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ONE-LOOP CORRECTION TO THE NEWTON LAW

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We have found the graviton contribution to the one-loop quantum correction to the Newton law. This correction results in interaction decreasing with distance as $1/r^3$ and is dominated numerically by the graviton contribution. The previous calculations of this contribution to the discussed effect are demonstrated to be incorrect.

1 Introduction

The problem of corrections to the equations of motion, arising in general relativity, is far from being new. The classical relativistic corrections to these equations were found long ago by Einstein, Infeld and Hoffmann [1], and by Eddington and Clark [2]. (A relatively simple derivation of these corrections is presented in the textbook [3].) Later this result was reproduced by Iwasaki by means of Feynman diagrams [4]. Thus, the problem of the classical relativistic corrections to the Newton law is solved finally.

Let us note that the general structure of the relativistic classical correction to the interaction potential of two bodies with masses m_1 and m_2, which would be of second order in the Newton gravitational constant k, is clear immediately. Indeed, the quantity km/c^2 (c is the velocity of light) has the dimension of length, so that with the account for the symmetry under the interchange $m_1 \leftrightarrow m_2$ the correction should be of the form

$$U_{cl} = a_{cl}\,\frac{k^2 m_1 m_2 (m_1 + m_2)}{c^2 r^2}. \tag{1}$$

The dimensionless constant a_{cl} as found in the above works equals $1/2$.

There is one more linear in k combination of constants which can be used for the construction of a power correction to the Newton potential.

We mean

$$\frac{k\hbar}{c^3} = l_p^2,$$

where $\hbar$ is the Planck constant, $l_p = 1.6 \cdot 10^{-33}$ cm is the Planck length. Clearly, such a correction, being of course of quantum nature, should look as follows:

$$U_{qu} = a_{qu} \frac{k^2 \hbar m_1 m_2}{c^3 r^3}. \tag{2}$$

One has to find the numerical constant a_{qu}. In spite of extreme smallness of the quantum correction, its investigation certainly has a methodological interest: this is a closed calculation of a higher order effect in the nonrenormalizable quantum gravity.

The reason why this problem allows for a closed solution is as follows. The Fourier-transform of $1/r^3$ is

$$\int d\mathbf{r} \, \frac{\exp(-i\mathbf{qr})}{r^3} = -\, 2\pi \ln q^2. \tag{3}$$

This singularity in the momentum transfer q means that the correction discussed can be generated only by diagrams with two massless particles in the t-channel. The number of such diagrams of second order in k is finite, and their logarithmic part in q^2 can be calculated unambiguously.

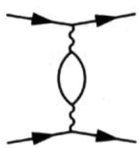


Figure 1: Photon (neutrino) loop

The corresponding diagrams with photons and massless neutrinos in the loop (see Fig. 1) were calculated in [5]–[8]. This contribution to the numerical factor a_{qu} is

$$a_{\gamma\nu} = -\frac{4 + N_\nu}{15\pi}, \tag{4}$$

where N_ν is the number of massless two-component neutrinos.

As to the contribution to the effect from the graviton exchange, it was considered in [9]–[15]. However, there is no quantitative agreement among the results of these works, even the predictions for the sign of the correction differ.

We believe that the correct result for the quantum correction to the Newton law will be sufficiently interesting from the theoretical point of view. This is the aim of our investigation. Among the previous works on the subject, the most detailed presentation of the calculation is given in [10, 15]. Our approach — the direct calulation of Feynman diagrams, the choice of the field operator for the gravitational field and of the gauge — is the same as in [9]–[12], [15]. It allows for a relatively detailed comparison of calculations of separate contributions to the effect. This comparison has demonstrated that in [9]–[12], [15] not all diagrams are taken into account, and the considered contributions are calculated incorrectly. Below, when discussing concrete diagrams, we will come back to the comparison with the previous works, including [13, 14]. And meanwhile, let us note an obvious error in [9]–[12], [15]: therein formula for the Fourier-transform of the function $1/r^3$ (see (3)) contains π^2, instead of π, and this error persists in the final answer as well.

Some of the considered diagrams contribute also to the classical relativistic correction. To check our calculations we computed in parallel these classical contributions and compared them with the corresponding results of [4]. As to these classical corrections, we have complete agreement with [4] for each diagram taken separately.

2 Simple loops

It is convenient to start with the diagrams where the Feynman integrals contain two denominators only.

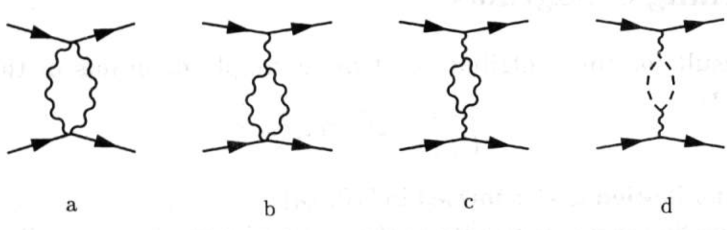


Figure 2: Simple loops

The simplest of them, Fig. 2a, is missed at all in [9]–[12], [15]. The result for this contribution to the quantum correction is

$$U_{qu1} = -\frac{22}{\pi}\frac{k^2 m_1 m_2}{r^3}. \tag{5}$$

The calculation of the next diagram, Fig. 2b, and that obtained from it by interchanging scalar particles, is also sufficiently simple and results in

$$U_{qu2} = \frac{26}{3\pi}\frac{k^2 m_1 m_2}{r^3}. \tag{6}$$

The result of [10] for this contribution differs from (6) only by a wrong power of π. The corresponding result of [15] is quite different.

