THE BFKL POMERON WITHIN PHYSICAL RENORMALIZATION SCHEMES AND SCALES

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Abstract

In this lecture the next-to-leading order (NLO) corrections to the QCD Pomeron intercept obtained from the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation are discussed. It is shown that the BFKL Pomeron intercept when evaluated in non-Abelian physical renormalization schemes with Brodsky-Lepage-Mackenzie (BLM) optimal scale setting does not exhibit the serious problems encountered in the modified minimal subtraction ($\overline{MS}$) scheme. The results obtained provide an opportunity for applications of the NLO BFKL resummation to high-energy phenomenology. One of such applications for virtual gamma-gamma total cross section shows a good agreement with preliminary data at CERN LEP.

БФКЛ-ПОМЕРОН В ФИЗИЧЕСКИХ СХЕМАХ ПЕРЕНОРМИРОВОК

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Аннотация

Обсуждаются следующие за ведущим вкладом поправки к интерсепту померона КХД, полученного из уравнения Балицкого-Фадина-Кураева-Липатова (БФКЛ). Показано, что интерсепт БФКЛ-померона при применении неабелевых физических схем перенормировок с оптимизирующей точкой перенормировки Бродского-Лепажа-Маккензи (БЛМ) не имеет серьёзных трудностей, возникающих в схеме минимального вычитания ($\overline{MS}$). Полученные результаты открывают широкие возможности для применения БФКЛ в физике высоких энергий. Приложение БФКЛ к описанию полных сечений виртуальных фотонов находится в хорошем согласии с предварительными данными ЛЭП в ЦЕРНе.

1 Motivation

The discovery of rapidly increasing structure functions in deep inelastic scattering (DIS) at HERA [1] at small-x is in agreement with the expectations of the QCD high-energy limit. The Balitsky-Fadin-Kuraev-Lipatov (BFKL) [2, 3] resummation of energy logarithms is anticipated to be an important tool for exploring this limit. The leading order (LO) BFKL calculations [2] predict a steep rise of QCD cross sections. Namely, the highest eigenvalue, ω^{max}, of the BFKL equation [2] is related to the intercept of the Pomeron which in turn governs the high-energy asymptotics of the cross sections: $\sigma \sim s^{\alpha_{I\!P}-1} = s^{\omega^{max}}$. The BFKL Pomeron intercept in the LO turns out to be rather large: $\alpha_{I\!P} - 1 = \omega_L^{max} = 12 \ln 2 (\alpha_S/\pi) \simeq 0.55$ for $\alpha_S = 0.2$; hence, it is very important to know the next-to- leading order (NLO) corrections. In addition, the LO BFKL calculations have restricted phenomenological applications because, *e.g.*, the running of the QCD coupling constant α_S is not included and the kinematic range of validity of LO BFKL is not known.

Recently the NLO corrections to the BFKL resummation of energy logarithms were calculated; see Refs. [4, 5] and references therein. The NLO corrections [4, 5] to the highest eigenvalue of the BFKL equation turn out to be negative and even larger than the LO contribution for $\alpha_S > 0.157$. At such circumstances the phenomenological significance of the NLO BFKL calculations seems to be rather obscure.

However, one should stress that the NLO calculations, as any finite-order perturbative results, contain both renormalization scheme and renormalization scale ambiguities. The NLO BFKL calculations [4, 5] were performed by employing the modified minimal subtraction scheme ($\overline{\rm MS}$) [6] to regulate the ultraviolet divergences with arbitrary scale setting.

In this work we consider the NLO BFKL resummation of energy logarithms [4, 5] in physical renormalization schemes in order to study the renormalization scheme dependence. To resolve the renormalization scale ambiguity we utilize Brodsky-Lepage-Mackenzie (BLM) optimal scale setting [7]. We show that the reliability of QCD predictions for the intercept of the BFKL Pomeron at NLO when evaluated using BLM scale setting within non-Abelian physical schemes, such as the momentum space subtraction (MOM) scheme [8, 9] or the Υ-scheme based on $\Upsilon \to ggg$ decay, is significantly improved as compared to the $\overline{\rm MS}$-scheme. This provides a basis for applications of NLO BFKL resummation to high-energy phenomenology. Certain aspects of this work were presented in Ref. [10].

