MUON COLLIDERS AND NEUTRINO FACTORIES: BASICS AND PROSPECTS

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The prospects of high energy physics are outlined with the advent of muon colliders and neutrino factories. Technical features of the novel accelerators are surveyed, with emphasis on optimization of the ionization cooling technique for muon beams. The ultimate luminosities of the muon collider are discussed as well as minimization of the detector background.

1. Introduction

Let us assume – Basic Physics (almost equal to Elementary Particle Physics in a wide sense, including Astro-Particle Physics) would not die – with all its machinery, huge teams, substantial cost, long lasting projects, etc. It is my belief: in the long term, it is a correct approach for Mankind.

And with this assumption let us consider the reasons for developing muon collider(s) and, technically speaking, its sub-step – neutrino factory(ies). Other colliders – photon-photon, photon-electron, lepton-hadron, photon-hadron – need special discussion. In all our considerations we will keep in mind “very” high energies.

Using existing technologies, it seems easier to construct a hadron (proton) collider for the same energy and for the same luminosity of fundamental interactions, as those in lepton-lepton collisions. To reach this goal, the hadron-hadron energy should be 10 times as high as the lepton-lepton one: hadrons bring to the collision their complex structure (quarks and gluons), whereas for lepton colliders (involving electrons/positrons or muons) we deal with fundamental (unstructured, to our present knowledge) incidental particles directly. Additionally, in hadron collisions the fundamental interactions are not at all monochromatic (100% energy spread in collision!), and each fundamental interaction is accompanied by many interactions of remnants of hadrons, which produced the given fundamental interaction.

Even in the frame of the Standard Model, muon colliders and $e^{+/-}$ (linear) colliders are not just technically different versions of lepton colliders of the same energy. This difference relates to much higher "parasitic" radiation in the electron case – in two senses:

1. The radiation of individual initial colliding particles is much higher for electrons (and increases with the energy). Tagging of such photons (and, moreover, measuring their energy) is almost impossible for options with the high luminosity-per-bunch collision, currently under development.

2. Coherent fields in the collision region are so high that the synchrotron radiation of counter-propagating electrons (+/–) takes off a substantial fraction of their energies.

Hence, instead of pure, say, electron-positron collisions with a narrow spectrum we would get a wide initial spectrum, a lot of parasitic photons and photon-photon and photon-electron collisions. Eliminating this background will be one of our major headaches (on background issues for muons – later!).

If we go to higher energies, the $e^{+/-}$ collider becomes more and more difficult technically – to prevent the growth of synchrotron radiation in the coherent field of the counter-rotating bunch and to keep the luminosity growing, we need to make the vertical size of the interaction spot one nanometer or smaller! There is no such a limitation for the muon collider (the rest mass is much greater!).

But, maybe, at energies of around 1 TeV and higher some *new physics* would appear, and the muon would be not just a heavier electron with completely new interactions. In this case the muon collisions would bring fundamentally complementary information to electron ones, and the muon collider becomes a <u>must</u>.

Muon storage rings required for muon colliders would give birth to excellent muon and electron neutrino/antineutrino beam sources – so-called Neutrino Factories. There, the relevant physics is very different from muon collider physics, but the accelerator technologies that should be used are similar in many aspects.

Why should Neutrino Factories be very useful?

1. Narrow (cooled) intense muon beams in storage rings of high enough energy produce narrow (with a transverse momentum of around 30 MeV/c) ν_e, ν_μ, anti-ν_e and anti-ν_μ beams, and enable very complete studies of neutrino interactions (behind perfect muon shielding!).

2. Such beams are perfect for long-distance neutrino studies (neutrino oscillations and related topics).

Accelerator technologies for colliders and factories are similar in many aspects:

ionization cooling (albeit weaker requirements for Neutrino Factories, an easier "first" step);

"fast" muon acceleration;

muon decay ring (without highest field requirement).

2. Muon Acceleration

We, at Novosibirsk, started – already in the 1960's[1–3] – to consider options for lepton colliders reaching energies of hundreds of GeV (and even higher) – linear electron-positron and muon colliders, in parallel.

At first glance, the main disadvantage of muons was their very short life-time (2.2 μs in the rest frame). Of course, the muon life-time grows proportionally to its energy, $E_\mu/E_{\mu 0}$, but it remains short. Hence, the cooling of muon beams and their acceleration to the energy required should be fast enough.

It is easy to estimate the average accelerating gradient required:

$$\frac{dN_\mu}{dE_\mu} = \frac{dN_\mu}{\frac{dE_\mu}{dx}\cdot dx} = -\frac{N_\mu}{\tau_{\mu 0}\cdot \frac{E_\mu}{E_{\mu 0}}\cdot c} = -\frac{N_\mu}{\tau_{\mu 0}\cdot \frac{E_\mu}{E_0}\cdot c}\ .$$

Hence,

$$N_\mu = N_{\mu 0}\left(\frac{E_{\mu-ini}}{E_{\mu-fin}}\right)^{\frac{E_{\mu 0}}{c\tau_{\mu 0}}\Big/\frac{dE_\mu}{ds}}\ ,$$

and the required gradient is

$$\frac{dE_\mu}{dx} = \frac{\frac{E_{\mu 0}}{c\tau_{\mu 0}}\cdot \ln\left(\frac{E_{\mu-fin}}{E_{\mu-ini}}\right)}{1-\frac{N_{\mu-fin}}{N_{\mu-ini}}}\ .$$

Hence, to lose only 10% upon acceleration from 200 MeV to 2 TeV, the average acceleration gradient should be 14 MeV/m – quite modest on a modern scale!

The limited muon lifetime entails the requirement to use the highest possible magnetic field B_{coll} in the collider itself. The number of turns useful for muon-muon luminosity in the collider (for an interaction region per turn) is

$$N_{lumi} = \frac{1}{2}\cdot\frac{ec\tau_{\mu 0}}{2\pi E_{\mu 0}}\cdot B_{coll}\ ,$$

and does not depend on the muon energy!

For the average collider field of 10 Tesla,

$$N_{lumi} = 1500.$$

And at a few TeV and 10 cycles per second, collisions of muon bunches become continuous in time!

In this paper, we consider a general scheme of the muon collider complex as presented in Fig. 1.

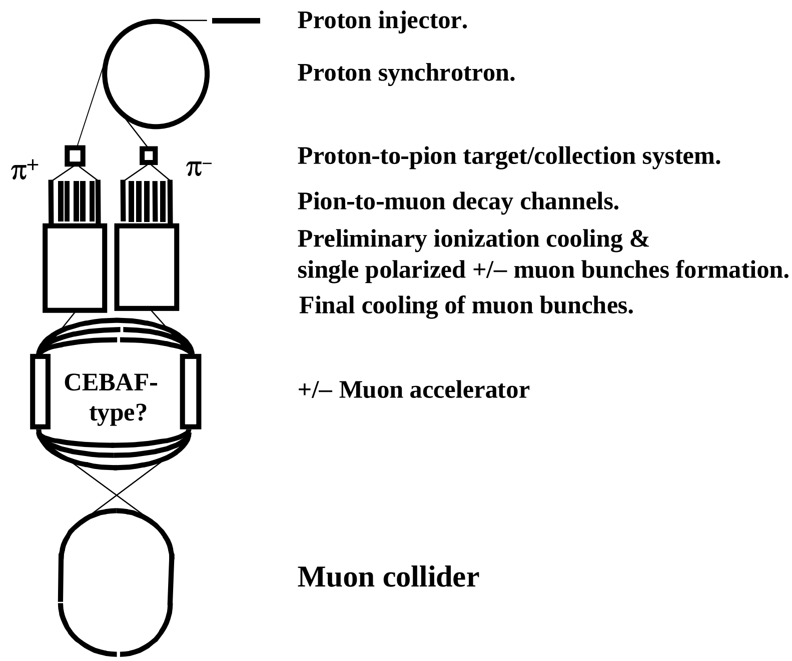


Figure 1. Muon collider complex – very schematic!

3. Ionization Cooling

But the crucial point is to compress a very large initial 6-emittance of muons to as small a volume as possible (and of course, with minimal intensity losses). In this paper, we use the definition of normalized emittances as

$$\varepsilon_{n\_x,y} = \theta^2_{x,y} \cdot \beta_{tran} \cdot \beta_\mu \gamma_\mu \ ,$$

$$\varepsilon_{nlong} = \Delta_E{}^2 \cdot \beta_{long} \cdot \beta_\mu \cdot \gamma_\mu \ ,$$

where $\Delta_E = \Delta E_\mu / E_\mu$.

And from the very beginning (in the 1960’s!) we realized that the only possibility for cooling muon beams is to develop and apply ionization cooling.

Earlier, a few authors[4] considered such cooling for compression of proton beams; but the conclusion was that the protons (as well as any strong interacting particles) would be lost because of nuclear interactions in the matter faster than their cooling process rate, so that under practical conditions ionization cooling is not useful.

This statement is even more correct for electrons (positrons) because of bremsstrahlung.

But just for muons − and for muons only! − ionization cooling is what we need: muons have normal ionization losses, but no strong interactions and negligible bremsstrahlung (below 1 GeV, where this cooling is of interest). And in our early reports (in 1969, 1970, 1971, 1980)[1–3,5] we always presented the ionization cooling as an essential part of the muon collider.

