DATA PROCESSING FOR TURN-BY-TURN BEAM POSITION MONITOR

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Abstract

A computer off-line program to analyze the data delivered by a turn-by-turn beam position monitor (BPM) of a storage ring, is developed. The program is a set of data processing algorithms elaborated for various beam diagnostic problems. They are: plotting of beam current and coordinates; fine spectral analysis of betatron and synchrotron oscillations; phase space diagram of non-linear betatron motion; analysis of slow beam vibration; determination of resolution of the BPM itself; etc. The program can also be used for processing of beam motion simulation data to compare the diagnostic results of a real motion and its models. The program operates in MS Windows environment and has numerous service options.

Some new or modernized algorithms for beam dynamics investigation are described. The results of usage of these algorithms for data obtained at the VEPP-4M collider, demonstrate potentialities of the program.

INTRODUCTION

Turn-by-turn intensity/position monitoring was developed at BINP for its storage rings. [1] A set of specialized control and processing on-line programs was used for the monitoring. [2] The programs operate in storage ring control computers.

It turns out that one more, universal, off-line processing program, input data of which are the turn-byturn arrays taken from the specialized programs as a result of beam monitoring, is rather useful.

A wide range of beam diagnostic problems arisen during commission, study and routine runs of the rings, can be solved using the program. Among them, are:

- precise measurement of frequency, amplitude and phase of betatron and synchrotron oscillations; [1,2]
- analysis of coherent betatron oscillation envelope and spectrum fine structure; [2]
- study of non-linear betatron motion on phase space diagram; [4,5]
- measurement of dynamic aperture; [5]
- statistical and spectral analysis of slow beam microvibration; [4]
- measurement of monitor's resolution.

The program consists of data processing algorithms used for the diagnostic tasks mentioned above, and due to its numerous service options and, this should be emphasized, due to possibility to use it for processing of beam motion simulation data to compare the diagnostic results of a real motion and its models makes such a program an effective instrument of beam dynamics investigation. In the program, simulation data can also be used for testing new algorithms being developed.

The program operates in MS Windows environment. MS Visual C++ 4.0 is used for building the code. The input data are four output arrays of a turn-by-turn four parallel channel BPM.

In this report, we describe some new algorithms and algorithms, development of which still continues. Results of usage of these algorithms for data obtained at the VEPP-4M e-,e+ collider, illustrate capability of the program.

1. MEASUREMENT OF BPM RESOLUTION

Essential parameters of a beam monitor are an absolute error and a resolution which characterizes scattering of the absolute error due to noise inherent to the monitor.

Natural monitor's noise constituents are thermal and Shottky noise of the electronics. Noise-like signals can come in a monitor from other devices. The beam signal itself may produce in the electronics noise-like extra signals.

Measured beam parameter itself has noise or noiselike components. For instance, beam position in a storage ring fluctuates due to Magnet Power Supply ripple. Due to this, BPM resolution is usually measured using a test signal, own noise of which is taken to be known. Such a resolution could be considered as an ultimate one if the test signal would have the same essential characteristics as the beam signal has. It is not so when, for instance, beam bunch duration is in the sub-nanosecond range.

We examined a possibility to measure BPM resolution, using beam signals themselves.

Beam position (x, y) can be calculated using the formula (the result (1.3) will be same if other formula is used):

 $x = M_x \cdot (u_1 - u_2 - u_3 + u_4) / (u_1 + u_2 + u_3 + u_4)$

 $y = M_v (u_1 + u_2 - u_3 - u_4)/(u_1 + u_2 + u_3 + u_4)$, (1.1) where u_i are four output BPM signals, M_x and M_v are scale coefficients of the pickup. Due to noise variation of u_i , values (x, y) are scattered around some average position ($\overline{x}, \overline{y}$). The dispersion (σ_x^*, σ_y^*) is a product of both the BPM noise as well as fluctuation of beam position and can not be used as a measure of the BPM resolution.

