STATIC SYNCHRO-BETATRON BEAM-BEAM EFFECT CAUSED BY CROSSING ANGLE

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Abstract

Beam-beam collision with a crossing angle results in a static head-tail deformation of colliding bunches. The static head-tail effect is cured by setting the collider working point well apart the integer resonances.

1 INTRODUCTION

In multi-bunch colliders, interaction region schemes with a crossing angle at the IP are used to simplify (and, in general, to increase) the parasitic crossing separation. This is achieved at the expense of some geometric luminosity reduction and, potentially, of synchro-betatron resonances enhancement.

In this paper we consider one of the factors limiting the usable range of the crossing angle. Crossing the IP at an angle, the head and the tail of the bunch are kicked in the opposite directions by the partner bunch on each revolution. Without the synchrotron oscillation, this would result in *different closed orbits of the bunch head and tail*. Obviously, the same is true for the case where the synchrotron oscillation is adiabatically slow compared to the betatron oscillation.

For correct treatment of this static synchro-betatron beam-beam effect with *finite* synchrotron tunes, we apply the complete synchro-betatron mapping [1] of the bunch variables over the arc passage and use the linearized beam-beam interaction of the finite-length bunches [2]. Then we find a periodic self-consistent solution to this strong-strong beam-beam problem.

2 ONE-DIMENSIONAL CASE

First consider the case where x-y coupling is negligible. Here the bunch deformation is confined to the crossing plane, let it be the horizontal crossing. Following [2], we outline the formalism for this one-dimensional x-case, then extend it to two dimensions x-y in the next section.

We use the so-called "hollow beam" model. It assumes that all particles of the bunch have equal synchrotron amplitudes and are evenly spread over the synchrotron phase, forming a ring in the synchrotron phase space. The ring is divided into N mesh elements, each characterized by its transverse dipole moment and its synchrotron phase. The dipole moment of the ith mesh, $1 \leq i \leq N$, is proportional to the transverse displacement x_i of the centroid of the particles populating this mesh, times the portion N_b/N of the bunch intensity, N_b, per mesh. The betatron motion will be described in terms of the normalized betatron variables, x_i and p_i, where p_i is the respective momentum. Thus $2N$ variables will be needed to characterize synchro-betatron motion in each bunch. They form a $2N$-vector X, where x_i and p_i are listed in the order corresponding to the mesh number, according to its synchrotron phase.

The synchro-betatron oscillations of N elements forming a bunch are represented by the $2N \times 2N$ matrix M, which maps the above vector over the collider arc,

$$M = C \otimes B, \quad B = \begin{pmatrix} \cos\mu_\beta & \sin\mu_\beta \\ -\sin\mu_\beta & \cos\mu_\beta \end{pmatrix},$$

where $\otimes$ denotes the outer product, B is the betatron oscillation matrix, C is the circulant matrix [1] with elements

$$C_{ij} = \frac{\sin N\varphi_{ij}}{N \sin\varphi_{ij}}, \quad \varphi_{ij} = \frac{1}{2}\Big(\mu_s - (N-i+j)\frac{2\pi}{N}\Big),$$

$1 \leq i,j \leq N$, and μ_β, μ_s are the betatron and synchrotron phase advances. With $N = 2m+1$, the eigenvectors and eigenvalues of matrix M exactly correspond to the first $-m,\ldots,m$ synchro-betatron harmonics with the tunes $\nu_\beta - m\nu_s, \ldots, \nu_\beta + m\nu_s$, $\nu_{\beta,s} = \mu_{\beta,s}/2\pi$.

Note that the synchrotron oscillation in the circulant matrix formalism transports the dipole moment values around the circle formed by the mesh elements with fixed synchrotron phases (i.e., fixed longitudinal positions in the bunch), rather than performing a permutation of the meshes themselves.