2 BFKL in Physical Renormalization Schemes

We begin with the representation of the $\overline{\rm MS}$- result of NLO BFKL [4, 5] in physical renormalization schemes. Although the $\overline{\rm MS}$-scheme is somewhat artificial and it lacks a clear physical picture, it can serve as a convenient intermediate renormalization scheme. The eigenvalue of the NLO BFKL equation at transferred momentum squared $t = 0$ in the $\overline{\rm MS}$-scheme [4, 5] can be represented as the action of the NLO BFKL kernel (averaged over azimuthal angle) on the LO eigenfunctions $(Q_2^2/Q_1^2)^{-1/2+i\nu}$ [4]:

$$\omega_{\overline{MS}}(Q_1^2,\nu) = \int d^2Q_2\ K_{\overline{MS}}(\vec{Q}_1,\vec{Q}_2)\left(\frac{Q_2^2}{Q_1^2}\right)^{-\frac{1}{2}+i\nu} = \\ = N_C\chi_L(\nu)\frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi}\left[1+r_{\overline{MS}}(\nu)\frac{\alpha_{\overline{MS}}(Q_1^2)}{\pi}\right], \qquad (1)$$

where

$$\chi_L(\nu) = 2\psi(1)-\psi(1/2+i\nu)-\psi(1/2-i\nu)$$

is the function related with the LO eigenvalue, $\psi = \Gamma'/\Gamma$ denotes the Euler ψ- function, the ν-variable is a conformal weight parameter [11], N_C is the number of colors, and $Q_{1,2}$ are the virtualities of the reggeized gluons.

The calculations of Refs. [4, 5] allow us to decompose the NLO coefficient $r_{\overline{MS}}$ of Eq. (1) into β-dependent and the conformal (β-independent) parts:

$$r_{\overline{MS}}(\nu) = r^{\beta}_{\overline{MS}}(\nu) + r^{conf}_{\overline{MS}}(\nu)\ , \qquad (2)$$

where

$$r^{\beta}_{\overline{MS}}(\nu) = -\frac{\beta_0}{4}\left[\frac{1}{2}\chi_L(\nu)-\frac{5}{3}\right] \qquad (3)$$

and

$$r^{conf}_{\overline{MS}}(\nu) = -\frac{N_C}{4\chi_L(\nu)}\left[\frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3+\left(1+\frac{N_F}{N_C^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right)-\chi_L''(\nu)+\right. \\ \left.+\ \frac{\pi^2-4}{3}\chi_L(\nu)-\frac{\pi^3}{\cosh(\pi\nu)}-6\zeta(3)+4\varphi(\nu)\right] \qquad (4)$$

with

$$\varphi(\nu) = 2\int_0^1 dx\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6}-{\rm Li}_2(x)\right],\ {\rm Li}_2(x) = -\int_0^x dt\frac{\ln(1-t)}{t}. \qquad (5)$$

Here $\beta_0 = (11/3)N_C - (2/3)N_F$ is the leading coefficient of the QCD β-function, N_F is the number of flavors, $\zeta(n)$ stands for the Riemann zeta-function, $\mathrm{Li}_2(x)$ is the Euler dilogarithm (Spence-function). In Eq. (4) N_F denotes flavor number of the Abelian part of the $gg \to q\bar{q}$ process contribution. The Abelian part is not associated with the running of the coupling [12] and is consistent with the correspondent QED result for the $\gamma^*\gamma^* \to e^+e^-$ cross section [13].

The β-dependent NLO coefficient $r^{\beta}_{\overline{MS}}(\nu)$, which is related to the running of the coupling, receives contributions from the gluon reggeization diagrams, from the virtual part of the one-gluon emission, from the real two-gluon emission, and from the non-Abelian part [12] of the $gg \to q\bar{q}$ process. There is an omitted term in $r^{\beta}_{\overline{MS}}(\nu)$ proportional to $\chi'_L(\nu)$ which originates from the asymmetric treatment of Q_1 and Q_2, it can be removed by the redefinition of the LO eigenfunctions [4].

The NLO BFKL Pomeron intercept then reads for $N_C = 3$ [4]:

$$\alpha_{I\!P}^{\overline{MS}} - 1 = \omega_{\overline{MS}}(Q^2, 0) = 12 \ln 2 \, \frac{\alpha_{\overline{MS}}(Q^2)}{\pi}\left[1 + r_{\overline{MS}}(0)\frac{\alpha_{\overline{MS}}(Q^2)}{\pi}\right], \quad (6)$$

$$r_{\overline{MS}}(0) \simeq -20.12 - 0.1020 N_F + 0.06692\beta_0 \, , \quad (7)$$

$$r_{\overline{MS}}(0)_{|N_F=4} \simeq -19.99 \, .$$

Physical renormalization schemes provide small and physically meaningful perturbative coefficients by incorporating large corrections into the definition of the coupling constant. One of the most popular physical schemes is MOM-scheme [8, 9], based on renormalization of the triple-gluon vertex at some symmetric off-shell momentum. However, in the MOM- scheme the coupling constant is gauge-dependent already in the LO, and there are rather cumbersome technical difficulties. These difficulties can be avoided by performing calculations in the intermediate $\overline{\mathrm{MS}}$-scheme, and then by making the transition to some physical scheme by a finite renormalization [8]. In order to eliminate the dependence on the gauge choice and other theoretical conventions, one can consider renormalization schemes based on physical processes [7], *e.g.*, V-scheme based on heavy quark potential. Alternatively, one can introduce a physical scheme based on the $\Upsilon \to ggg$ decay using the NLO calculations of Ref. [14].