But it was only when we presented a reasonably complete theoretical consideration of ionization cooling as an inherent part of the muon collider (1981)[6] that the whole approach attracted interest, and now many groups in different labs throughout the world are actively developing different options for various stages of the muon collider. And even an International collaboration for a Muon Collider was established and is now operating.

Unfortunately, there is almost zero support from corresponding financial agencies — around the Globe!

Maybe, this is just a reflection (or a part) of the general crisis in basic science support.

The principal idea of ionization cooling is quite simple (at least, for the transverse part of 6-emittance): friction force due to ionization losses is directed opposite to the full velocity of the muons, but only the lost momentum parallel to the equilibrium orbit is restored by an external electric field (practically, some accelerating RF field).

As usually, *Life* is more complicated, than the *Idea*:

1. For high luminosity we need to cool all the six phase-space dimensions, including the longitudinal one; but in the longitudinal direction, "natural" ionization cooling is relatively very slow, or even negative (heating instead of cooling);
2. Besides the useful energy losses, muons experience multiple scattering on nuclei and electrons of the stopping media, as well as strong fluctuations of the ionization losses.

And, of course, the cooling time should not be very long – not to lose too many muons by decay.

Let us evaluate (in the first approximation) the ionization decrements and the equilibrium 6-emittance of a muon beam under ionization cooling.

For the case of cooling due to "full energy losses" of any origin, the increment of six-dimension density (or the sum of decrements for all three emittances) is equal to

$$\delta_{\Sigma 0} = \frac{2P_{fr}}{p_\mu v_\mu}\left(1 - \frac{p_{\mu long}}{2v_\mu} \cdot \frac{dv_\mu}{dp_{\mu long}}\right) + \frac{1}{v_\mu} \cdot \frac{dP_{fr}}{dp_{\mu long}},$$

(energy losses of the "equilibrium particle" are assumed to be compensated by an external source).

The power of ionization energy losses by a charged particle is

$$P_{fr} = 4\pi r_e^2 m_e c^3 N_e \cdot \frac{1}{\beta_\mu} \cdot \left\{ \ln\left[\frac{2m_e c^2 \beta_\mu^2}{I \cdot \left[1 + \frac{2}{\sqrt{1-\beta_\mu^2}} \cdot \frac{m_e}{M_\mu} + \left(\frac{m_e}{M_\mu}\right)^2\right] \cdot \left(1-\beta_\mu^2\right)} \right] \right\} - \beta_\mu^2,$$

(here I is the effective ionization potential of atoms involved in the collisions).

Consequently, the sum of decrements, expressed now in cm^{-1} of cooling matter, is (for small angles)

$$\delta_{\Sigma 0} = \frac{P_{fr}\sqrt{1-\beta_\mu^2}}{M_\mu c^3 \beta_\mu^3} \cdot \left(1+\beta_\mu^2\right) + \frac{\left(1-\beta_\mu^2\right)^{3/2}}{M_\mu c^3 \beta_\mu^2} \cdot \frac{dP_{fr}}{d\beta_\mu}.$$

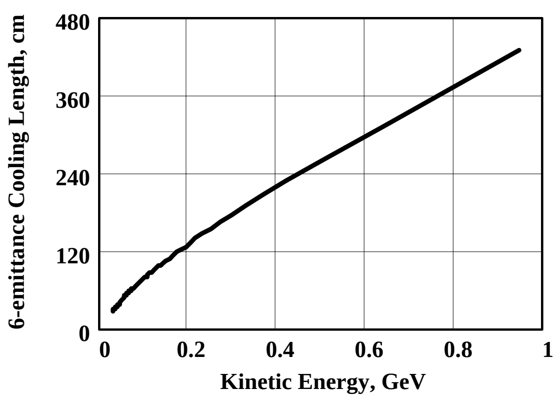


Figure 2. The "cooling length" for the 6-emittance (i.e., the product of all 3 emittances) in lithium (Li).

As seen in Fig. 2, to cool the 6-emittance *a million times*, say, at 200 MeV (kinetic energy), we need to travel through about 15 meters of lithium (with a "continuous" energy recovery). In this case, 10% of muons decay at 200 meters.

But – there are longitudinal problems! Figure 3 shows the "natural" longitudinal decrement as a function of the cooling energy.

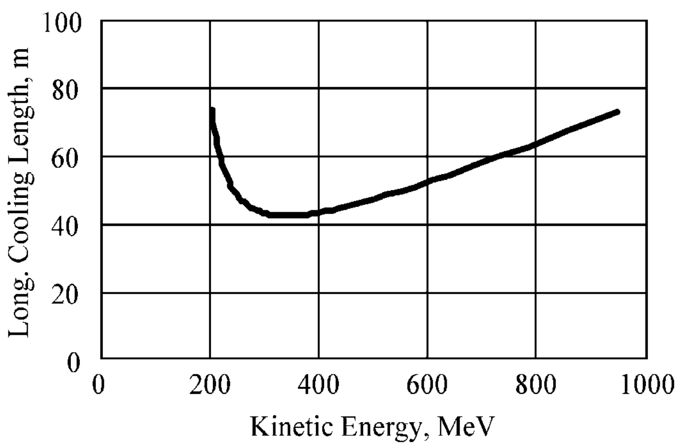


Figure 3. Inverse longitudinal decrement.

You can see that the "natural" longitudinal cooling is really too slow. And at lower energies the longitudinal de̲crement even converts into a fast in̲crement!

Hence, it is obligatory to effectively redistribute the sum of decrements in favour of the longitudinal degree of freedom (κ_{long} is a fraction of the full 6-emittance decrement transferred to the longitudinal degree of freedom) – this is a real challenge for inventors!

The transverse cooling asymptotically sqeezes the muon angular spread down to the equilibrium one, i.e., to the multiple scattering angle acquired at one transverse emittance cooling length:

$$\theta^2_{x,y\_eq} = 4\pi r_\mu^2 N_e (Z+1) L_C \cdot \frac{1}{\gamma_\mu^2 \beta_\mu^4} \cdot \left(\frac{1-\kappa_{long}}{2} \cdot \delta_{\Sigma 0} \right)^{-1} .$$

If we forget the velocity dependence of energy losses, and other complications, $\theta^2{}_{eq}$ is very transparent:

$$\theta_{eq}^2 = \frac{m_e}{M_\mu} \cdot (Z+1) \cdot \frac{3}{1-\kappa_{long}} \cdot \frac{\sqrt{1-\beta_\mu^2}}{\left(1+\beta_\mu^2\right)} .$$

It depends on *Z*, but not on the average electron density or the focusing (if uniform); its dependence on the cooling energy is presented in Fig. 4.

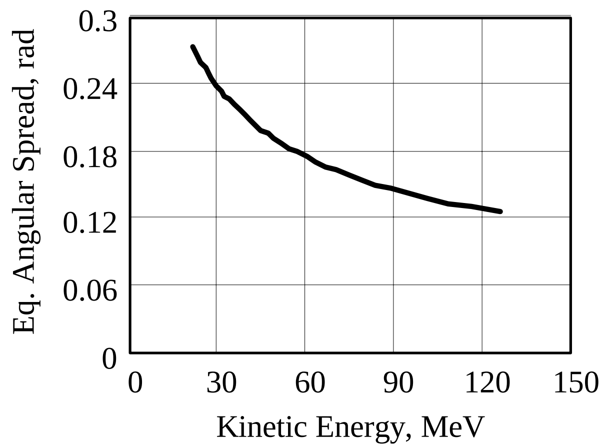


Figure 4. The equilibrium angular spread resulting from ionization cooling.

The corresponding relative energy spread at equilibrium (due to balance of fluctuations of the ionization losses and the longitudinal cooling) would be

$$\Delta_{E\_eq}^2 = \frac{2\pi r_\mu^2 N_e \left(2-\beta_\mu^2\right)}{\kappa_{long}\delta_{\Sigma 0}} ,$$

as shown in Fig. 5.

As you see, the equilibrium angles and energy spread at energies of interest – upon full cooling! – are not small at all! Hence, the usual paraxial and monochromatic beam optics should work poorly – a big additional headache!

And to reach small enough emittances at the final stage of cooling – and, hence, to reach an acceptable collider luminosity – at this stage we need to use very strong focusing in all three directions – as strong as practically possible!

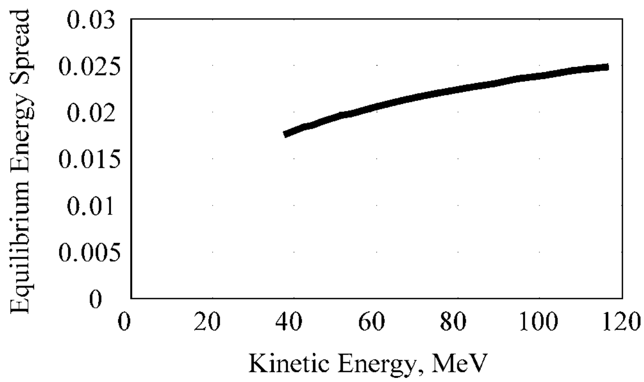


Figure 5. Equilibrium relative energy spread (κ_{long} is assumed at its optimum, around 0.25).

