearest neighbors.

If $\epsilon_{def} \sim Ry$, $c_s/v \sim 0.1$ and $\bar{\omega}_a \sim 10^{13}\ s^{-1}$ is about the Debye temperature in energy units, then:

$$\epsilon \simeq 10^{13} eV/s,$$

$$\frac{dE}{dl} \simeq 10^7 eV/cm,$$

which is a little less than loss by radiation of magnons.

4.2 Radiation due to the atomic shell deformation by monopole magnetic field

In strong magnetic field an atomic shell loses its spherical symmetry and takes a form of an ellipsoid of revolution with elongation along the field direction. Let evaluate the value of distortion. The size of the area of electron localization λ_H in the magnetic field is defined by the uncertainty ratio $\lambda_H^2 = \frac{\hbar c}{He}$.

The electron localization on the Compton wavelength

$$\lambda_c = \frac{\hbar}{m_e c} \simeq 5 \cdot 10^{-11}\ cm$$

corresponds to a so called critical field:

$$H_c = \frac{m_e^2 c^3}{\hbar e} \simeq 4 \cdot 10^{13}\ Oe. \tag{32}$$

Thus, we can define the localization size as

$$\lambda^2 = \lambda_c^2 \frac{H_c}{H}, \tag{33}$$

what we compare with Bohr radius:

$$r_B = \frac{\hbar^2}{me^2} \simeq 5 \cdot 10^{-9}\ cm .$$

An atom size in the direction orthogonal to the magnetic field ($H \geq H_c$) is defined as λ_H . At fields appreciably below H_c the size is determined by a coulomb field of a nucleus and corresponds to the size of the Bohr orbit. As evaluation of the atom size in the field $H << H_c$, but enough strong, we use a following construction (which has the rights asymptotics):

$$\frac{1}{r^2} = \frac{1}{\lambda_H^2} + \frac{1}{r_B^2} . \tag{34}$$

So at $\lambda_H >> r_B$ we obtain:

$$R^2 = r_B^2 (1 - \frac{r_B^2}{\lambda_H^2}) . \tag{35}$$

On a critical radius of the sound waves $\lambda_c = c_s / \bar{\omega}_a$ the field is of the order of 10^7 Oe. Then

$$\lambda_H^2 \simeq \lambda_c^2 \cdot 10^7 >> r_B^2,$$

and we have

$$R^2 = r_B^2 (1 - 10^{-2}) . \tag{36}$$

We assume that the equilibrium density is defined by the atom size in corresponding direction. Apparently, in the plane orthogonal to the field atoms density is in r_B^2/r^2 times greater, than in absence of the field. It causes a tension

$$P = \frac{1}{\chi} \cdot \frac{r_B^2}{\lambda_H^2}, \tag{37}$$

Here $\chi \simeq 1/Y$ is a compressibility, and Y is a Young's modulus.

$$\chi = \frac{1}{V} \frac{\delta V}{\delta P} \simeq 10^{-11} (N/m^2)^{-1} . \tag{38}$$

For simplicity we consider a single dimension case. Let direct an external pressure $P(t)$ along the normal to a half-infinite border between vacuum and material with density of ρ_g and the Young's modulus of Y). The displacement ζ_0 of a layer under action of pressure P is

$$a = \frac{P}{\rho \lambda}, \tag{39}$$

or

$$\zeta_0 = a\tau^2 = \frac{P\lambda}{\rho v_g^2} = \frac{P\lambda}{Y} \frac{c^2}{v_g^2} . \tag{40}$$

Here c is the speed of the longitudinal sound wave:

$$C^2 = \frac{Y}{\rho_g}. \tag{41}$$

The energy $\mathcal{E}_a$, transferred to the acoustic Cherenkov radiation by the force of magnetic pressure on a distance of λ_c is

$$\mathcal{E}_a \simeq PS\zeta_0 = \frac{P^2 \lambda S}{Y} \frac{c^2}{v_g^2}. \tag{42}$$

Substituting Eq.(37) into Eq.(42) we find

$$\mathcal{E}_2 = \left(\frac{r_B^2}{\lambda_H^2}\right)^2 \frac{V}{Y\chi^2} \frac{c^2}{v_g^2} = YV \left(\frac{r_B^2}{\lambda_H^2}\right)^2 \frac{c^2}{v_g^2}, \tag{43}$$

where $V = S\lambda$ is the volume determined by the critical wavelength of acoustic radiation.