A finite renormalization due to the change of scheme can be accomplished by a transformation of the QCD coupling [8]:

$$\alpha_S \to \alpha_S\left[1 + T\frac{\alpha_S}{\pi}\right], \quad (8)$$

where T is some function of N_C, N_F and, for the MOM- scheme, of a gauge parameter ξ. Then the NLO BFKL eigenvalue in the MOM-scheme can be represented as follows:

$$\begin{aligned} \omega_{MOM}(Q^2,\nu) &= N_C\chi_L(\nu)\frac{\alpha_{MOM}(Q^2)}{\pi}\Big[1+r_{MOM}(\nu)\,\frac{\alpha_{MOM}(Q^2)}{\pi}\Big], \quad (9)\\ r_{MOM}(\nu) &= r_{\overline{MS}}(\nu)+T_{MOM}. \end{aligned}$$

For the transition from the $\overline{\rm MS}$-scheme to the MOM-scheme the corresponding T-function has the following form [8]:

$$\begin{aligned} T_{MOM} &= T^{conf}_{MOM}+T^{\beta}_{MOM}, \quad (10)\\ T^{conf}_{MOM} &= \frac{N_C}{8}\Big[\frac{17}{2}I+\xi\frac{3}{2}(I-1)+\xi^2(1-\frac{1}{3}I)-\xi^3\frac{1}{6}\Big],\\ T^{\beta}_{MOM} &= -\frac{\beta_0}{2}\Big[1+\frac{2}{3}I\Big], \end{aligned}$$

where $I=-2\int_0^1 dx\,\ln(x)/[x^2-x+1]\simeq 2.3439$.

Likewise, one can obtain for the V-scheme [7]:

$$T_V = \frac{2}{3}N_C-\frac{5}{12}\,\beta_0\,, \quad (11)$$

and, by the use of the results of Ref. [14], for the Υ- scheme:

$$T_\Upsilon = \frac{6.47}{3}\,N_C-\frac{2.77}{3}\,\beta_0\,. \quad (12)$$

One can see from Table 1 that there is no a strong renormalization scheme dependence, though the problem of a large NLO BFKL coefficient remains. A large size of the perturbative corrections leads to a significant renormalization scale ambiguity.

3 Optimal Renormalization Scale Setting

The renormalization scale ambiguity problem can be resolved if one can optimize the choice of scales and renormalization schemes according to some sensible criteria. In the BLM optimal scale setting [7], the renormalization scales are chosen such that all vacuum polarization effects from the QCD β-function are resummed into the running couplings. The coefficients of the

Table 1: Scheme-transition function and the NLO BFKL coefficient in physical schemes

Scheme		$T = T^{conf} + T^{\beta}$	$r(0) = r^{conf}(0) + r^{\beta}(0)$	$r(0)$ $(N_F = 4)$
M	$\xi = 0$	$7.471 - 1.281\beta_0$	$-12.64 - 0.1020N_F - 1.214\beta_0$	-22.76
O	$\xi = 1$	$8.247 - 1.281\beta_0$	$-11.87 - 0.1020N_F - 1.214\beta_0$	-21.99
M	$\xi = 3$	$8.790 - 1.281\beta_0$	$-11.33 - 0.1020N_F - 1.214\beta_0$	-21.44
V		$2 - 0.4167\beta_0$	$-18.12 - 0.1020N_F - 0.3497\beta_0$	-21.44
Υ		$6.47 - 0.923\beta_0$	$-13.6 - 0.102N_F - 0.856\beta_0$	-21.7

perturbative series are thus identical to the perturbative coefficients of the corresponding conformally invariant theory with $\beta = 0$. The BLM approach has an important advantage of resumming the large and strongly divergent terms in the perturbative QCD series which grow as $n![\alpha_S\beta_0]^n$, *i.e.*, the infrared renormalons associated with coupling constant renormalization. The renormalization scales in the BLM approach are physical in the sense that they reflect the mean virtuality of the gluon propagators [7].

