Temperature regime in neutron production target for BNCT

T.A.Vsevolozhskaya

Budker Institute of Nuclear Physics, Novosibirsk, Russia

One of the problems in creation of the neutron production target for BNCT consists in necessity to spread the proton beam about the whole target surface to avoid the local overheating, caused by inhomogeneous distribution of energy deposition. Defocusing of the beam up to the target size creates some technical complications. More attractive looks the target moving across the beam, or the beam sweeping across the target.

Temperature of target heating, caused by the energy deposition under a particle beam passing through, is defined by the thermal conductivity equation $\frac{\partial T}{\partial t} = a^2 \Delta T + \frac{q(x,y,z,t)}{C_p}$ (1) with proper boundary conditions, one of which is that of practically no heat evacuation from the inlet target surface, i.e. $\frac{\partial T}{\partial z}_{z=0} = 0$. Function $q(x, y, z, t)$ in (1) describes the specific power deposition in dependence on coordinates and time.

In case of a permanent particle beam the temperature in target is growing until the equilibrium is achieved between the power deposition and evacuation. The temperature stabilization time τ_∞ is important to be known to make estimations on efficiency of moving target. When τ_∞ is small compared to a time of material exposure under the beam, $\tau_b \cong 2x_0 / u_0$ with $2x_0$ standing for the beam size in the motion direction and u_0 - for velocity, the target motion gives practically no effect. If, by the opposite, τ_∞ is much more than τ_b, only the last defines the maximum temperature.

To solve the equation (1) the operational method of complex variable function theory is convenient to be used. Time dependence of specific power deposition has a form $q(x,y,z,t) = q_0(x,y,z) \cdot \eta(t)$, where $\eta(t)$ is the unit function, equal to zero when $t < 0$, and to unity when $t \geq 0$, whose image is $1/\mathrm{p}$.

If the transverse size of the beam is much more than the target depth, the dependence of energy deposition on transverse coordinates may be neglected, so that $q_0(x,y,z) \cong q_0(z)$. Besides this $q_0(z)$ may be considered equal to a constant value q_{00} within the mean range of particles $\delta \cong 15\mu m$, and to zero outside it. Thus the temperature image $U(z,p)$ is got in a form:

$$U(z,p) = -\frac{1}{p\sqrt{p} \cdot aC_\rho} \int_0^{min\{z,\delta\}} q_0(z') \cdot sh \frac{(z-z')\sqrt{p}}{a} \cdot dz' + A_0 . ch \frac{z\sqrt{p}}{a} .$$

Here A_0 is defined with the use of boundary condition at the outlet target surface $z = \Delta$. In the ideal case of zero temperature maintained here, i.e. $U(\Delta, p) = 0$, the temperature image at $z = 0$ is found as:

$$U(0,p)=\frac{q_{00}}{C_p p^2}\cdot\frac{\cosh\frac{\Delta\sqrt{p}}{a}-\cosh\frac{(\Delta-\delta)\sqrt{p}}{a}}{\cosh\frac{\Delta\sqrt{p}}{a}}.$$

In the limit $p\to 0$ this defines the equilibrium temperature, $T(z,\infty)=\left[pU(z,p)\right]_{p=0}$, at inlet surface $z=0$ as:

$$T(0,\infty)=\frac{q_{00}}{C_p}\cdot\frac{\delta\cdot(2\Delta-\delta)}{2a^2}.$$

Here C_p stands for the heat capacity and a^2 – for temperature conductivity coefficient.

With $q_{00}\cong 65$ kJ/g, which corresponds to the heat flux of 1 kJ/cm^2/s, $\Delta=0.5$ мм and $\delta=15\mu$м in molybdenum target $T(0,\infty)=36^\circ C$.

Temperature dependence on time is described as:

$$T(0,t)=\frac{q_{00}}{a^2C_p}\left[\frac{(2\Delta-\delta)\cdot\delta}{2}-2\Delta^2\left(\frac{2}{\pi}\right)^3\sum_{k=1}^{\infty}\frac{1}{(2k-1)^3}\cdot\sin\frac{(2k-1)\pi\delta}{2\Delta}\cdot\exp\left(-\frac{(2k-1)^2\pi^2a^2t}{4\Delta^2}\right)\right].$$

The first term in the sum ($k=1$) forms more than 80% of the whole sum even at $t=0$. Its characteristic time $\tau_{\infty,1}=\frac{4\Delta^2}{\pi^2a^2}$ defines in the main the temperature stabilization time $\tau_\infty\cong\tau_{\infty,1}$. The effective rate of heating appears to be by a ratio $\sim\frac{\pi^2\delta}{4\Delta}$ less than its initial value $\left.\frac{\partial T(0,t)}{\partial t}\right|_{t=0}=\frac{q_{00}}{C_p}$, which maintains during a very short time $\tau_0\sim\frac{4\delta^2}{\pi^2a^2}$ only. For $\delta=15\mu$м in molybdenum target this time is equal to $\sim 1.7\mu$ s, while the stabilization time with $\Delta=0.5$ мм is equal to $\tau_\infty\cong 1.86$ мs.