At $Y = 10^{11} N/m^2, \frac{\lambda_H^2}{r_B^2} = 10^{-3}$ and $V = 10^{-23}\ m^3, \chi \simeq 10^{-11}$ we obtain:

$$\mathcal{E}_2 = 10^{-17} J \text{ or } 10^8\ eV/cm.$$

For para- and ferromagnetics all above is valid under condition of an adiabatic spin flip in the monopole field. To satisfy it the spin precession frequency in the monopole field should be much higher, than the inverse interaction time $\frac{1}{\tau} \simeq \frac{v_g}{a}$:

$$\omega = \frac{\mu_b H}{\hbar} >> \frac{v_g}{a}. \tag{44}$$

Here a - a lattice constant. Otherwise in expression for monopole power loss should enter the factor of the order $(\omega\tau)^2$, which can appreciably decrease the monopole energy losses in the para- and ferromagnetics.

5 Interaction of the massive monopole with a superconductor

The monopole leaves "a tail"- a magnetic field string, as it transverses a superconductor. It is the Abrikosov vortex, which is filled by a normal phase and by a monopole magnetic flux. A vortex core radius is determined by a coherent length ξ, and a tube effective radius of magnetic flux which is the London penetration depth λ_L

$$\lambda_L = \sqrt{\frac{mc^2}{\mu n_s e^2}}. \tag{45}$$

Thus, energy loss of the slow monopole in the superconductor is made up from two components:
- Braking by magnetic string tension,
- Loss due to destruction of the condensate of Cooper pairs and formation of the core of the normal phase.

The magnetic string loss is

$$\frac{dW_m}{dl} = \frac{1}{8\pi}\frac{\Phi_0^2}{\pi\lambda_L^2} . \tag{46}$$

Using a typical value of $\lambda_L = 10^{-5}cm$, and $\Phi_0 = 4 \cdot 10^{-7}\ G \cdot cm^2$ we obtain

$$\frac{dW_m}{dl} \simeq 10^{-5}\ erg/cm \simeq 10^7\ eV/cm . \tag{47}$$

The losses due formation of the core of the normal phase is

$$\frac{dW_n}{dl} = \Delta E n_s \pi \xi_c^2 , \tag{48}$$

here ΔE is the gap width, n_s is the density of superconducting carriers. For typical suprconductor parameters we estimate the value of the energy losses to be of the order of $10^7\ eV/cm$. To be accurate, it is necessary to include surface energy on the core border of the Abrikosov vortex into $\frac{dW_n}{dl}$. However, it is clear, that the answer will remain within the same order of magnitude.

At the London length $\lambda_L = 10^{-5}\ cm$ the pressure of a magnetic field is relatively small and the losses on the Cherenkov acoustic radiation can be neglected.

6 Monopole interaction with usual (bulk) conductors

When the monopole transverse a normal massive conductor, he loses energy on the Joule heat. Besides that, the braking force acts on it as well as in superconductor from the side of a magnetic field of the induced currents.

The thickness of a skin layer in conductor is described by expression

$$\delta = \sqrt{\frac{c^2\tau}{2\pi\sigma\mu}} , \tag{49}$$

where σ is the conductivity, μ the permeability and τ period (pulse duration) of the magnetic field.

We can express τ as

$$\tau = \frac{\delta}{v} , \tag{50}$$

where v is the monopole velocity. Substituting Eq.(50) into Eq.(49) gives the expression

$$\delta_c = \frac{c^2}{2\pi\sigma\mu v} . \tag{51}$$

This length have a simple physical meaning - at distances less then δ_c the monopole field can be considered as free with an accuracy to the permeability of a matter, and at distances of δ_c and more it will form the magnetic tail of the monopole, as an analogue of the string in superconductor. Due to the finite conductivity of a matter the tail gradually dissipates and energy of the magnetic field transforms into Joule heat.