For these final stages the best option for ionization cooling up to now (in my personal opinion, of course) seems to be the use of lithium rods with a strong current along the rod for as strong transverse focusing as possible (the scheme is presented in Fig. 6).

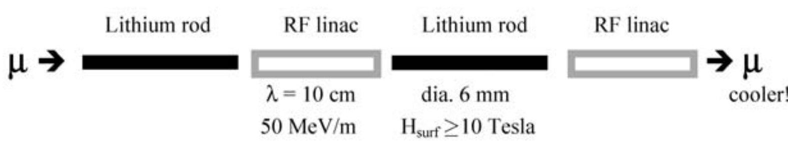


Figure 6. The scheme of the final stages of transverse cooling.

To arrange the strongest possible transverse focusing, my preference is to use current-carrying liquid-lithium rods, which focus muon beams by means of the azimuthal magnetic field gradient, limited by magnetic field on the surface (10 Tesla or somewhat higher – pulsed operation mode):

$$\beta_{li} = \sqrt{\frac{p_\mu c \cdot a_{li}}{eH_{\max}}}\ .$$

For the parameters under discussion, $\beta_{li} \to 1$ cm – quite good!

The whole device is just a very long lithium lens (in total), developed at Novosibirsk for positron and antiproton collection decades ago, and still in use now (INP, CERN, FNAL). The improved – and the first liquid – version is now under developement at Novosibirsk (Fig. 7).

Figure 7. Liquid lithium rod – operating pre-prototype for final cooling .

The radius of the rod and surface field at the final stage should provide an acceptance, say, 2-3 times as large as the final muon emittance; for a final lithium rod diameter of about 6 mm, and 10 Tesla on the surface in this case, the resulting transverse emittance is presented in Fig. 8. Since high repetition rates are necessary, we need to use liquid lithium to remove the Ohmic heat.

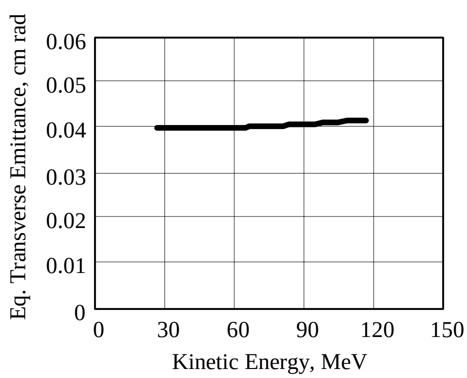


Figure 8. Normalized transverse emittance after final cooling.

But in order to reach the highest possible luminosity for any (affordable) number of muons (as we will see a bit later), we need the equilibrium normalized 6-emittance upon final cooling to be as small as possible:

$$\varepsilon_{neq\_6} = \varepsilon^2_{neq\_tran}\varepsilon_{neq\_long} = \theta^4_{eq} \cdot \Delta^2_{Eeq} \cdot \beta^2_{tran\_loc} \cdot \beta_{long} \cdot \beta^3_{\mu cool}\gamma^3_{\mu cool} \ .$$

We see here again, that β^2_{tran} and β_{long} in the cooling matter should be as small as possible. The equilibrium emittance upon final cooling being limited, we need to cool all degrees of freedom; hence a reasonable fraction κ_{long} of the sum of decrements should be redistributed to the longitudinal degree of freedom.

It is still not clear, which option for the final cooling arrangements would give the smallest normalized 6-emittance at equilibrium. At the moment I like the following "helical option," see Figs. 9 and 10.

The magnetic field H_{helix} and the radius should correspond to each other in a natural way:

$$R_{helix} = \frac{p_\mu c}{eH_{helix}} .$$

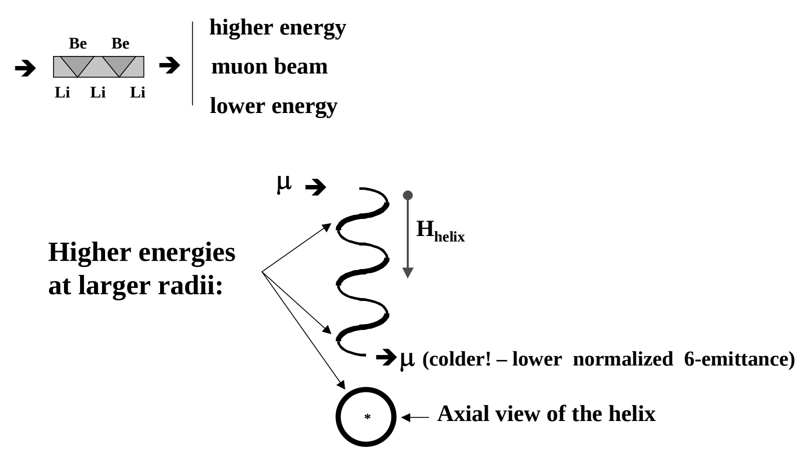


Figure 9. Schematic view of one section for simultaneous transverse and longitudinal cooling (helical option).

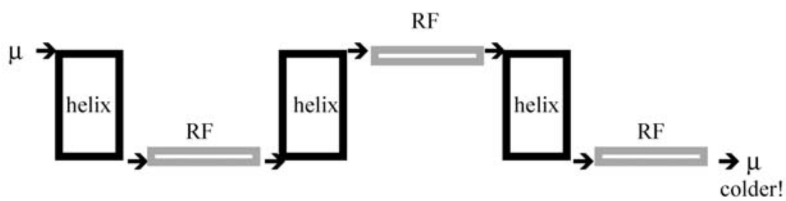


Figure 10. The general scheme of the final steps of cooling. (The momentum dispersion should "extract" decrement from both transversal degrees – serially!)

In this case, the muons with mean momentum p_μ would move along the center of the rod and all the acceptance of the rod would be used efficiently. But of course, the helix curvature should be high enough to produce a momentum dispersion that transfers, via berillium "teeth" (located at the outer part of the helix), a large enough fraction of one of the transverse decrements into the longitudinal one. Practically speaking, for an average muon kinetic energy around 150 MeV, to transfer a longitudinal decrement $\delta_{long} = \kappa_{long} \cdot \delta_{\Sigma 0}$ (with averaging over several steps of that kind), for $\kappa_{long} = 0.25$, we need to apply H_{helix}= 7 Tesla, providing that R_{helix}= 10 cm.

To get the longitudinal emittance (and, consequently, the 6-emittance) minimal at the final stage of ionization cooling, we need at this stage an RF system operating at the shortest possible wavelength (10 cm?) and a high accelerating gradient (30 MeV/m on the average?), thus providing the smallest effective β_{long} (5 cm?). In this case, it is possible to reach the emittances shown in Fig. 11.

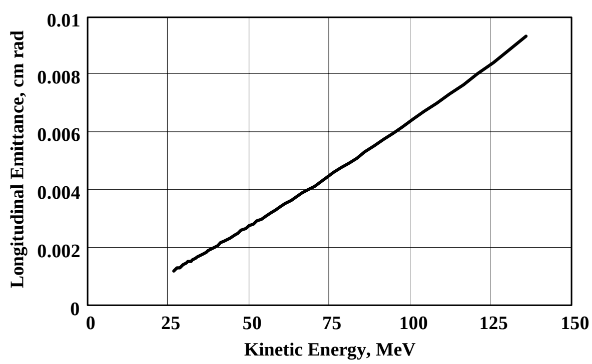


Figure 11. Equilibrium longitudinal normalized emittance.

As a result, the final cooling would provide the 6-emittance shown in Fig. 12.

We need the smallest final normalized beam 6-emittance. For such a high angular and energy spread (see above), to progress in this direction we need to find something unconventional.

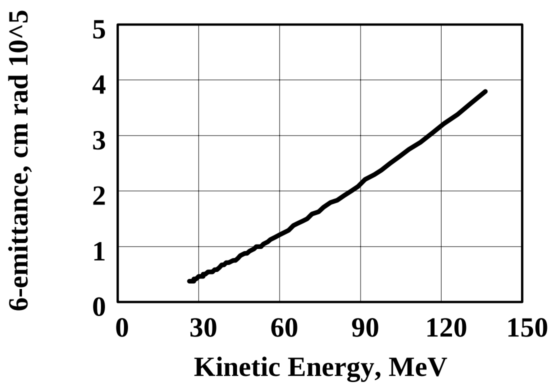


Figure 12. Equilibrium normalized 6-emittance.

4. Matching Problems

One of the most difficult problems is proper matching of focusing in the sequential moderation/acceleration sections (from the exit of one moderator section to the entrance of the next one). High momentum spreads can lead to unacceptably high chromatic and non-linear aberrations (resulting in modest beta-functions!). The focal lengths of most familiar individual lenses – short solenoids or quadruple doublets – are proportional to the square of the momentum of the particles they are focusing. But the focal lengths of lithium and plasma lenses are just proportional to the momentum; hence, their use should make it possible to reach low enough chromatic aberrations in matching sections more easily.

The following option that seemed promising to me is shown in Fig. 13. In the table below the drawing, p is the current muon momentum; E_{kin} is the corresponding kinetic energy; β_{tran} is the current beta-function; L_{frac} is the length of section fractions; H_{max} is the magnetic field on the surface of the focusing element; R_{curr} is the radius of the focusing element; I_{foc} is the peak current in the element (all the numbers are rough and need careful optimization).