The BLM scale setting [7] can be applied within any appropriate physical scheme. In the present case one can show that within the V-scheme (or the $\overline{\rm MS}$-scheme) the BLM procedure does not change significantly the value of the NLO coefficient $r(\nu)$. This can be understood since the V-scheme as well as $\overline{\rm MS}$-scheme are primarily adjusted to the case when, in the LO, there are dominant QED (Abelian) type contributions, whereas, in the BFKL case, the LO gluon-gluon (non-Abelian) interactions are important.

Therefore, from the point of view of BLM scale setting, one can separate QCD processes into two classes specifying whether gluons are involved into the LO or not. Thus one can expect that in the BFKL case it is appropriate to use a physical scheme which is adjusted to non-Abelian interactions in the LO. One can choose the MOM- scheme based on the symmetric triple-gluon vertex [8, 9] or the Υ-scheme based on $\Upsilon \to ggg$ decay. The importance of taking into account this circumstance for vacuum polarization effects can be seen from the "incorrect" sign of the β_0-term for $r_{\overline{MS}}$ in the unphysical $\overline{\rm MS}$-scheme (Eq. (7)).

Adopting BLM scale setting, the NLO BFKL eigenvalue in the MOM-

Table 2: The NLO- BFKL-Pomeron intercept in the BLM scale setting within non-Abelian physical schemes

Scheme		$r_{BLM}(0)$ ($N_F = 4$)	$\alpha_{I\!P}^{BLM} - 1 = \omega_{BLM}(Q^2, 0)$		
			$Q^2 = 1$ GeV^2	$Q^2 = 15$ GeV^2	$Q^2 = 100$ GeV^2
M	$\xi = 0$	-13.05	0.134	0.155	0.157
O	$\xi = 1$	-12.28	0.152	0.167	0.166
M	$\xi = 3$	-11.74	0.165	0.175	0.173
Υ		-14.01	0.133	0.146	0.146

scheme is

$$\omega_{BLM}^{MOM}(Q^2,\nu) = N_C\chi_L(\nu)\frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi}\left[1 + r_{BLM}^{MOM}(\nu)\frac{\alpha_{MOM}(Q_{BLM}^{MOM\,2})}{\pi}\right], \tag{13}$$

$$r_{BLM}^{MOM}(\nu) = r_{MOM}^{conf}(\nu)\,. \tag{14}$$

The β-dependent part of the $r_{MOM}(\nu)$ defines the corresponding BLM optimal scale

$$Q_{BLM}^{MOM\,2}(\nu) = Q^2\exp\left[-\frac{4r_{MOM}^{\beta}(\nu)}{\beta_0}\right] = Q^2\exp\left[\frac{1}{2}\chi_L(\nu) - \frac{5}{3} + 2\left(1+\frac{2}{3}I\right)\right]. \tag{15}$$

Taking into account the fact that $\chi_L(\nu) \to -2\ln(\nu)$ at $\nu \to \infty$, one obtains at large ν

$$Q_{BLM}^{MOM\,2}(\nu) \;=\; Q^2\frac{1}{\nu}\exp\left[2\left(1+\frac{2}{3}\,I\right) - \frac{5}{3}\right]. \tag{16}$$

At $\nu = 0$ we have $Q_{BLM}^{MOM\,2}(0) = Q^2(4\exp[2(1 + 2I/3) - 5/3]) \simeq Q^2\,127$. Note that $Q_{BLM}^{MOM\,2}(\nu)$ contains a large factor, $\exp[-4T_{MOM}^{\beta}/\beta_0] = \exp[2(1 + 2I/3)] \simeq 168$, which reflects a large kinematic difference between MOM- and $\overline{\rm MS}$- schemes [15, 7], even in an Abelian theory.

Analogously, one can implement the BLM scale setting in the Υ-scheme (Table 2).

Figures 1 and 2 give the results for the eigenvalue of the NLO BFKL kernel. We have used the QCD parameter $\Lambda = 0.1$ GeV which corresponds to $\alpha_S = 4\pi/[\beta_0\ln(Q^2/\Lambda^2)] \simeq 0.2$ at $Q^2 = 15$ GeV^2. Also, the generalizations [16, 17, 18] of the β -function in the running coupling and of flavor number for continuous treatment of quark thresholds have been used.