Expansion of the model to the case of two noninteracting bunches is straightforward by using a $4N \times 4N$ matrix,

$$M_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes M \ .$$

The linearized beam-beam interaction is described by a $4N \times 4N$ matrix M_{bb} consisting of consecutive short kicks and drifts between interactions of macroparticles sitting in each mesh, and assumed to be rigid Gaussian disks [3]. For example let us consider the interaction of two bunches each consisting of 3 elements. Since matrix M_2 makes a transformation from the IP to the IP, the first step is the longitudinal unfolding of the bunch. Fig. 1 shows the position of the particles in their mesh elements before the first interaction. The next step is the interaction between particles 1,3,4 and 6, which is linear in relative distance. For instance, the kick given to the first particle is

$$\Delta p_1 = -\frac{2\pi\xi}{3}[(x_1 - x_6) + (x_1 - x_4)] \ ,$$

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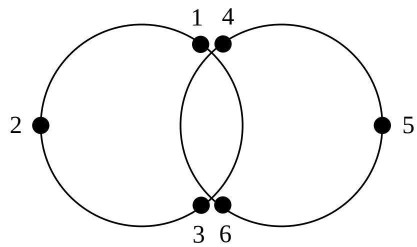

Figure 1: Position of macroparticles in the synchrotron phase space.

where x and p are the particle's coordinate and momentum, and ξ is the beam-beam parameter. Next follows the free drift and interaction of particles 2,4,6 and 1,3,5; next the drift and interaction of the "tail" particles Nos. 2 and 5, and finally return to the IP. Generalization of the algorithm to the case of $N > 3$ is evident.

The complete one-turn matrix M_t is the product of the arc matrix and the beam-beam matrix, $M_t = M_{bb}M_2$. Its eigenvalues and eigenvectors completely characterize the synchro-betatron modes of the beam-beam system and can be obtained numerically using a computer algebra system.

The crossing angle α is added to our variables before the beam-beam interaction stage and subtracted after it; it is introduced in terms of vector $X_0 = \alpha(0, 1, ..., 0, 1)$. To find the static deformation of the colliding bunches, we write the 1-turn periodicity condition

$$M_{bb} \cdot (M_2 \cdot X + X_0) - X_0 = X \ . \qquad (1)$$

Solving this equation for X yields initial conditions for each mesh element in the bunch corresponding to its closed orbit.

In the following part of this section we will discuss dependence of this solution on the machine parameters. In the illustrations we use $N = 5$ mesh divisions in each bunch, centered at the longitudinal positions

$$z_i = a\cos\big((2i-1)\pi/5\big), \quad 1 \le i \le 5,$$

numbering is cyclic starting from the bunch head, see Fig. 1. With the synchrotron amplitude $a = \sqrt{2}\sigma_z$, the variance of z is equal to the Gaussian bunch length. We are interested in parameters close to those of the KEK B-factory, particularly, we take $a = 8$ mm maximum.

The crossing angle enters proportionally in our linear model. We take $\alpha = 11$ mrad (horizontal) in what follows.

In the following set of figures the phase-space positions of the five mesh elements in the bunch are shown joined in the plot, the missing side of the pentagon corresponds to the bunch head.

The synchrotron tune ν_s determines separation of the particles 1 and 5 (2 and 4) with the same longitudinal coordinates in the transverse phase space, see Fig. 2. We chose $\nu_s = 0.01$ in the following examples.

The beam-beam parameter ξ should enter almost linearly, if the dynamic tune $\tilde{\nu}$ would be kept fixed by adjusting the machine tune ν_β with variation of ξ, according to $\cos\tilde{\nu} = \cos\nu - 2\pi\xi\sin\nu = const$. In practice ν is fixed, and this ξ-dependence is shown in Fig. 3. The ξ-dependent spread of the bunch distortion in fact corresponds to enhancement of the effective transverse emittance, if we have in mind the nonlinearity of the real beam-beam kick.

The bunch length represented in this "hollow beam" model by the amplitude a also enters almost linearly when $a \ll \beta^*$, Fig. 4. But for the case $a \sim \beta^*$ the deformation of the bunch shape becomes nontrivial.