Figure 14 illustrates the chromatic aberration of similarly structured matching sections using plasma lenses and solenoidal lenses. The beta-function at the first 4 cm in the second Li-Be helix for a nominal muon momentum (1.00) and ± 10% deviation for the cases of plasma lenses (left) and solenoidal lenses (right), for the same corresponding focal lengths with optimization.

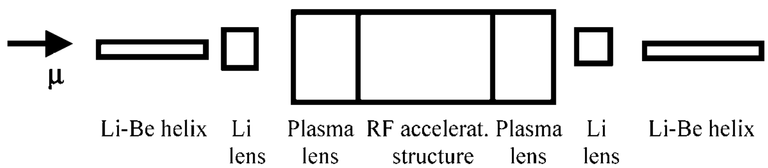


Figure 13. Schematics of a matching section between two consecutive cooling sections (geometrically not to scale!).

p, MeV/c	70					70 141 200					200 70
E_{kin} , MeV	21					21 71 121					121 21
β_{tran} , cm	0.7		3			95 84 190			3		1.1 0.7
L_{frac}, cm	85	1	1	8	20	125	35	1	2	1	85
H_{max} , T	20	0	20	0	0.3	0	0.5	0	20	0	20
R_{curr} , cm	0.4		1		4.5	7	7		1		0.4
I_{foc}, MA	0.4		1		0.1		0.2		1		0.4

The short and strong lithium lenses at the exit and at the entrance of Li-Be helices are necessary to make the beta-function a few times larger than in the helices and to ease low-aberration functioning of plasma lenses with longer focal lengths. But using them (instead of plasma lenses) at much higher beta-values inside accelerating structures is impossible – the multiple scattering results in unacceptable emittance growth.

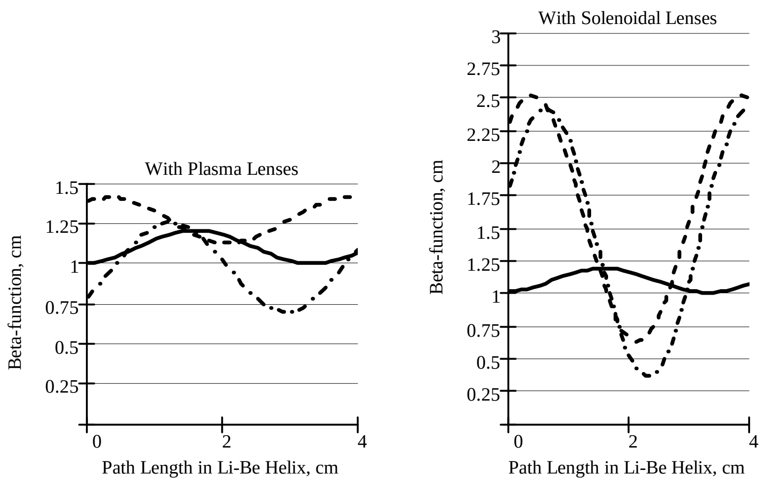


Figure 14. Resulting chromatic aberrations: solid line for nominal momentum, dashed line for +10% , dash-dotted line for –10% deviation.

In Figure 15 an option for the same kind of end matching section (for the exit of the cooling system) is presented. As seen from Fig. 16, the resulting chromatic aberrations are acceptably small – quite comfortable for the acceleration and emittance gymnastics further needed.

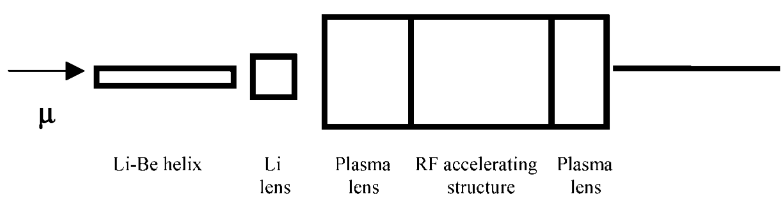


Figure 15. The matching structure of the last cooling/accelerating section (not to scale geometrically!). All the notations are the same as in Fig. 12.

pc, MeV/c	70					70 141 200		200
E_{kin}, MeV	21					21 71 121		
β_{tran}, cm	0.7		3.3			95 84 190		190 90
L_{sect}, cm	85	1	0.7	2	20	125	3.4	100
H_{max}, Ts	20	0	20	0	0.3	0	0.5	
R_{curr}, cm	0.4		0.9		4.5	7	7	
I_{foc}, MA	0.4		0.9		72		0.2	

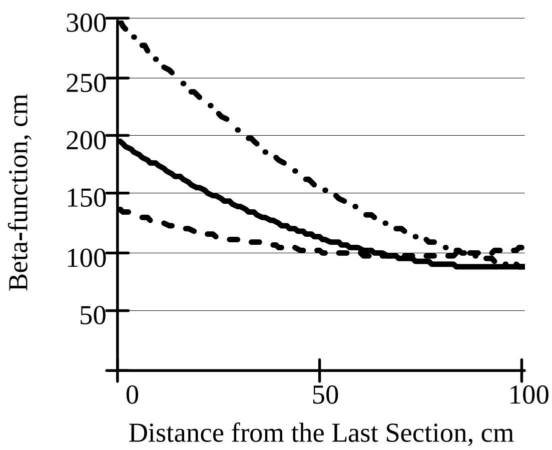


Figure 16. The beta-function upon the exit of the last cooling/accelerating section (the notation is the same as in Fig. 13).

5. Collider Luminosity

If at the cooling stage the normalized transverse emittance $\varepsilon_{neqtran}$ and the longitudinal emittance $\varepsilon_{neqlong}$ were reached and were kept constant at all the stages including the collision, the bunch length at collision is limited by $\varepsilon_{neqlong}$ and by the maximal acceptable energy spread ΔE_{max}; the transverse beta-functions are made equal to the bunch length, and the collider magnetic field

H_{coll} provides for muons N_{lumi} effective turns prior to the luminosity e-fold reduction due to muon decay, the ultimate luminosity would be

$$L_{\mu\mu\,\max\max} = \frac{N_\mu^2}{4\pi} \cdot \frac{\gamma_{coll}^2}{\varepsilon_{neqtran} \cdot \varepsilon_{neqlong}} \cdot \frac{\Delta E_{\max\_coll}}{E} \cdot N_{lumi} \cdot f_0 \ .$$

This is seen from the following obvious chain of equations:

$$L_{\mu\mu\_\max\max} = \frac{N_\mu^2}{4\pi\sigma_{tran\_coll}^2} \cdot N_{lumi}\, f_0 = \frac{N_\mu^2}{4\pi} \cdot \frac{\beta_{tran\_coll}}{\sigma_{tran\_coll}^2 \beta_{tran\_coll}} \cdot N_{lumi}\, f_0$$

$$= \frac{N_\mu^2}{4\pi} \cdot \frac{1}{\varepsilon_{tran\_coll}\sigma_{long\_coll}} \cdot N_{lumi}\, f_0 = \frac{N_\mu^2}{4\pi} \cdot \frac{\gamma_{coll}^2}{\varepsilon_{neqtran}\varepsilon_{neqlong}} \cdot \frac{\Delta E_{\max coll}}{E} \cdot N_{lumi}\, f_0 \,,$$

(we always assume that $\beta_{tran\_coll} = \sigma_{long\_coll}$). The luminosity is shown in Fig. 17.

And for convenient cooling energies the luminosity would reach

$$L_{\mu\mu\ \max\max} \sim 0.5 \cdot 10^{37}\ \text{cm}^{-2}\text{sec}^{-1} \ .$$

But if we calculate the beta-value at collision, assumed (as always here) to be equal to the muon bunch length, we would get 5 microns(!) – impractically short.

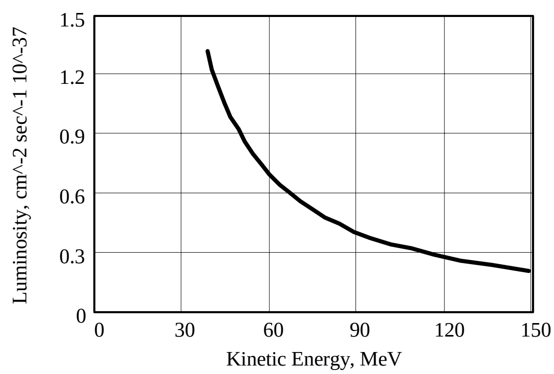


Figure 17. The luminosity of a "super-maximal" collider (E_μ= 2 TeV + 2 TeV, N_μ=1·10^{12}, H_{coll}=10 T, f_0=15 s^{-1}, with a fraction of the sum of cooling decrements transferred to the longitudinal direction κ_{long}= 0.25), with the equilibrium emittances reachable as the ultimate limit in the cooling process (see above), as a function of the muon kinetic energy at the cooling stage

Hence, we need to use a different limitation. If we limit additionally the bunch length $\sigma_{longcoll}$, the following formula for "practical" maximum luminosity should be valid:

$$L_{\mu\mu\max} = \frac{N_\mu^2}{4\pi} \cdot \frac{\gamma_{coll}^{3/2}}{\varepsilon_{neq6}^{1/2}\sigma_{longcoll}^{1/2}} \cdot \left(\frac{\Delta E_{\max}}{E}\right)^{1/2} N_{life}\, f_0 \,.$$

It is obtained from another chain of equations:

$$L_{\mu\mu\_\max} = \frac{N_{\mu\mu}^2}{4\pi} \cdot \frac{1}{\sigma_{tran\_coll}^2} \cdot N_{lumi} f_0 = \frac{N_{\mu\mu}^2}{4\pi} \cdot \frac{1}{\varepsilon_{tran\_coll} \cdot \varepsilon_{long\_coll}^{1/2} \cdot \sigma_{long\_coll}^{1/2}} \cdot \left(\frac{\Delta E_{\max\_coll}}{E} \right)^{1/2} \cdot N_{lumi} f_0$$

$$= \frac{N_{\mu\mu}^2}{4\pi} \cdot \frac{\gamma_{coll}^{3/2}}{\varepsilon_{neq6}^{1/2} \cdot \sigma_{long\_coll}^{1/2}} \cdot \left(\frac{\Delta E_{\max\_coll}}{E} \right)^{1/2} \cdot N_{lumi} f_0 .$$

With these conditions, the luminosity graph is shown in Fig. 18.