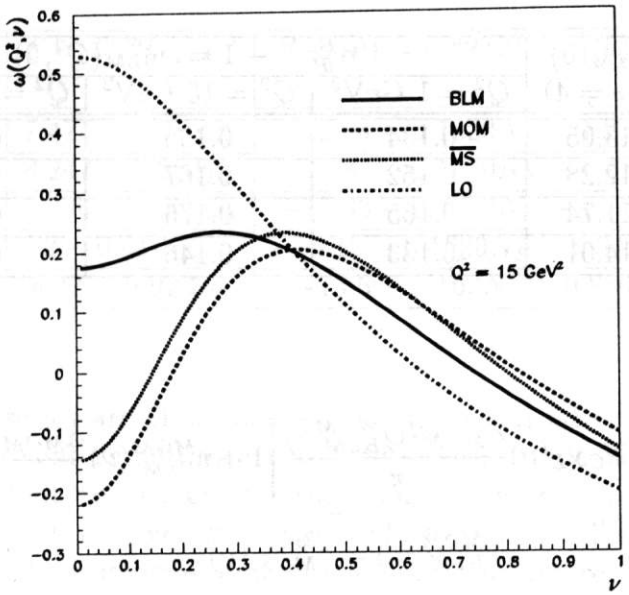

Figure 1: ν-dependence of the NLO BFKL eigenvalue at $Q^2 = 15$ GeV^2: BLM (in MOM-scheme) – solid, MOM-scheme (Yennie gauge: $\xi = 3$) – dashed, $\overline{\text{MS}}$-scheme – dotted. LO BFKL ($\alpha_S = 0.2$) – dash-dotted.

One can see from Fig. 1, that the maximum which occurs at non-zero ν is not as pronounced in the BLM approach compared to the $\overline{\text{MS}}$-scheme, thus it does not serve as a good saddle point at high energies.

One of the striking features of this analysis is that the NLO value for the intercept of the BFKL Pomeron, improved by the BLM procedure, has a very weak dependence on the gluon virtuality Q^2. This agrees with the conventional Regge theory where one expects a universal intercept of the Pomeron without any Q^2-dependence. The minor Q^2-dependence obtained, on one side, provides near insensitivity of the results versus precise value of Λ, and, on the other side, leads to the approximate scale and conformal invariance. Thus one may use conformal symmetry [11, 19, 20] for the continuation of the present results to the case $t \neq 0$.

Therefore, by applying the BLM scale setting within non-Abelian physical

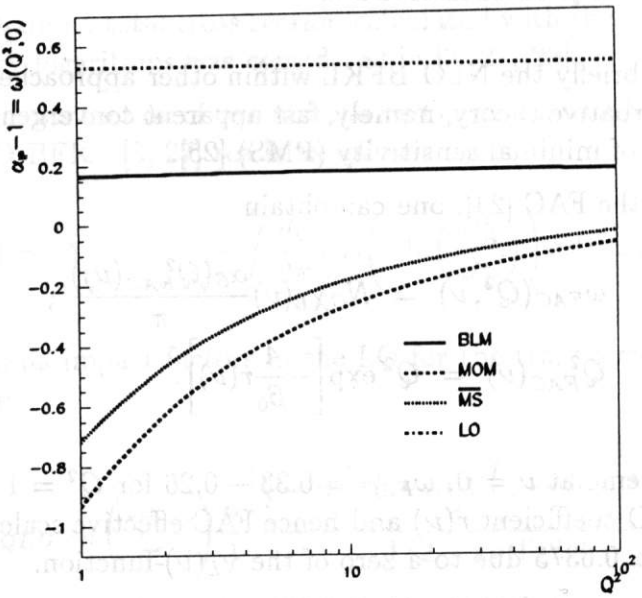

Figure 2: Q^2-dependence of the BFKL Pomeron intercept in the NLO. The notation is as in Fig. 1

schemes (MOM- and Υ- schemes), we do not face the serious problems [21, 22, 23] which were present in the $\overline{\text{MS}}$-scheme, e.g., oscillatory cross section disbehavior based on the saddle point approximation [21], and a somewhat puzzling analytic structure [22] of the $\overline{\text{MS}}$-scheme result [4, 5].

Since the BFKL equation can be interpreted as a "quantization" of the renormalization group equation [19], it follows that the effective scale should depend on the BFKL eigenvalue ω, associated with the Lorentz spin, rather than on ν. Thus, strictly speaking, one can use the above effective scales as function of ν only in "quasi-classical" approximation at large Q^2. However, the present remarkable Q^2-stabilty leads us to expect that the results obtained with LO eigenfunctions may not change considerably for $t \neq 0$ due to the approximate conformal invariance.

4 Other Approaches to Perturbation Theory Optimization

Now we consider briefly the NLO BFKL within other approaches to the optimization of perturbative theory, namely, fast apparent convergence (FAC) [24] and the principle of minimal sensitivity (PMS) [25].