Distribution of equilibrium temperature over target depth is found from solution of equation $\frac{\partial^2T}{\partial z^2}+\frac{q_0(z)}{C_pa^2}=0$ with above defined boundary conditions. It has a form of Fourier series $T(z,\infty)=\sum_{n=1}^{\infty}T_n\cdot\cos\nu z$, where $\nu=\frac{(2n-1)\pi}{2\Delta}$, $T_n=\frac{q_{0n}}{C_pa^2\nu^2}$ and $q_{0n}=2q_{00}\cdot\frac{\sin\nu\delta}{\nu\Delta}$.

As a real approach to the heat evacuation scheme let's consider the cooling of target with the turbulent flow of liquid metal over the outlet target surface. The boundary condition here is determined through an achievable value of the specific heat transfer $Q=\lambda_2 Nu\cdot(T-T_0)/l$, defined with the heat conductivity of liquid λ_2, temperature difference between the target surface and liquid, $T-T_0$, characteristic size of the liquid metal stream l and the Nusselt coefficient Nu. With T_0 put zero the boundary condition at $z=\Delta$ is got in a form $\lambda_1\left.\frac{\partial T}{\partial z}\right|_{z=\Delta}=-\lambda_2 Nu\cdot\frac{T}{l}$, where λ_1 is the heat conductivity of target material.

Temperature image at inlet surface with this new condition at the outlet one gets a form:

$$U(0,p)=\frac{q_{00}}{C_\rho p^2}\cdot\frac{\cosh\frac{\Delta\sqrt{p}}{a}-\cosh\frac{(\Delta-\delta)\sqrt{p}}{a}+\frac{\lambda_1}{\lambda_2}\cdot\frac{l\sqrt{p}}{aNu}\cdot\left[\sinh\frac{\Delta\sqrt{p}}{a}-\sinh\frac{(\Delta-\delta)\sqrt{p}}{a}\right]}{\cosh\frac{\Delta\sqrt{p}}{a}+\frac{\lambda_1}{\lambda_2}\cdot\frac{l\sqrt{p}}{aNu}\cdot\sinh\frac{\Delta\sqrt{p}}{a}}.$$

This defines the equilibrium temperature $T(z,\infty)=[pU(z,p)]_{p=0}$ as being equal to $T(0,\infty)\cong\frac{q_{00}\delta}{C_\rho a^2}\cdot\left(\frac{2\Delta-\delta}{2}+\frac{l}{Nu}\cdot\frac{\lambda_1}{\lambda_2}\right)$ at $z=0$ and to $T(\Delta,\infty)\cong\frac{q_{00}\delta}{C_\rho a^2}\cdot\frac{l}{Nu}\cdot\frac{\lambda_1}{\lambda_2}$ at $z=\Delta$.

For liquid metal, flowing through the channels, the Nusselt coefficient is found as $Nu\cong 4.5+0.014Pe^{0.8}$, where $Pe=\frac{u_0 l}{a^2}$, u_0 - flow velocity and l - channel diameter. For target of the molybdenum, cooled with the liquid gallium, flowing through the channels of diameter $l=2$ mm with velocity $v_0=10$ m/s, one gets $Nu=14.1$. With $q_{00}\cong 65$ kJ/g the equilibrium temperature at outlet surface $T(\Delta,\infty)$ is equal to $45^\circ C$, while at the inlet one - to $81^\circ C$, more than two times higher as compared to the ideal case $T(\Delta,t)\equiv 0$.

Temperature dependence on time is still described by means of expression:

$T(0,t)=T(0,\infty)-\frac{q_{00}}{a^2C_\rho}\cdot 2\Delta^2\sum_{k=1}^{\infty}B_k\cdot\exp\left(-\frac{\phi_k^2a^2t}{\Delta^2}\right)$. However ϕ_k now are the roots of equation: $\cos\phi-\frac{\lambda_1}{\lambda_2}\cdot\frac{l}{\Delta\cdot Nu}\cdot\phi\sin\phi=0$, i.e. $\phi\cdot tg\phi\cong 0.81$ for values of l and v_0 accepted above. The first of roots, which defines in the main the characteristic time of temperature stabilization, is $\phi_1\cong 0.506\cdot\frac{\pi}{2}$, i.e. about two times less than in the ideal case. Time of temperature stabilization becomes equal to $\tau_\infty\cong 7.2$ ms.

The equilibrium temperature distribution in target with new boundary condition still has a form of series $T(x,y,z)=\sum_{m=1}^{\infty}T_m(x,y)\cos\mu_m z$ with μ_m being the roots of equation $\lambda_1\mu\sin\mu\Delta=\frac{\lambda_2 Nu}{l}\cdot\cos\mu\Delta$.