In such a case, $L_{\mu\mu\ \max} \sim 0.5 \cdot 10^{35}$ cm^{-2}sec^{-1} for the same parameters as above, and $\sigma_{longcoll}$ = 3 mm – also not bad!

As we see, "in more practical circumstances" the equilibrium normalized 6-emittance ε_{neq6} enters the maximum luminosity directly. And the goal of final ionization cooling really is to make it minimal. But not only.

For the finally achieved 6-emittance we need to control the partition of the transverse and longitudinal emittances – and optimize this partition together with the muon collider optics, keeping in mind the monochromaticity and polarization requirements, etc., – hence a "deep emittance gymnastics" is necessary.

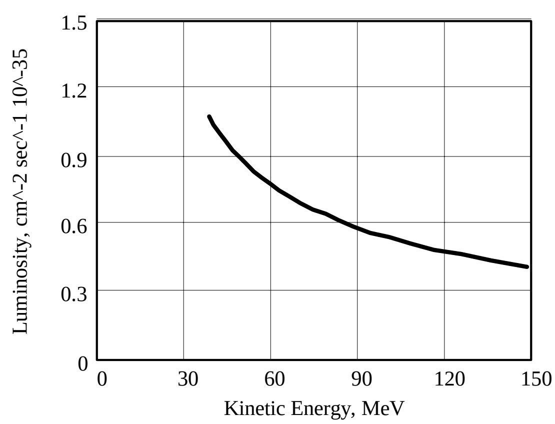


Figure 18. The luminosity of a "maximal" collider (E_μ= 2 TeV + 2 TeV, N_μ=1·10^{12}, H_{coll}=10 T, f_0=15 s^{-1}, fraction of the sum of cooling decrements transferred to the longitudinal direction κ_{long}= 0.25, σ_{long}= 3 mm) with the equilibrium emittances reachable as the ultimate limit in the cooling process (see above), as a function of cooling kinetic energy.

For the purpose of emittances gymnastics, we can use a combination of:

dispersive elements,

septum elements,

RF accelerating/decelerating structures,

delay lines,

(but not ionization components, which damage the 6-density by scattering!).
Such a transformation should be arranged at some convenient energy of the muon beam.

An option is presented in Fig. 19.

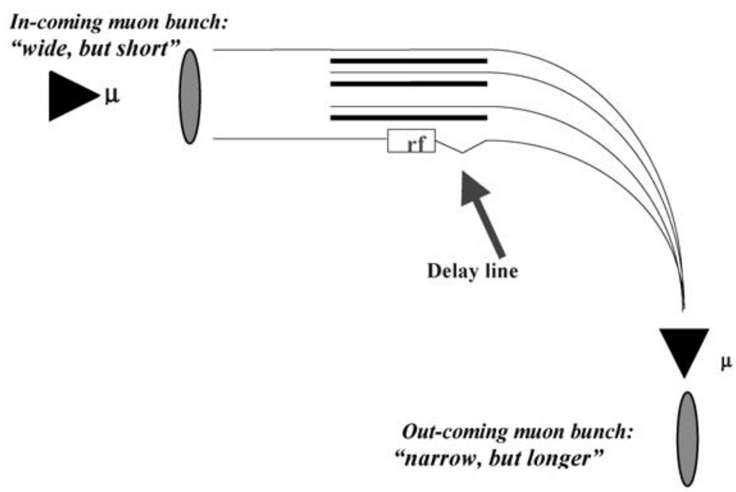


Figure 19. An example of "bunch gymnastics" needed to maximize the luminosity at a very high energy.

The "monochromatic" collider option (as in the case of the "low energy Higgs Factory") could require muon bunch rearrangement in the opposite direction.

Table 1 presents data for a collection of "ultimate colliders" for $5 \cdot 10^{12}$ muons per bunch (luminosity per detector). Here ζ_{eff} is the factor indicating the reduction of the initial polarization due to the energy spread in the collider. The first row presents parameters for the so-called Higgs Factory, which would be of interest at low mass, hence for a very narrow Higgs boson. The last row presents parameters of an "ultimate muon collider" currently conceivable.

Similar options were considered in a Muon Collaboration Report under a somewhat different approach; see Table 2 for baseline parameters of high- and low-energy muon colliders. Higgs/year assumes a cross section $\sigma = 5\times10^4$ fb; a Higgs width $\Gamma = 2.7$ MeV; 1 year = 10^7 s.

Table 1: Collider options for 5×10^{12} muons per bunch (luminosity per detector).

$2E_{coll}$	**N_μ** 10^{12}	**R_{coll} m**	$\frac{\Delta E_{max}}{E}$	**ν_r**	**V_{0coll} MV**	**ζ_{eff}**	**$L_{\mu\mu}$ cm $^{-2}$ s $^{-1}$**
100 GeV	5	25	$3*10^{-5}$	20	0.1	1	$7*10^{32}$
1 TeV	5	180	$3*10^{-4}$	30	100	0.8	$2*10^{35}$
4 TeV	5	720	$2.5*10^{-4}$	30	1000	0.8	$7*10^{35}$
10 TeV	5	1800	$1*10^{-4}$	30	1000	0.8	$1.5*10^{36}$
10 TeV	**5**	**1800**	**$1*10^{-3}$**	**300**	**1000**	**0**	**$6*10^{36}$**

Table 2. Baseline parameters for high- and low-energy muon colliders.

CoM energy (TeV)	3	0.4	0.1		
p energy (GeV)	16	16	16		
p's / bunch	2.5×10^{13}	2.5×10^{13}	5×10^{13}		
Bunches/fill	4	4	2		
Rep. rate (Hz)	15	15	15		
p power (MW)	4	4	4		
μ / bunch	2×10^{12}	2×10^{12}	4×10^{12}		
μ power (MW)	28	4	4		
Wall power (MW)	204	120	81		
Collider circum. (m)	6000	1000	350		
Ave. bending field (T)	5.2	4.7	3		
Rms $\Delta p/p$ %	0.16	0.14	0.12	0.01	0.003
6-D $\varepsilon_{6,N}$ $(\pi m)^3$	1.7×10^{-10}	1.7×10^{-10}	1.7×10^{-10}	1.7×10^{-10}	1.7×10^{-10}
Rms ε_n (π mm-mrad)	50	50	85	195	290
β^* (cm)	0.3	2.6	4.1	9.4	14.1
σ_z (cm)	0.3	2.6	4.1	9.4	14.1
σ_r spot (μm)	3.2	26	86	196	294
σ_θ IP (mrad)	1.1	1.0	2.1	2.1	2.1
Tune shift	0.044	0.044	0.051	0.022	0.015
n_{turns} (effective)	785	700	450	450	450
Luminosity $cm^{-2}s^{-1}$	7×10^{34}	10^{33}	1.2×10^{32}	2.2×10^{31}	10^{31}
Higgs / year			1.9×10^3	4×10^3	3.9×10^3

6. Polarized Muons

A high degree of polarization is very important for extracting full physics information from muon collider experiments. Hence, first of all, it is worthwhile to find a way to produce highly polarized intense muon beams.[5,8] We assume that positive and negative pions generated by different proton bunches can be accumulated.

A sketch of a possible option for a protons-to-pions multi-channel conversion system, followed by multi-channel pion-to-muon decay channels, is presented in Fig. 20. It might be reasonable to arrange a sectioned target (using additional channels). This could be especially useful at high proton energy – around 100 GeV.

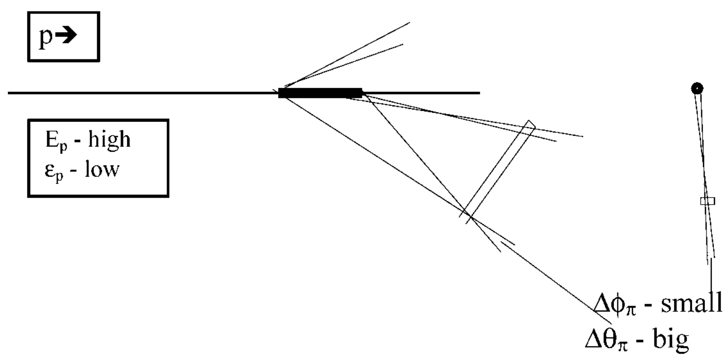


Figure 20. Schematic of a multi-channel proton-to-pion conversion system.