It is easily to show, that for typical parameters of a conductor current-carrying ability of a metal layer with thickness δ_c is enough to keep the monopole flux in a tube with a radius of the order δ_c.

As a critical length known, it is easily to evaluate monopole energy losses in conductor. In the cross section of diameter of the critical length magnetic field is

$$B_c = \frac{\Phi_0}{\pi \delta_c^2} \ . \tag{52}$$

Accordingly, the monopole energy loss due to the tail formation (friction in the magnetic field) is

$$\frac{dW_m}{dl} = \frac{\Phi_0^2}{8\pi^2\delta_c^2} = \frac{\Phi_0^2\sigma^2\mu^2 V_g^2}{2c^4} \ . \tag{53}$$

Examining Eq.(53), we can see that the monopole energy losses in the conductor are proportional to a square of a monopole speed. At speed of $v = 10^{-4}c$ the loss is

$$\frac{dW_m}{dl} \simeq 10^{-5} \ erg/cm \simeq 10^7 \ eV/cm \ . \tag{54}$$

7 Magnetic track formation by the slowly moving monopole in ferromagnetic

It is expected that a slow monopole, moving through a transversely magnetized ferromagnetic film, should leave a distinctive track of magnetization in it. We can use this phenomenon to design of an effective detector of the massive cosmic monopoles. To do this we make a close look at the mechanism of magnetic track formation.

7.1 Quasistatic approximation.

Consider a thin layer of light-axis hard ferromagnetic magnetized perpendicularly to surfaces along a light axis. It is easy to see that the external magnetic field is absent (double layer!), but the surface density of a magnetostatic energy of such configuration is rather large.

Therefore such a magnetization configuration appears more expedient energetically, when the magnetic film is split into a system of magnetic domains. Magnetizations of the domains are also orthogonal to the surface and is directed along the easy axis, and have opposite signs in neighbouring domains [26].

The domain characteristic size is determined by a condition of minimal total energy per an unit of the film surface. For ferrogarnet films a characteristic scale of a domain structure is about 1/100 mm. And the total energy of a film decreases more than 1000 times by creation of the domain structure.

However, if the anisotropy constant K_u is reasonably large, and the effective field of anisotropy exceeds the value of demagnetizing field

$$\frac{2K_u}{I_s} \geq \frac{I_s}{\mu_0}, \quad K_u \geq \frac{1}{2}\frac{I_s^2}{\mu_0}, \tag{55}$$

then the system is in a metastable state. Therefore for domain formation it is necessary to have a magnetic bubble with magnetization opposite to the film magnetization and parallel to the demagnetizing field. In Eq.(55) the parameter I_s is magnetization of the film material and μ_0 is permeability.

It turned out, that the size of such magnetic bubble should be finite and not very small. For simplicity we consider a magnetic bubble of cylindric shape of radius r , with axis orthogonal to the film surface and its magnetization is opposite to the film magnetization. Then the total energy of the magnetic bubble can be found as

$$\mathcal{E} = (1-2N)\frac{I_s^2}{\mu_0}\pi r^2 h - \gamma 2\pi r h \,, \tag{56}$$

where N is a magnetization factor of the domain, r a radius of the domain, h the film thickness, γ the density of surface energy of the domain wall.

The magnetic bubble size can be evaluated from the condition

$$\frac{\partial \mathcal{E}}{\partial r} = 0\,. \tag{57}$$

If we use for the unmagnetization factor the following approximation

$$N = \frac{2}{3}\frac{r}{h}\,, \tag{58}$$

then as solutions Eq.(57) we have two values r of the domain radius. The first value is **the radius of collapse** r_c, below which the state of the domain-magnetic bubble is unstable and collapses

$$r_c = \frac{h}{4} - \left[\left(\frac{h}{4}\right)^2 - \frac{\mu_0\gamma h}{2I_s^2}\right]^{1/2} = \frac{h}{4}\cdot\left[1-\left(1-\frac{4r_0}{h}\right)^{1/2}\right] \tag{59}$$