The betatron tune is a crucial parameter for the static synchro-betatron effect. Since this effect is a closed orbit distortion different over the bunch length, we can expect a $\cot\pi\tilde{\nu}$ dependence. Thus, setting the machine tune far away the integer, we can have a considerable reduction in this effect. To demonstrate this, we give the top and bottom rows in Figs. 2–4 figures. The horizontal betatron tune actually used at KEKB is $\{\nu\} = 0.52$,

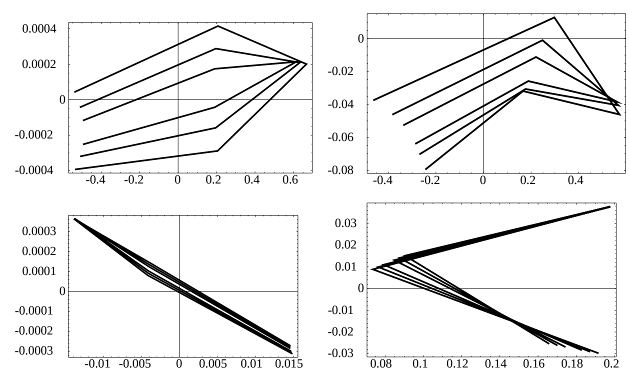

Figure 2: Transverse phase space x' (mrad) vs x (mcm) past collision. The synchrotron tune is varied, $\nu_s = 0.01$; 0.02; 0.03, other parameters are: $a = 8$ mm, $\xi = 0.05$. Top: $\{\nu\} = 0.08$, bottom: $\{\nu\} = 0.52$; left: $\beta^* = 70$ cm, right: $\beta^* = 0.7$ cm.

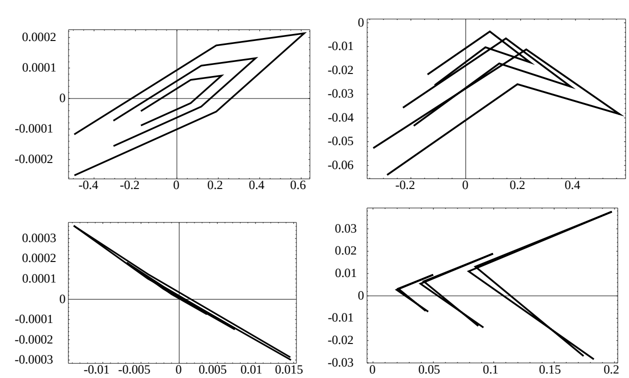

Figure 3: Transverse phase space x' (mrad) vs x (mcm) past collision. The beam-beam parameter is varied, $\xi = 0.05$; 0.025; 0.0125, other parameters are: $\nu_s = 0.01$, $a = 8$ mm. Top: $\{\nu\} = 0.08$, bottom: $\{\nu\} = 0.52$; left: $\beta^* = 70$ cm, right: $\beta^* = 0.7$ cm.

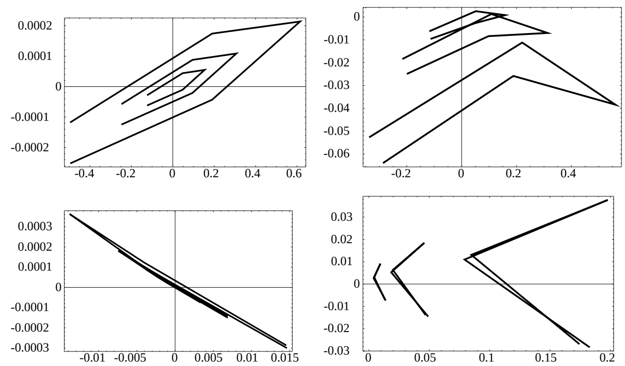

Figure 4: Transverse phase space x' (mrad) vs x (mcm) past collision. The bunch length is varied, $a = 2$; 4; 8 mm, other parameters are: $\nu_s = 0.01$, $\xi = 0.05$. Top: $\{\nu\} = 0.08$, bottom: $\{\nu\} = 0.52$; left: $\beta^* = 70$ cm, right: $\beta^* = 0.7$ cm.

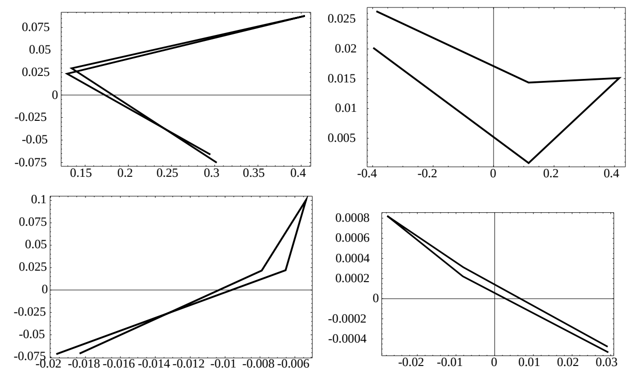

Figure 5: Vertical phase space y' (mrad) vs y (mcm) past collision. Top: $\{\nu_y\} = 0.08$, bottom: $\{\nu_y\} = 0.56$; left: $r_3 = 0.1$ cm^{-1}, right: $r_4 = 1$. Other parameters are: $\{\nu_x\} = 0.52$, $\nu_s = 0.01$, $\xi_{x,y} = 0.05$, $\beta_x^* = 60$ cm, $\beta_y^* = 0.7$ cm, $a = 8$ mm.