In each pion-collecting straight channel, using one-dimensional "thin surface current-carrying lenses" in doublets for the initial matching of focusing, it is necessary to direct pions of a wide spectrum into many independent channels. In each channel, in the θ-direction the beam transversal emittance is large, but in the ϕ-direction it is quite small. These beams can easily be transported away from the target area, and the following channel gymnastics will be performed in reasonably free space.

The next step is to arrange the energy dispersion in this smaller emittance direction in each channel, and then to direct each of the ± 5% momentum spread pion beams into additional separate strong-focusing decay channels.

Such narrow momentum spread pion beams (i.e., with a very small emittance in one direction), upon passing about 2 decay lengths (proportional to the pion energy in each channel, around $15\beta_\pi\gamma_\mu$ meters), generate muon beams of momentum spread about ± 30% (see Fig. 21), with a strong correlation of the muon spin direction and its momentum.

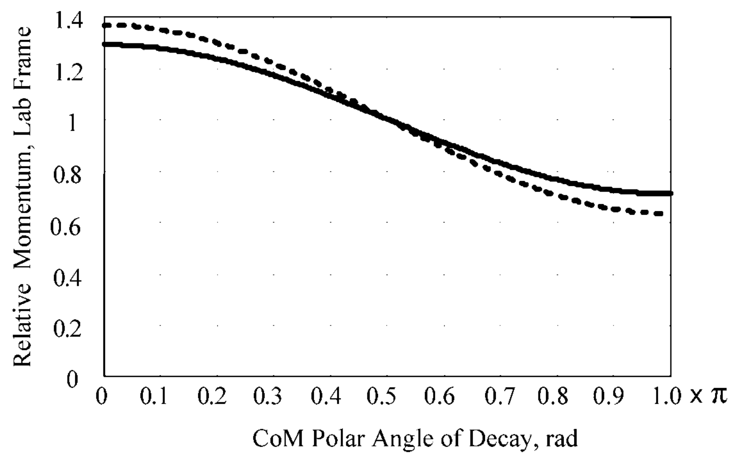


Figure 21. Relative muon momentum in the lab system vs the polar angle of decay θ in the CoM system, for pion kinetic energies of 300 MeV (solid line) and 60 MeV (dotted line).

Consequently, for every particular muon beam, we cut away the middle 30% of the muon spectrum and direct the upper and the lower parts with opposite helicities into two separate sub-channels. At the next phase, we shift the energy in each muon channel by RF acceleration/deceleration to the energy optimal for ionization cooling (below 100 MeV kinetic energy). And then, upon preliminary cooling, we combine all the muons from "upper sub-channels" into one longitudinally-polarized bunch, and all the muons from "lower sub-channels" into another bunch of opposite helicity, each with a 70% degree of polarization. This procedure, if it appears useful, could be arranged in a few stages. Then, all 4 bunches (μ^+ and μ^-) will be cooled down to the lowest 6-emittance.

Afterwards, we can reverse the helicity of the "lower" bunches at a later stage upon acceleration up to 45 GeV (by applying an additional non-accelerating full turn) and then combine the two bunches into one (one μ^+ bunch and one μ^- bunch) with a 6-emittance twice as high as that reachable at the final cooling.

The helicity reversal of muons happens because of their anomalous magnetic moment. Positive relative spin-to-velocity rotation is very slow at the low energy (e.g., at the cooling stage), thus not damaging the initial muon beam's degree of polarization; but it becomes faster proportionally to the muon energy, and at 45 GeV each full turn of the muon trajectory results in reversal of the muon helicity. Let us keep in mind that all the muon spin motion proceeds in the median plane of the collider.

Helicities of colliding bunches are modulated at relative frequency ν_{spin},

$$\nu_{spin} = \frac{\mu_{anom}}{\mu_0}\gamma_\mu = \frac{E[\mathrm{Gev}]}{90}.$$

Because of this modulation, at integer spin resonances the helicity always remains the same in the collision process. At half-integer resonances the helicity reverses at consequent turns. At intermediate energies, the modulation of spin-at-collision proceeds with the non-integer fraction of ν_{spin}.

The relative helicities of muon bunches at the interaction region (from ++/–– to +–/–+) can be controlled by choosing a proper injection path (e.g., by an additional non-accelerating turn of one beam at, say, 45 GeV).

At high energy, when $\nu_{spin} >> 1$, a non-complete coherence of the spin rotation becomes important, and this effect can lead to the loss of polarization degree due to beam energy spread. The loss becomes significant if the spin frequency difference in the beam reverses the relative spin orientation at a half of the synchrotron oscillation period. The effective polarization degree loss factor ζ_{eff} (compared to the initial degree) can be expressed as

$$\zeta_{eff} = 1 - \frac{1}{2}\left(2\pi\frac{\nu_{spin}}{\nu_{synch}}\Delta_{Ecoll}\right)^2,$$

where Δ_{Ecoll} is the muon beam energy spread in the collider, ν_{synch} is the relative synchrotron frequency (this evaluation is meaningful if ζ_{eff} is not very far from 1; otherwise the polarization degree goes to zero).This ζ_{eff} was used in Table 2 of collider options.

7. Background

We talked about a cleaner interaction of “point-like” particles in the case of muon-muon collisions (at very high energies) – compared to hadron-hadron and electron-positron (in linear collider) collisions. That is correct.

But all the muons decay inside a collider: every muon produces an electron (positron) with an energy of 1/3 of the muon energy, on the average. High energy electrons appear in a very high field (above 10 Tesla). They hit the inner wall of the vacuum chamber – and produce showers.

While passing the high magnetic field (prior to hitting the inner wall) they produce many high energy photons of the synchrotron radiation. These hit the outer wall of the vacuum chamber and again produce showers.

In usual colliders and detectors, they give an additional heavy heat load to the cryogenics, produce additionally a lot of neutrons and radioactivity, and “provide” detector(s) with a strong background and radiation load.

But, in principle, there is a very attractive solution (see Fig. 22) – to switch from the normal collider optics with vertical and horizontal focusing quads to skew-quads (Option I) (again alternating, as always in strong focusing) – no harm for collider operation. Plus, to remove superconducting dipole coils from the median plane.

The other option (Option II in Fig. 22) to solve the same problem is to use combined-function strong-focusing dipoles. The choice of option should be the subject of more careful studies.

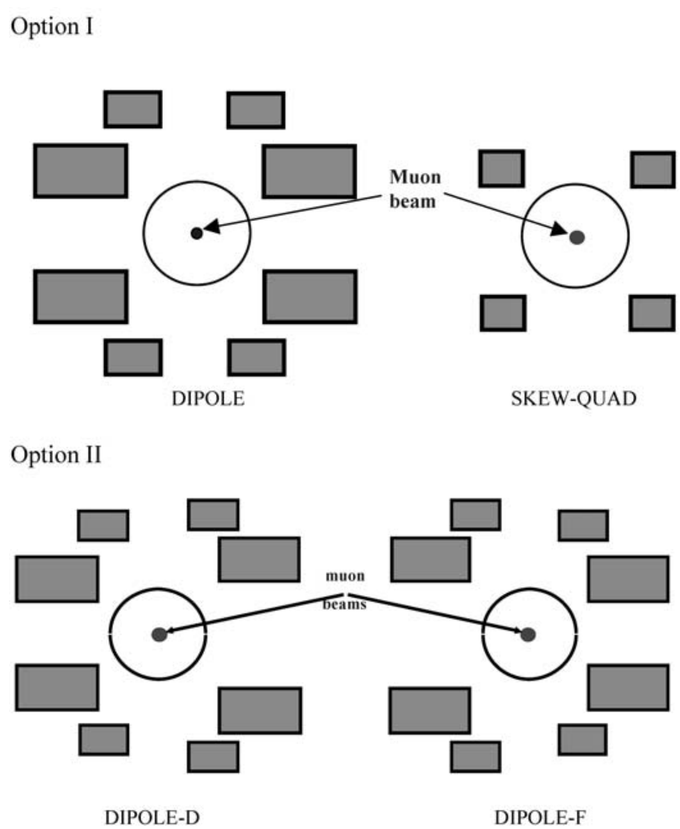


Figure 22. Schematic sketch of superconducting magnet coils for an “open median plane” collider.

In a muon collider with an open median plane all the decay electrons go to the center and all the SR photons go off the center of the collider.

Of course, the Interaction Region(s) and Detector(s) should be designed with the same idea – to provide that all "decay background" particles could miss sensitive components of the detector.

8. Examples of Projects

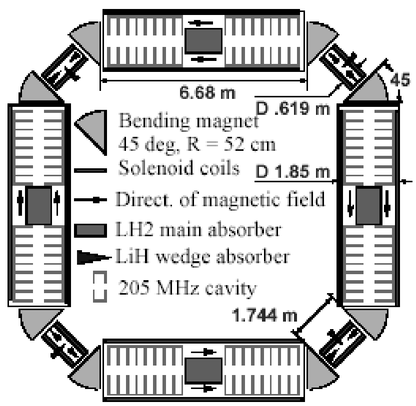
There are many publications presenting different views on the preparation of muon beams, cooling and acceleration, and several pre-projects of Neutrino Factories and Muon Colliders. I will not try to analyze all these options – the figure legends below make them clear.