And the second solution gives us **the equilibrium radius** r_{eq}

$$r_{eq} = \frac{h}{4}\cdot\left[1+\left(1-\frac{4r_0}{h}\right)^{1/2}\right]\,. \tag{60}$$

At $h >> r_0$ we can find from Eq.(59) and Eq.(60)

$$r_c|_{h>r_0} \to \frac{r_0}{2} \tag{61}$$

$$r_{eq}|_{h>r_0} \to \frac{h}{2} \tag{62}$$

As one can see, we have introduced here some characteristic radius

$$r_0 = 2\mu_0\gamma/I_s^2$$

which defines the minimum radius of collapse. From non-negativity of the underroot expression in Eq.(59) we obtain, that the minimum width of a ferrolayer, in which a cylindrical domain can exist, is

$$H_{min} = 4r_0 = 8\frac{\mu_0\gamma}{I_s^2} \tag{63}$$

It is necessary to emphasize, that in a zero external magnetic field, the cylindrical magnetic domain is unstable and turns into a stripe domain [27].

Let us evaluate the orders of magnitudes:

$$\gamma \simeq 1\ erg/cm^2,$$

$$I_s^2 \simeq\ 10^6 erg/cm^3\ .$$

So r_0 will be of the order 10^{-6} *cm* or 10 nm. And the collapse radius is yet less for "thick" enough films! Thus, at film thickness about 100 nm we shall have a cylindric magnetic domain with characteristic size of the order of 30 nm. What is the monopole field at such distance?

$$H = \frac{\Phi_0}{4\pi r_0^2} \simeq 2 \cdot 10^3\ Oe\ , \tag{64}$$

that is enough without any doubt for re-magnetization of a material with coercive force of the order of 1 Oe.

The existence of minimum radius hinders the fluctuational creation of microscopic domain — magnetic bubbles, which could create a quasi-periodic domain structure of the magnetic film. In this sense, a homogenically magnetized film can be quite stable against splitting into structure of magnetic domains. All abovementioned is true for films with high mobility of domain walls. Films with low wall mobility are even more stable, and at the same time, domains with radius less then the collapse radius can exist in them, in principle. So, in the Co/Pt films the movement of domain walls is suppressed. And in the 20 nm film the transverse domains of cylindric form with diameter of the order of 50-100 nm are obtained. We note here, that the coercive force of the light-axis Co/Pt film is of the order of 1-2 kOe.

7.2 Track dynamics — domain formation

However, these speculations are true only in static, for very slow monopoles only. As we noted before, the characteristic speed of a monopole is $v \simeq 10^{-4} - 10^{-3}c$, and for our consideration let us assume $v = 10^{-4}c$. The time of monopole interaction with an electron τ can be defined as the time, during which a field higher than some critical field H_c interacts with the electron

$$\tau \simeq \frac{r_c}{v} \simeq \frac{1}{v}\sqrt{\frac{\Phi_0}{4\pi H_c}} . \tag{65}$$

At H_c of the order $3 \cdot 10^3$ Oe we have $\tau \sim 3 \cdot 10^{-12}$ s. It means, that the spin-flip of the magnetic in the "track" takes place during the interaction time. For such spin-flip, the adiabatic condition is necessary — the frequency of spin precession in the overturning field should be much larger than the inverse time of the interaction:

$$\omega = \frac{\mu_B H}{\hbar} \gg \frac{1}{\tau} . \tag{66}$$

It is possible to derive from here the minimal magnetic field which is appropriate for the adiabatic mode, and the track radius:

$$H \gg H_c = \frac{\hbar}{\mu_B \tau} = \frac{4\pi\hbar^2 v^2}{\mu_b^2 \Phi_0} , \tag{67}$$

$$R_t \ll r_c = \sqrt{\frac{\Phi_0}{4\pi H_c}} . \tag{68}$$

In our case at $v \simeq 10^{-4}c$ we get

$$H_c \simeq 10^7 Oe$$

$$r_c \simeq 10^{-7} cm,$$

and for $v \simeq 10^{-6}c$ we have

$$H_c \simeq 10^3 Oe$$

$$r_c \simeq 10^{-5} cm.$$

It is obvious, that the conditions of adiabatic and even resonant spin flip are not fulfilled, while $r_c < r_0$, that corresponds to the monopole speed $v \simeq 10^{-6}c$.