Let's consider another, not dipole (1.1) but quadrupole combination of the signals:

 $q = M_{q'}(u_1 - u_2 + u_3 - u_4)/(u_1 + u_2 + u_3 + u_4).$ (1.2) For linear approximation of the pickup position characteristics, or for a beam placed near the center, such a combination does not depend on beam position. Assuming that the beam is thin, taking in (1.1) and (1.2) $u_1 \approx u_2 \approx u_3 \approx u_4$ one can easily obtain for the dispersions:

$$\sigma_q = \sigma_x, \ \sigma_y, \quad \text{if} \ M_q = M_x, \ M_y. \tag{1.3}$$

The dispersion σ_q does not depend on beam position. Thus, the BPM resolution (σ_x, σ_y) can be obtained as a dispersion σ_q of some set of measurements with $M_q = M_{\lambda x}, M_{y}$.

Note that the average beam position is calculated using the same set. Besides, the beam position fluctuation can be obtained as difference $(\sigma_{xy}^* - \sigma_{qxy})$.

The histograms (1024 samples) produced by the program, shown on Fig.1.1, present the resolution of the VEPP-4M BPM and the beam vertical fluctuation (beam intensity is $6 \cdot 10^{10}$ e, bandwidth is 60 Hz).



Fig.1.1. BPM resolution (a) and beam fluctuation (b).

2. PRECISE MEASUREMENT OF BETATRON PARAMETERS

Precise measurement of betatron frequency, amplitude and phase discovers some new facilities of beam diagnostics, such as measurement of beta function, chromaticity, cubic non-linearity, analysis of coherent betatron oscillation envelope and spectrum fine structure, etc.

Betatron frequency (non-integer part) is usually obtained by Fast Fourier Transform (FFT) of an array of N turn-by-turn beam coordinate samples x_k . FFT gives a complex spectrum array Φ_j :

$$\Phi_j = \sum_{k=0}^{N-1} x_k \cdot \exp(-i \cdot 2\pi k j/N), \qquad (2.1)$$

 $j = 0, 1, ..., N/2 - 1.$

Betatron frequency Q ($0 \le Q < 0.5$) is corresponded by the frequency $Q_m = m/N$ of a maximal harmonic Φ_m of the spectrum:

$$Q = Q_m + \varepsilon_Q, \qquad (2.2)$$

where $|\varepsilon_{Q}| = 1/2N$ is FFT discreteness error.

For many purposes, it is necessary to determine oscillation amplitude a and phase φ as well as frequency Q:

 $a = |\Phi_m| + \varepsilon_a, \ \varphi = -\arctan(\ln \Phi_m / \operatorname{Re} \Phi_m) + \varepsilon_{\varphi}$ (2.3)

But a problem arise, if one try to calculate a and φ using FFT. When the frequency error ε_Q can be decreased by increasing quantity of samples N, the amplitude error ε_a/a may reach -35%, the phase one ε_{φ} may reach $\pm \pi/2$ with any N (see Fig.2.1).

Refinement of FFT can be obtained by using various algorithms. [1,3] These algorithms refine the frequency

value. Then, accurate values of the amplitude and phase can be calculated.



Using simulation, two refinement algorithms were studied: the FFT + Dichotomy one (DA) [1,2] and the FFT Interpolation one (IA) [3]. A signal and a noise were used as:

 $x_k = \exp[i \cdot (2\pi kQ + \varphi)] + \eta_k$, k = 0, 1, ..., N, (2.4) where η_k is a "white noise" discrete function with maximal fluctuation $\pm 3\sigma$. Damping of the oscillations was introduced by multiplying the signal by factor:

$$D = \exp(-k/\alpha N). \tag{2.5}$$

The results are shown on Fig.2.2, where the plots of the errors are presented. Each point is the result of averaging of 40 simulations. When the noise becomes more than some threshold the errors increase rapidly, as the signal spectral peak can not be found among noise ones.



Fig.2.2. Accuracy of the DA (a) and the IA (b).