8.1. Ring Options of Ionization Coolers

From the very beginning we in Novosibirsk considered[2,3] as a "natural option" ring coolers with short ionization regions located in low beta-function regions (similar to collision regions in colliders) – to minimize the influence of multiple scattering. Since that time, we have shifted to a different approach (as presented above), at least for final cooling.

But ring cooler options have now become popular. Two options (by V. Balbekov and of R. Palmer) are presented in Fig. 23 and Fig. 24; the structures of these coolers are easy to understand from their legends.

One of the most difficult problems is injection of beams with a high 6-emittance in the ring. A possible option is to use a helical-type cooler at the very initial cooling stage, when the whole aperture is accessible for passing the muon beam, and to use the ring part at a later stage.



Circumference	36.963 m
Nominal energy	
at short SS	250 MeV
Bending field	1.453 T
Norm. field gradient	0.5
Max. solenoid field	5.155 T
RF frequency	205.69 MHz
Accelerating gradient	15 MeV/m
Main absorber length	128 cm
LiH wedge absorber	14 cm
Grad. of energy loss	0.75 MeV/cm

Figure 23. Dipole ring (V. Balbekov).

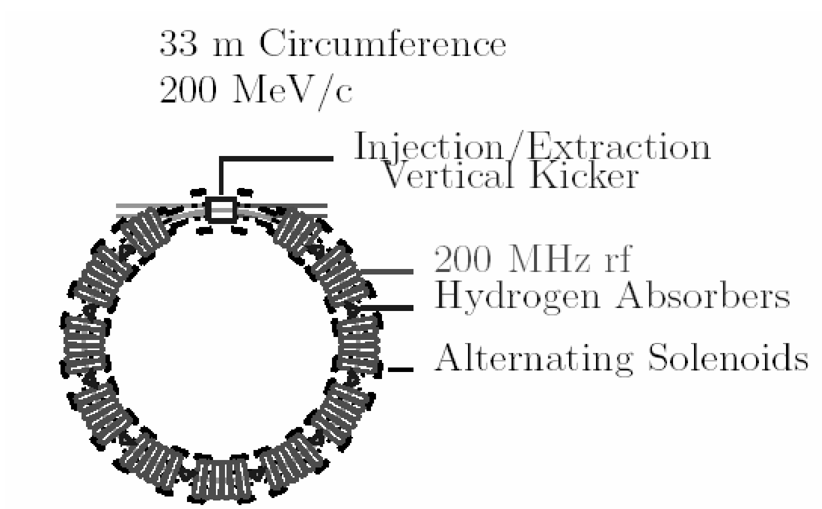


Figure 24. Bent-solenoid ring (R. Palmer).

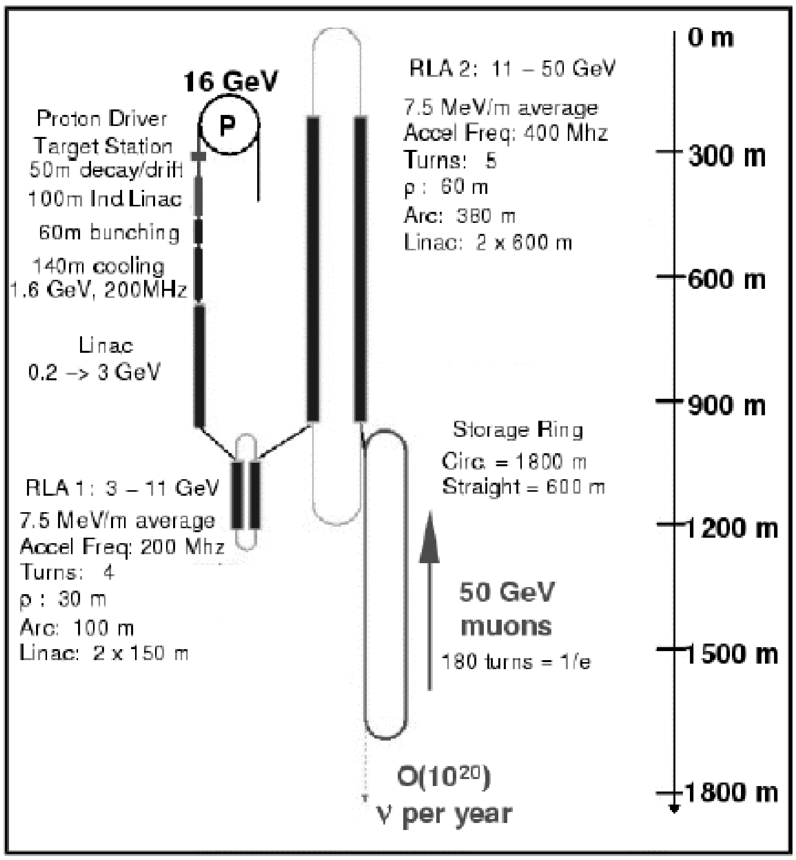


Figure 25. The Neutrino Factory. A muon-based neutrino factory is another option for the field (USA based collaboration).

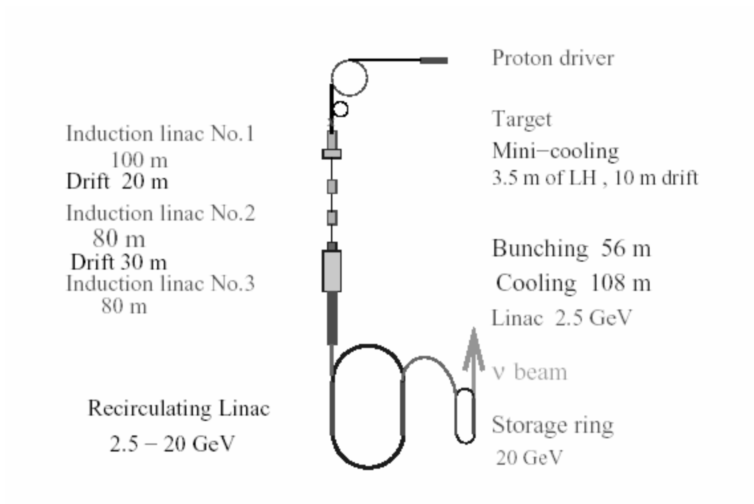


Figure 26. Schematic of the Neutrino Factory, version Study-II.

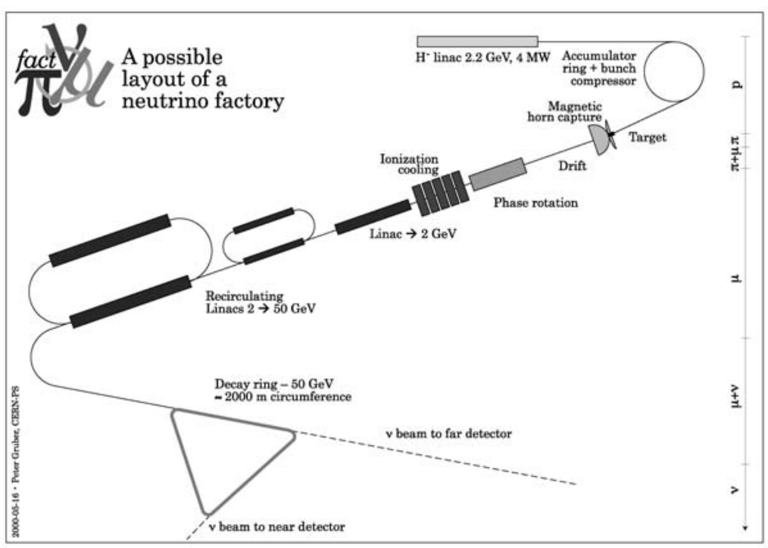


Figure 27. Schematic layout of the CERN scenario for a Neutrino Factory.