We shall consider the influence of a conductivity of the film matter now. The reason is that the monopole magnetic flux is being frozen into the cylindric area around the track axis and then diffuses radially. The radius of the "flux pipe" and the diffusion factor of the flux are determined by the film conductivity. The radius of the flux pipe δ_c, as follows from above, we can find:

$$\delta_c = \frac{c^2}{2\pi\sigma\mu v} . \tag{69}$$

As was noted already, this length has a simple physical meaning. At distances less than δ_c the monopole field can be considered as free. At distances of the order of δ_c and more, the magnetic tail of the monopole is formed, which is an analogue of a string in a superconductor. Due to the finite conductivity of matter the tail gradually expands and the energy of a magnetic field converts into heat. At the monopole speed about $v \simeq 10^{-4}c$, the flux pipe has the radius of the order of $10^{-5}cm$. The flux pipe expands with time as:

$$R(t) = \delta_c\sqrt{\frac{t}{\tau}} \ . \qquad (70)$$

Thus the magnetic moment of the track is conserved, as well as the frozen flux. It is easy to calculate the average intensity of the magnetic field in the flux pipe immediately after monopole flight

$$H = \frac{\Phi_0}{\pi\delta_c^2} \sim 10^3 Oe \ . \qquad (71)$$

Characteristic time of the monopole interaction with an electron in the conductor is:

$$\tau \simeq \frac{\delta_c}{v} \simeq 10^{-11} - 10^{-10}\ s \ , \qquad (72)$$

and the field, providing the adiabtic inversion of the magnetic spin in a track will be:

$$H_c \simeq \frac{\hbar}{\mu_B\tau} \simeq 10^4\ Oe \ . \qquad (73)$$

Thus, the field frozen in the conductor H_c can affect appreciably the process of spin-flip in the track and provide the adiabatic spin-flip of electrons in the magnetic at monopole speeds below 10^{-4} c. Besides, it can render a certain influence on dynamics of the domain walls in conducting films with high mobility.

7.3 Track readout

So, the domain induced by the monopole flight has the characteristic size of the order of 50 nm and magnetization about several thousand Oe. Then the domain magnetic flux Φ_d will be of the order:

$$\Phi_d = \pi r^2 I_s \simeq \Phi_0 = 2\cdot 10^{-7}\ G\cdot cm^2$$

For readout of such a flux we can use the high sensitive fluxmeter on the basis of superconducting quantum interference devices — SQUIDs or magnetooptical devices on the basis of Kerr effect (the rotation of the light polarization plane at reflection from a surface of ferromagnet which is magnetized perpendicular to the surface). Note that the second way is technically easier and does not require cryogenic maintenance. In the last case for detector realization we need surfaces, covered with a thin layer of light-axis magnetized magnetic media plus a magnetooptic device for readout of the film transverse magnetization (direction of the domain magnetization!) with a system of the high-resolution positioning.

A similar technique has emerged recently in an almost ready shape, suitable for a detector design with minimum adjustment. These are the magneto-optic recording technologies used in modern magneto-optic disks [28] and the appropriate readout devices.

8 Conclusion

This paper analyzes the mechanisms of the energy losses of the massive slow moving monopole in matter. In general this study applies to the magneto-ordered matter — ferromagnets, as well as conductors in normal and in superconducting state. New effective mechanisms of energy losses of slow magnetic monopole are considered:
— Cherenkov excitation of spin waves at monopole passage through the magneto-ordered magnetic,
— Excitation of Cherenkov acoustic radiation due to polarization of the matter and due to the atomic shell deformation of the matter by the magnetic field of the monopole.