As to the diagrams Fig. 2c,d with the polarization operator of graviton, we have nothing to add here to works [9]–[12]. The calculation is based on the effective Lagrangian corresponding to the sum of these diagrams, with gravitons and vector ghosts, obtained by 't Hooft and Veltman [16]:

$$L = -\frac{1}{16\pi^2}\ln|q^2|\left(\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right). \tag{7}$$

This contribution to the quantum correction is

$$U_{qu3} = -\frac{43}{30\pi}\frac{k^2 m_1 m_2}{r^3}. \tag{8}$$

Let us mention that diagrams 2c,d were computed in other variables, $\psi_{\mu\nu} = h_{\mu\nu} - 1/2\delta_{\mu\nu}h$, in Refs. [14, 17, 18], and in the source description of gravity (due to Schwinger) in [5].

3 Triangle diagrams

Our result for the contribution of more simple diagrams of the type Fig. 3a is

$$U_{qu4} = \frac{28}{\pi}\frac{k^2 m_1 m_2}{r^3}. \tag{9}$$

This contribution is also missed in [10, 15].

These diagrams contribute to the classical correction as well. An extra proof of our normalization for the seagull vertex is the agreement with the corresponding classical result of [4].

a b

Figure 3: Triangle diagrams

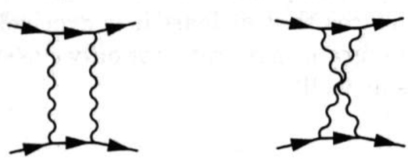
Much more tedious is the calculation of diagrams of the type Fig. 3b. It results in

$$U_{qu5} = -\frac{29}{3\pi}\frac{k^2 m_1 m_2}{r^3}. \tag{10}$$

The corresponding result of [15] differs from (10) only by sign and wrong power of π. The result of [10] for this contribution is quite different.

4 Box diagrams

The box diagrams generate contributions of two types. The first one is of the same origin as that of previous diagrams: q^2 serves as a small-momenta cut-off for integrals divergent at large momenta. This contribution originates from the confluence of the vertices on one "matter" line (which effectively results in a triangle graph) or on both.



a b

Figure 4: Box diagrams

In fact, the "triangle" contributions from the s- and u-channel diagrams cancel, and those from the double confluence add up and result in

$$U_{qu6} = -\frac{8}{\pi}\frac{k^2 m_1 m_2}{r^3}. \tag{11}$$

As to the irreducible part of the box diagrams, it is infrared divergent with q^2 serving as a high-momenta cut-off. This contribution is

$$U_{qu7} = -\frac{23}{3\pi}\frac{k^2 m_1 m_2}{r^3}. \tag{12}$$

For the box diagrams as well, we have checked that our results for thus generated classical corrections agree completely with those of [4].

The box contributions to the quantum correction are missed at all in [9]–[12], [15], though diagrams Fig. 4a,b are considered in [19] from another point of view.

On the other hand, neither in [13], nor in [14] we could find any mention of the "infrared" contribution of the type (12). In fact, in [14] the problem of classical and quantum corrections was treated in different variables, $\psi_{\mu\nu} = h_{\mu\nu} - 1/2\delta_{\mu\nu}h$. It can be easily demonstrated that the expressions for the box diagrams are exactly the same in both variables, ψ and h. However the box contributions, as calculated in [14], disagree both with the classical ones obtained in [4] (which are demonstrated explicitly in [4] to be the same in both variables, ψ and h) and with our results for the quantum correction, be it (11), or (12), or the sum of (11) and (12).

At last few words more on Ref. [13]. The approach advocated therein looks quite interesting and promising. However, the results for the quantum correction presented in [13] do not agree with ours (neither do they agree with those of [10, 11, 12, 15, 16]). Due to the lack of details in [13], we cannot say with certainty what is the origin of the disagreement. Still, an impression arises that at least it is overlooked in [13] that the irreducible triangle diagrams generate not only classical corrections, but quantum corrections as well.

5 Conclusions

Summing up all the contributions obtained, (5), (6), (8), (9), (10), (11), (12), we arrive at the following result for the Newton potential, with the discussed quantum correction due to the two-graviton exchange included:

$$U(r) = -\frac{k m_1 m_2}{r}\left(1 + \frac{121}{10\pi}\frac{k\hbar}{c^3 r^2}\right). \tag{13}$$

Let us note that the derived overall correction enhances, but not suppresses the common Newton attraction.

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