It is natural, that IA's frequency accuracy depends strongly on difference between a real signal and the model one, for which the interpolation formula has been derived. The plot (b) illustrates this, where the curves plotted by circles are calculated for the model signal (2.4), and the curves plotted by triangles for the same signal multiplied by factor D.

For the program mentioned in Introduction the DA was preferred to the IA, as a more suitable for real signals.

A precise measurement of betatron frequency of damping oscillations discovers, for instance, a possibility of reconstruction of their envelope. The array x_k is divided on $N \cdot Q$ segments, a maximal value of each is an element of the envelope array.

An example of using of the envelope reconstruction algorithm is given on Fig.2.3.



Fig.2.3. Fast damping of vertical betatron oscillations.

Vertical betatron oscillations with a fast damping of the beam at the VEPP-4M are shown. The envelope after the first kick is proportional to $\exp(-t/\tau_1)$, the envelope after one more kick is proportional to $\exp(-t^2/\tau_2^2)$. Cubic non-linearity can be calculated using measured value of τ_2 .

3. PHASE SPACE TRAJECTORIES OF NON-LINEAR BETATRON OSCILLATIONS

A conventional method of observation of non-linear betatron motion phase space trajectory is based on turnby-turn beam position measurement by two BPMs.

We discovered, that the information required to construct the turn-by-turn momentum at azimuth s_0 can be obtained from the turn-by-turn coordinate at the same azimuth, measured by a single BPM. Periodicity of the both gives a possibility to do it.

An empirical method for phase space trajectory construction based on spectral analysis of single BPM's data was developed. [4] The method was successfully used for investigation of non-linear beam dynamics at the VEPP-4M collider. [4,5]

Study of the problem of non-linear phase space trajectory construction is in progress. Some new results are presented below.

Let's consider the coordinate x_k and momentum x'_k , at some azimuth s_0 at k-th turn. The coordinate x_{k+1} and momentum x'_{k+1} at the next turn can be calculated using x_k and x'_k and the single turn matrix:

$$x_{k+1} = x_k \cos \mu + x'_k \beta \sin \mu, \qquad (3.1)$$

$$x'_{k+1} = x'_{k} \cos \mu - x_{k} \cdot \beta^{-1} \sin \mu + \zeta(x_{k}), \quad (3.2)$$

where $\mu = 2\pi Q$ is betatron phase advance per one turn, β is the beta function value at the azimuth s_0 , ζ is the effect of non-linear force per turn. The initial momentum x'_0 can be calculated immediately from (3.1): $x'_0 = (x_1 - x_0 \cdot \cos \mu)/\beta \sin \mu$. (3.3)

Thus, if an array of turn-by-turn beam coordinates measured by a BPM, is available, the expression (3.2) can be used as a recurrent formula to calculate the turnby-turn momentum.

Three parameters are to be known to do this: μ , β , and ζ . Betatron phase advance per one turn $\mu = 2\pi Q$ can be measured precisely using the algorithm described in the section 2. An error in the value of β is not critical, it results only in scaling of the momentum.

A problem is evaluation of ζ . A non-linearity of accelerator lattices usually consists of a sextupole component proportional to x^2 , and an octupole one

proportional to x^3 . If to use the approximation $\zeta = Ax^2 + Bx^3$, coefficients A and B can be roughly determined using correlation technique or be obtained from a measured value of cubic non-linearity [5]. The problem of ζ 's evaluation needs further investigation.

Note, that the spectral method does not require any information about non-linearity parameters.

To compare the two methods, computer simulation was used. The result of calculation by the program of one of the trajectories simulated is shown on Fig.3.1.

 \circ - Tracking + - Spectral method \triangle - Iteration method



Fig.3.1. Non-linear phase space trajectory.

These are the plots of phase space trajectory on coordinate-momentum and phase-action planes. The curve plotted by circles is obtained by particle tracking in the VEPP-4M non-linear model lattice near sextupole resonance $3Q_x = 26$. The curve plotted by crosses is constructed by the spectral method [4] mentioned above. The curve plotted by triangles is the result of the recurrent method with the approximation $\zeta = Ax^2$. One can see a fine closeness of the curves.

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