The interaction of monopoles with films of magnetic materials is considered, in particular the interaction of slow monopoles with thin films of light-axis magnetics with high mobility of domain walls (materials with cylindric magnetic domains) is discussed. It is shown, that at passage of the slow massive monopole through a magneto-hard magnetic film, the track-domain with the characteristic size of the order of 50 nm and magnetization of about several thousand Oe can be formed. Thus the magnetic flux of the track appears to be about the value of the flux quantum. For readout of such a flux, detectors using fluxmeter on the basis already widely known SQUIDs can be used. However, for registration of traces of slow cosmic monopoles in magnetic matter, the experimental devices using the Kerr magneto-optical effect are more preferable, in our opinion. They recently have emerged in a shape suitable for detector design provided appropriate adjustment.

The authors are grateful to all colleagues for helpful and lively discussions of this work in various places and institutes. Special thanks go for stimulating interest and useful remarks to L.M. Barkov and I.B. Khriplovich (both Budker INP), and G. Tarle (Univ. of Michigan).

References

[1] P.A.M. Dirac, Proc. Roy. Soc. **A133** (1931) 60.

[2] D.E. Groom, Phys. Rep. **140** (1986).

[3] H.V. Klapdor-Kleingrothaus, A. Staudt, *Teilchenphysik ohne Beschleuniger.* Stuttgart, 1995.

[4] V.M. Vurodov, V.P. Martem'janov, *Private communication about old unpublished paper.* March 1998.

[5] L.M. Barkov, I.I. Gurevich, M.S. Zolotarev et al., Zh. Eksp. Teor. Fiz.**61** (1971) 1721; JETP 61 (1971) 1721.

[6] A.M. Poljakov, Pis'ma Zh. Eksp. Teor. Fiz. **20** (1974) 430; JETP Lett. 20 (1974) 194.

[7] G. t'Hooft, Nucl. Phys. B **79** 276 (1974).

[8] B. Cabrera, Phys. Rev. Lett. **48** (1982) 1378; R.D. Gardner et al., Phys. Rev. **44** (1991) 622.

[9] V.A. Rubakov, Pis'ma v ZhETF **33** (1981) 644; Nucl. Phys. **B203** (1982) 311.

[10] C. Callan, Phys. Rev. D **25** (1982) 2141.

[11] I.A. Belolaptikov et al., LANL e-print archive, astro-ph/9802223.

[12] MACRO collaboration, Phys. Lett. **B406** (1997) 249.

[13] F.C. Adams et al., Phys. Rev. Lett. **70** (1993) 2511.

[14] PDG Review of Particle Properties, Phys. Rev. D **54** (1996).

[15] L3 Collaboration, Phys. Lett. **B345** 609 (1995).

[16] D0 Collaboration, S. Abachi et al., Preprint Fermilab Pub-98/095-E (1998).

[17] I.M. Frank, *Vavilov—Cherenkov radiation.* M.: Nauka, 1988.

[18] D.D. Kirchniz, V.V. Losjakov, Pis'ma Zh. Eksp. Teor. Fiz. **42** (1985) 226.

[19] V.P. Zrelov,*Vavilov—Cherenkov radiation and its application to high energy physics.* Atomisdat, 1968.

[20] *Dirac monopole.* Ed. B.M. Boltovsky and Yu.D. Usachev, Mir, 1970.

[21] S.D. Drell et al., Phys. Rev. Lett. **50** (1983) 644.

[22] F.Kajino et al., Phys. Rev. Lett. **52** (1984) 1373.

[23] P.V. Vorob'ev, I.V. Kolokolov, Preprint BINP 98-16, Novosibirsk, 1998, hep-ph/9806495; P.V. Vorob'ev, I.V. Kolokolov, Pis'ma Zh. Eksp. Teor. Fiz. **67** (1998) 866.

[24] A.G. Gurevich, *Magnetic resonance in ferrites and anti-magnetics.* M.: Nauka, 1973.

[25] A.I. Ahiezer, V.G. Bar'jakhtar, S.V. Peletminski, *Spin waves.* M.: Nauka, 1967.

[26] S. Tikadsumi,*Physics of ferromagnetic.* Mir, 1987.

[27] A. Eshenfelder,*Physics and engineering of cylindrical magnetic domains.* Mir, 1983.

[28] E. Beitzig et al., Appl. Phys. Lett. **61** (1992) 142.