For a beam, moving about the target with velocity u_0 in transverse direction x, for instance, the particle density distribution along this direction is to be taken into account, while in another one the beam still may be considered as homogeneous: $T_m(x,y)\cong T_m(x)$.

In the coordinate frame where the beam rests an equation for $T_m(x)$ reads:

$$\frac{\partial^2T_m}{\partial x^2}-\frac{u_0}{a^2}\frac{\partial T_m}{\partial x}-\mu_m^2T_m+\frac{q_m(x)}{a^2C_\rho}=0.$$

Here $q_m(x)$ are the coefficients of specific power expansion in a series $q(x,z)=\sum_{m=1}^{\infty}q_m(x)\cos\mu_m z$ with above defined μ_m. With supposition made about the specific power dependence on z one gets $q_m(x)=q_0(x)\cdot\frac{4\sin\mu_m\delta}{2\mu_m\Delta+\sin 2\mu_m\Delta}$, where $q_0(x)$ stands for specific power at the inlet target surface.

Solution for m - component of equilibrium temperature is got as:

$$T_m(x) = \frac{1}{a^2 C_\rho (D_1 - D_2)} \left[\int_x^\infty q_m(x') \exp[D_1(x - x')] dx' + \int_{-\infty}^x q_m(x') \exp[D_2(x - x')] dx' \right],$$

with D_1, D_2 being the roots of characteristic equation $D^2 - \frac{u_0}{a^2} D - \mu_m^2 T_m = 0$, i.e. $D_{1,2} = \frac{u_0}{2a^2} \pm \sqrt{\left(\frac{u_0}{2a^2}\right)^2 + \mu_m^2}$.

Let $q_0(x)$ be an infinite series of rectangles of height q_{00} and width Δx, distant from each other by X_0 between the centers. Within the rectangle, $-\frac{\Delta x}{2} \le x \le \frac{\Delta x}{2}$, the solution has a form:

$$T_m(x) = \frac{q_{0m}}{a^2 C_\rho (D_1 - D_2)} \left\{ \frac{1}{D_1} \left[1 - e^{-D_1 \cdot \left(\frac{\Delta x}{2} - x\right)} + 2 \sinh \frac{D_1 \Delta x}{2} \cdot \frac{e^{-D_1 \cdot (X_0 - x)}}{1 - e^{-D_1 \cdot X_0}} \right] - \frac{1}{D_2} \left[1 - e^{D_2 \cdot \left(\frac{\Delta x}{2} + x\right)} - 2 \sinh \frac{D_2 \Delta x}{2} \cdot \frac{e^{D_2 \cdot (X_0 + x)}}{1 - e^{D_2 \cdot X_0}} \right] \right\}$$

and between them, $\frac{\Delta x}{2} \le x \le X_0 - \frac{\Delta x}{2}$:

$$T_m(x) = \frac{2 q_{0m}}{a^2 C_\rho (D_1 - D_2)} \left\{ \frac{1}{D_1} \sinh \frac{D_1 \Delta x}{2} \cdot \frac{e^{-D_1 \cdot (X_0 - x)}}{1 - e^{-D_1 \cdot X_0}} + \frac{1}{D_2} \sinh \frac{D_2 \Delta x}{2} \cdot \frac{e^{D_2 \cdot x}}{1 - e^{D_2 \cdot X_0}} \right\}.$$

In a limit of extremely large velocity u_0 both these expressions transform into $T_m(x) \cong \frac{q_{0m}}{a^2 C_\rho \mu_m^2} \frac{\Delta x}{X_0}$, which corresponds to the homogeneously distributed power deposition of specific value, decreased as compared to q_{00} in proportionality to the ratio $\frac{\Delta x}{X_0}$. Condition for such a transformation is the μ_m smallness as compared to $\frac{u_0}{a^2}$: $\mu_m^2 << \left(\frac{u_0}{2a^2}\right)^2$. In the case of molybdenum target cooled with gallium, for the first root of equation $\mu\Delta \cdot \tan \mu\Delta \cong \frac{Nu \cdot \Delta}{l} \cdot \frac{\lambda_2}{\lambda_1}$, $\mu_1 \Delta \cong 0.506 \cdot \frac{\pi}{2}$, this is fulfilled at $u_0 > 20$ cm/s.

When the water is used for target cooling instead of the liquid gallium by the same geometric parameters and flow velocity, the equation for μ_m definition reads $\mu\Delta \cdot \tan \mu\Delta \cong 0.172$. Time of temperature stabilization, defined with the first root $\mu_1 \Delta \cong 0.403$, is equal to ~ 28 мсек. Equilibrium temperature at inlet target surface is by ~6.8 times higher than in the ideal case, and three times higher as compared to the liquid gallium cooling.