By the use of the FAC [24], one can obtain

$$\omega_{FAC}(Q^2,\nu) = N_C\chi_L(\nu)\frac{\alpha_S(Q^2_{FAC}(\nu))}{\pi}, \quad (17)$$

$$Q^2_{FAC}(\nu) = Q^2\exp\left[-\frac{4}{\beta_0}r(\nu)\right]. \quad (18)$$

In the $\overline{\rm MS}$-scheme at $\nu = 0$, $\omega_{FAC} = 0.33 - 0.26$ for $Q^2 = 1 - 100$ GeV^2. However, the NLO coefficient $r(\nu)$ and hence FAC effective scale, each have a singularity at $\nu_0 \simeq 0.6375$ due to a zero of the $\chi_L(\nu)$-function.

In the PMS approach [25] the NLO BFKL eigenvalue reads as follows

$$\omega_{PMS}(Q^2,\nu) = N_C\,\chi_L(\nu)\frac{\alpha_{PMS}(Q^2(\nu))}{\pi}\left[\frac{1+(C/2)\alpha_{PMS}/\pi}{1+C\alpha_{PMS}/\pi}\right], \quad (19)$$

where the PMS effective coupling α_{PMS} is a solution of the following transcendental equation

$$\frac{\pi}{\alpha_{PMS}} + C\ln\left(\frac{C\alpha_{PMS}/\pi}{1+C\alpha_{PMS}/\pi}\right) + \frac{C/2}{1+C\alpha_{PMS}/\pi} = \frac{\beta_0}{4}\ln\left(\frac{Q^2}{\Lambda^2}\right) - r(\nu), \quad (20)$$

with $C = \beta_1/(4\beta_0)$ and $\beta_1 = 102 - 38N_F/3$. At $\nu = 0$ one obtains in the $\overline{\rm MS}$-scheme $\omega_{PMS} = 0.23 - 0.20$ for $Q^2 = 1 - 100$ GeV^2 but, by the same reason as in the FAC case, the PMS effective coupling has a singularity at ν_0. Thus, the application of the FAC and PMS scale setting approaches to the BFKL eigenvalue problem leads to difficulties with the conformal weight dependence, which is an essential ingredient of the BFKL calculations. The unphysical behavior of the FAC and PMS effective scales for jet production processes has been noted in Refs. [26].

The problem can be resolved by the expansion of $\chi_L(\nu)$-function near its zero to avoid unphysical behavior of the optimization procedure.

5 Application for Gamma-Gamma Scattering

The gamma-gamma total cross section calculated with the resummation of the leading energy logarithms was considered in [3, 27, 28].

The total cross section of two unpolarized gammas with virtualities Q_A and Q_B in the LO BFKL [3, 27] reads as follows:

$$\sigma(s,Q_A^2,Q_B^2) = \sum_{i,k=T,L} \frac{1}{\pi Q_A Q_B} \int\limits_0^\infty \frac{d\nu}{2\pi} \cos\left(\nu \ln\left(\frac{Q_A^2}{Q_B^2}\right)\right) F_i(\nu) F_k(-\nu) \left(\frac{s}{s_0}\right)^{\omega(Q^2,\nu)}, \tag{21}$$

with the gamma impact factors in the LO for the transverse and longitudinal polarizations:

$$F_T(\nu) = \alpha_{QED}\,\alpha_S \left(\sum_q e_q^2\right) \frac{\pi}{2} \frac{\left[\frac{3}{2}-i\nu\right]\left[\frac{3}{2}+i\nu\right]\Gamma\left(\frac{1}{2}-i\nu\right)^2 \Gamma\left(\frac{1}{2}+i\nu\right)^2}{\Gamma(2-i\nu)\Gamma(2+i\nu)}, \tag{22}$$

$$F_L(\nu) = \alpha_{QED}\,\alpha_S \left(\sum_q e_q^2\right) \pi \frac{\Gamma\left(\frac{3}{2}-i\nu\right)\Gamma\left(\frac{3}{2}+i\nu\right)\Gamma\left(\frac{1}{2}-i\nu\right)\Gamma\left(\frac{1}{2}+i\nu\right)}{\Gamma(2-i\nu)\Gamma(2+i\nu)}, \tag{23}$$

where Regge scale parameter s_0 is proportional to a hard scale $Q^2 \sim Q_A^2, Q_B^2$, Γ being the Euler Γ-function, and e_q is the quark electric charge.

In the NLO BFKL case one should obtain the formula analogous to LO BFKL (Eq. 21). It has been demonstrated in Ref. [29] that the infrared singularities at the NLO are cancelled out for impact factors of colorless particles. Therefore, in the NLO both the kernel of the BFKL equation and impact factors are infrared safe which confirm a self-consistence of such factorization scheme.