3 TWO-DIMENSIONAL CASE

With the account of the local x-y coupling at the IP the horizontal crossing angle can cause vertical bunch deformations which are critical for a flat-beam collider.

For a coupled lattice we replace the 2×2 betatron matrix B of the previous section by the 4×4 matrix B_4 expressed via the Twiss matrices of normal modes u, v,

$$B_4 = R_4^{-1}\, N\, R_4, \quad N = \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix},$$

where the decoupling matrix R_4 is symplectic. Following Edwards and Teng [4], we write R_4 in terms of a 2×2 coupling matrix R and its symplectic conjugate R^+:

$$R_4 = \begin{pmatrix} \mu I & -R^+ \\ R & \mu I \end{pmatrix}, \quad R = \begin{pmatrix} r_1 & r_2 \\ r_3 & r_4 \end{pmatrix},$$

$$R^+ = -SR^T S = \begin{pmatrix} r_4 & -r_2 \\ -r_3 & r_1 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

where $\mu = \sqrt{1 - \det R}$ and superscript T means matrix transpose. We will refer to r_{1-4} as *coupling parameters*.

To understand the role of different coupling parameters we can consider the case where the horizontal orbit kick is applied at the distance l past the IP, and let L_4 be this drift matrix. The vertical closed orbit distortion projected onto the IP is determined by the matrix $Q = (I - B_4)^{-1} L_4^{-1}$,

$$\begin{aligned} Q_{32} \;\; = \;\; & -\mu\big(r_1\beta_u - (r_2 + r_1 l)\alpha_u + r_2 l\gamma_u\big)\cot\pi\nu_u \\ & -\big((r_4 - r_3 l)\beta_v + (r_2 - r_1 l)\alpha_v\big)\cot\pi\nu_v\,. \end{aligned}$$

The exact solution of the self-consistent problem is available from the 2-dimensional analog of Eq. (1).

In practice, r_1 and r_2 are minimized by monitoring the vertical beam size at the IP, e.g. from the specific luminosity data, while r_3 and r_4 are the most difficult for measurement. In Fig. 5 we give an illustration of their effect on the vertical bunch distortion at the IP caused by the horizontal crossing angle. We take values of r_3 and r_4 which are not observable from the beam size monitoring at the IP. With the KEKB parameters we get the effect comparable to the design vertical size. Again setting the vertical tune far from the integer resonances results in suppression of this unwanted effect by almost one order of magnitude.

4 CONCLUSION

Beam-beam collision with a crossing angle results in a static head-tail deformation of colliding bunches, proportional to the crossing angle and dependent on the bunch length, synchrotron and betatron tunes, and the beam-beam parameter. Because of residual x-y coupling at the interaction point, this deformation is not confined to the crossing plane.With the account of nonlinearity of the beam-beam kick, this effect leads to an increase in the effective emittances in the both transverse planes and potentially degrades the luminosity, especially for flat colliding beams.

The static head-tail effect is strongly reduced by setting the collider working point well apart the integer resonances, this is a way to relax the requirements on residual coupling at the IP. In practice this means that both betatron tunes should be slightly above a half-integer, and reasonably far from the main coupling resonance.

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5 REFERENCES

[1] V.V. Danilov, E.A. Perevedentsev, *Nucl. Instr. Meth.* **A 391**, 77 (1997).

[2] E.A. Perevedentsev and A.A. Valishev, *Phys. Rev. ST-AB* **4**, 024403 (2001).

[3] K. Hirata, "Coherent betatron oscillation modes due to beam-beam interaction," Nucl. Instrum. Methods A **269**, 7 (1988).

[4] D. Edwards and L. Teng, IEEE Trans. Nucl. Sci., **20**, No. 3, 885 (1973).