While exact NLO impact factors of gamma are not known yet [30] one can use the LO impact factors of Eqs. (22-23) [3, 13, 27] implying that the main NLO corrections come from the NLO BFKL subprocess rather than from the impact factors [31, 32]. Thus, in the NLO BFKL one can have Eq. (21) but with $\omega(Q^2,\nu)$ taken in the NLO. To imply the BLM procedure to the total cross section one can see that one can imply the BLM procedure directly to the NLO BFKL eigenvalue $\omega(Q^2,\nu)$ within the accuracy up to the next-to-next-to-leading order (NNLO) and higher subleading terms.

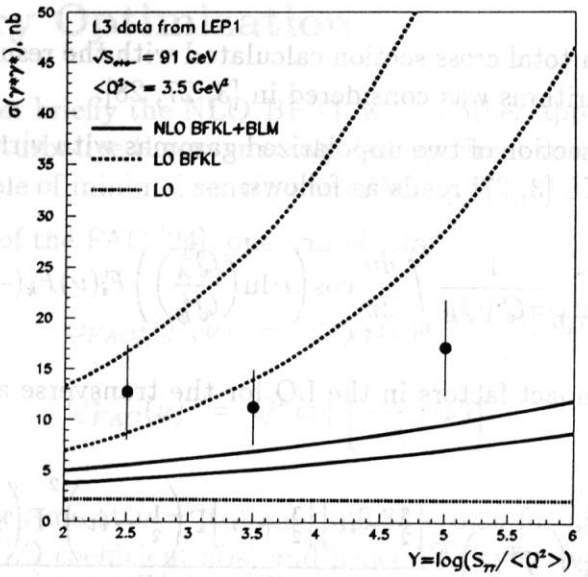

Figure 3: The NLO BFKL Pomeron vs preliminary L3 data on virtual gamma-gamma cross section (with subtracted quark-box contribution) at energy 91 GeV of the e^+e^- collisions. Solid curves: NLO BFKL in BLM; dashed: LO BFKL, and dotted: LO contribution. Two different choices of the Regge scale: $s_0 = Q^2/2$ and $s_0 = 2Q^2$.

For numerical calculations the NLO BFKL eigenvalue $\omega(Q^2, \nu)$ in the MOM-scheme (Yennie gauge: $\xi = 3$) has been used.

In Figs. 3 – 5 the comparison of BFKL predictions in the LO and NLO BFKL [31, 32] improved by the BLM procedure with L3 Collaboration data [33, 34] from CERN LEP is shown. Different curves reflect uncertainty with the choice of the Regge scale parameter which indicates when the asymptotic regime starts. At infinite collision energies, the cross sections do not depend on this scale parameter s_0. For present calculations, two variants have been choosen $s_0 = Q^2/2$ and $s_0 = 2Q^2$, where for symmetric virtuality case $Q^2 = Q_A^2 = Q_B^2$. One can see from Figs. 3 – 5 that the LO BFKL predictions overestimate the L3 data, while the agreement of the NLO BFKL improved by the BLM procedure is reasonably well, especially at higher energies of LEP2 $\sqrt{s_{e^+e^-}} = 183 - 189$ GeV (Figs. 4, 5). One can notice also that sensitivity of the NLO BFKL results with respect to the Regge parameter s_0 is much smaller than in the case of the LO BFKL. Recent OPAL Collaboration data

[35] are also in a good agreement with the NLO BFKL predictions.

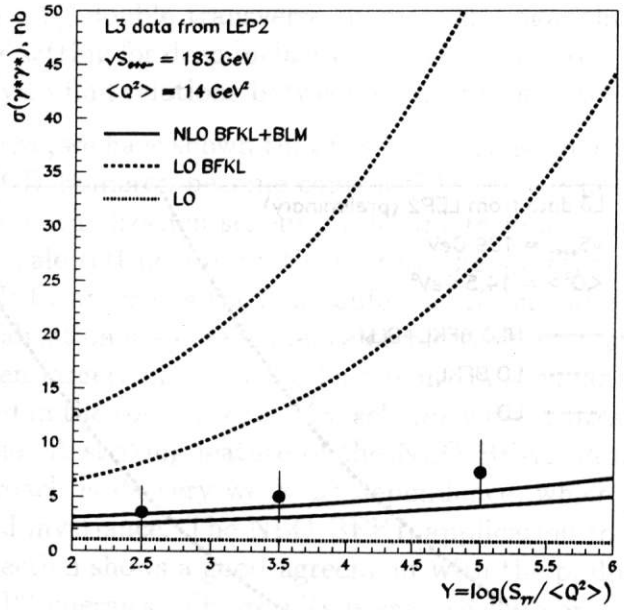

Figure 4: The same as Fig. 3, but for energy 183 GeV of e^+e^- collisions

The gamma-gamma scattering is attractive from viewpoint that it is theoretically more controllable rather than hadron-hadron and lepton-hadron collisions where non-perturbative hadronic structure functions are involved. In addition, in the gamma-gamma scattering the unitarization (screening) corrections due to multiple Pomeron exchange would be less important than in hadron collisions. It was shown in Refs. [36] that the unitarization corrections in hadron collisions can lead to higher value of the (bare) Pomeron intercept than the effective intercept value. Since the hadronic data fit yields about 1.1 for the effective intercept value [37, 38], then the bare Pomeron intercept value should be above this value. So that, in case of small unitarization corrections in the gamma- gamma scattering at large Q^2 one can accomodate the NLO BFKL Pomeron intercept value 1.13 - 1.18 along with larger unitarization corrections in hadronic scattering [36], where it can lead to a smaller effective Pomeron intercept value about 1.1 for hadronic collisions.

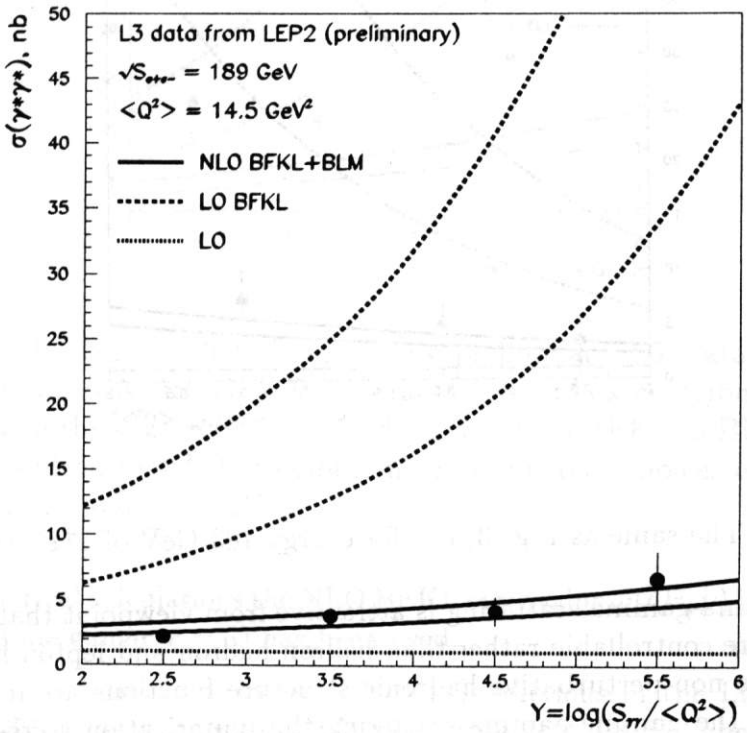

Figure 5: The same as Figs. 3,4, but for energy 189 GeV of e^+e^- collisions

6 Summary

There have been a number of recent papers which analyze the NLO BFKL predictions in terms of rapidity correlations [39, 40], t-channel unitarity [41], angle-ordering [42], double transverse momentum logarithms [43, 44, 45, 46] and BLM scale setting for deep inelastic structure functions [47]. This requires a further study to find relations between such approaches.

To summarize, we have shown that the NLO corrections to the BFKL equation for the QCD Pomeron become controllable and meaningful provided one uses physical renormalization schemes relevant to non-Abelian gauge theory. BLM optimal scale setting automatically sets the appropriate physical renormalization scale by absorbing the non-conformal β-dependent coefficients. The strong renormalization scale dependence of the NLO corrections to BFKL resummation then largely disappears. This is in contrast to the unstable NLO results obtained in the conventional $\overline{\rm MS}$- scheme with arbitrary choice of renormalization scale. A striking feature of the NLO BFKL Pomeron intercept in the BLM approach is its very weak Q^2-dependence, which provides approximate conformal invariance. The NLO BFKL application to the total gamma-gamma cross section shows a good agreement with the preliminary L3 data at the CERN LEP2 energies. The results presented here open new windows for applications of NLO BFKL resummation to the high-energy phenomenology.

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Note added at Proof. After presentation of this lecture, the OPAL and L3 Collaborations at the CERN LEP accumulated statistics at higher energies and finalized their data [48]. The final OPAL and L3 data [48], although presented in a different way, show even better agreement [49] with our earlier predictions [31, 32].

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