8.3. Muon Collider Complex – Options

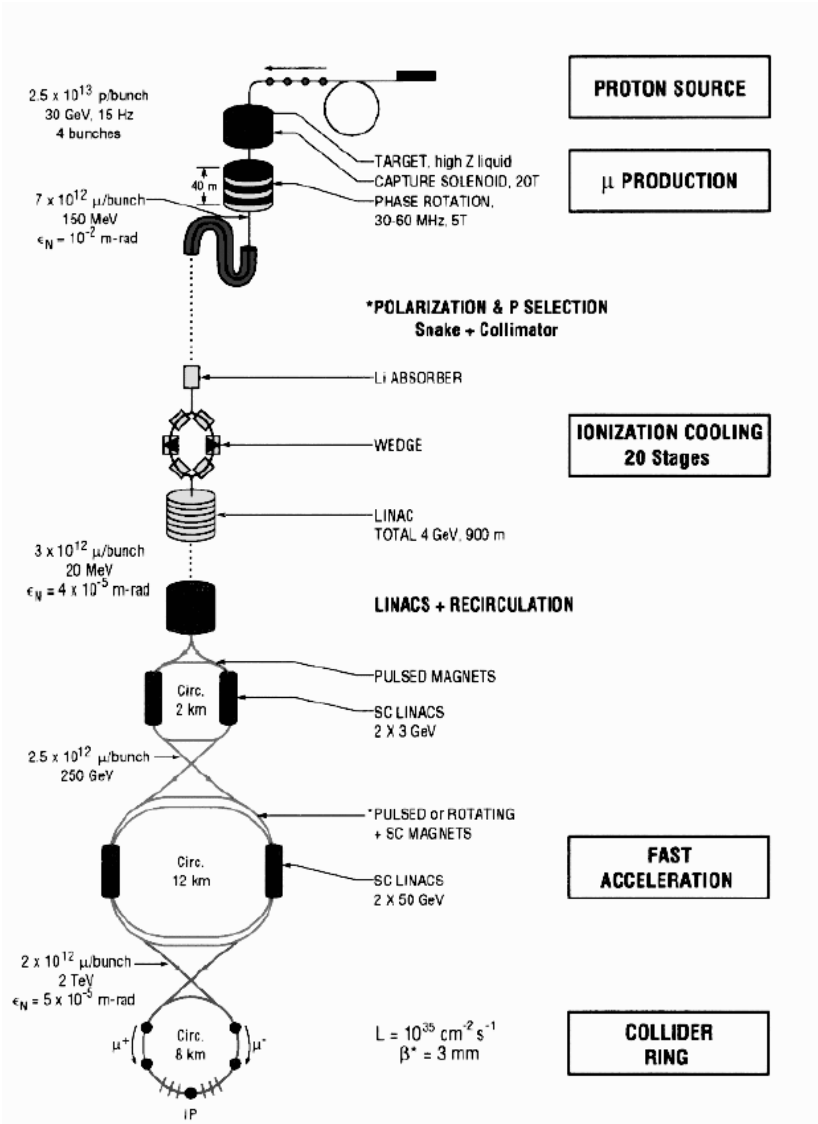


Figure 28. Muon Collider – an option from the Muon Collaboration Review.

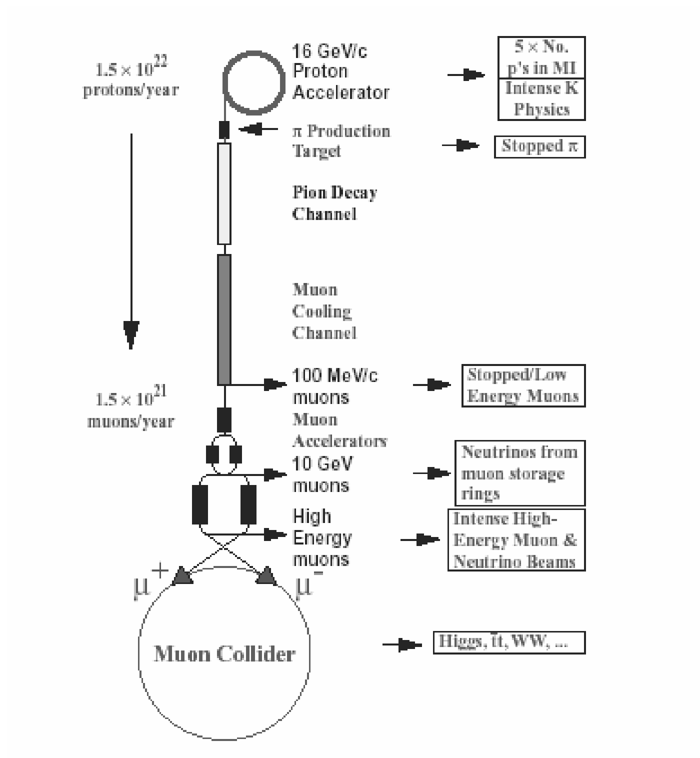


Figure 29. Muon Collider – another option from the Muon Collaboration Review.

9. Some Historical Remarks

As I have mentioned before, many of the topics hot nowadays were under active discussion at Novosibirsk many years ago – starting from the 1960's.[1,2,3] As an example, here is an extract from my talk at the 1971 International Seminar on High Energy Physics Prospects at Morges – the pre-ICFA meeting after the 1971 Accelerator Conference at CERN. My talk was, as others, quite informal, but Professor Yves Goldschmidt-Clermont (CERN) immediately forced me to convert it into printed form at CERN. Here are the muon-related extracts:

CERN/D.Ph.II/YGC/mmg
21.9.1971

Morges Seminar 1971

Intersecting Storage Rings at Novosibirsk

A.N. Skrinsky

....

....

....

$\mu^+ \mu^-$ possibilities

These experiments at hundreds GeV energy region will be available, only when several very difficult things will be discovered (developed):

1. To have a very large number of protons with tens GeV energy in rather short bunches. It is necessary to have about 10^{14} or even 10^{15} protons in about 10 sec in several meters long bunch. It is interesting, that the muon accelerator will be at the same time a very high intensity generator for all types of neutrinos up to the maximum accelerator energy. About 1/4 of all the accelerated muons may go into useful neutrinos. The neutrino beam shall have a diameter of about 10 cm behind the complete shielding.

To produce with maximum efficiency muons with 1 GeV or less energy, using nuclear cascade, strong focusing in the target and in decay channel. It seems possible to have 0.1 or even more useful muon per proton.

3. To cool muons in special hundred-kilogauss pulsed storage ring, using ionization energy losses. If the targets are in places with very small η-function, the final emittance of muon beam should be small enough to be injected into the main muon accelerator with small aperture and to be well compressed in interaction points.

To accelerate muons rapidly in some accelerators. If the muons are accelerated to their rest energy in a time, several times less than their life time at rest, most of the muons will be accelerated up to the required energy. It is possible to use a linear accelerator, or to use a synchrotron with more than a hundred kilogauss and magnetic field with a short rise time. In the last case, the accelerator will be at the same time the colliding beams ring. In the ring with such a magnetic field it is possible to have several thousands of useful turns of muon beams.

If all of these conditions are satisfied, it seems to be possible to have an average luminosity 10^{31} cm^{-2} sec^{-1} and may be a bit more, which should be sufficient.

It is interesting, that the muon accelerator will be at the same time a very high intensity generator for all types of neutrinos up to the maximum accelerator energy. About 1/4 of all the accelerated muons may go into useful neutrinos. The neutrino beam shall have a diameter of about 1O cm behind the complete shielding.

(In modern wording – NEUTRINO FACTORY !)

Later, the muon colliders and neutrino factories, based on ionization cooling, were very briefly presented in my introductory talk “Accelerator and Detector Prospects of High Energy Physics” at XX High Energy Physics International Conference, Madison, 1980.[3]

The road to muon-based neutrino factories and muon colliders is still long. But the harvest should be very rich.

References

1. Budker G.I., in Proceedings of the 7th International Conf. on High Energy Accelerators, Yerevan (1969) p. 33; extract in *Physics Potential and Development of* $\mu+\mu-$ *Colliders: Second Workshop*, ed. D. Cline, AIP Conf. Proc. **352**, 4 (1996).
2. Skrinsky A.N., presented at the International Seminar on Prospects of High-Energy Physics, Morges, 1971 (printed at CERN, unpublished); extract in *Physics Potential and Development of* $\mu^+\mu^-$ *Colliders: Second Workshop*, ed. D. Cline, AIP Conf. Proc. **352**, 6 (1996).
3. Skrinsky A.N., "Accelerator and Instrumentation Prospects of Elementary Particle Physics," in *Proceedings of the XX International ("Rochester") Conference on High Energy Physics, Madison, 1980*, New York, 1981, v.2, p.1056-1093; and in *Uspekhi Fiz. Nauk*, Moscow, 1982, **138**, 1, pp.3-43; translated at Soviet Physics Uspekhi **25** (9), September 1982, pp. 639-661.
4. Kolomensky A.A., Atomnaya Energiya **19**, 534, (1965); Ado Yu.M., Balbekov V.I., Atomnaya Energiya **39**, 40 (1971).
5. Skrinsky A.N. and Parkhomchuk V.V., *Sov. J. Part. Nucl.* **12**, 223-247 (1981).
6. Neuffer D., *Particle Accelerators* **14**, 75 (1983).
7. Skrinsky A.N., "Ionization Cooling and Muon Collider," in Proceedings of 9th ICFA Beam Dynamics Workshop: Beam Dynamics and Technology Issues for Muon-Muon Colliders, Montauk, NY (1995); Nuclear Instruments and Methods **A 391**, 188-195 (1997).
8. Skrinsky A.N., "Polarized muon beams for muon collider," in *Proceedings of the Symposium on Physics Potential and Development of* $\mu^+\mu^-$ *Colliders*, San Francisco (1995); Nuclear Physics B, Proceedings Supplement, v. 51A, November 1996, pp. 201-203.
9. Palmer R.B., Neuffer D. and Gallardo J., "A Practical High-Energy High-Luminosity $\mu^+\mu^-$ Collider," Advanced Accelerator Concepts: 6th Annual Conference, ed. P. Schoessow, AIP Conf. Proc. **335**, 635 (1995);
   Neuffer D. and Palmer R.B., "Progress Toward a High-Energy, High-Luminosity $\mu^+\mu^-$ Collider," The Future of Accelerator Physics: The Tamura Symposium, ed. T. Tajima, AIP Conf. Proc. **356**, 344 (1996).
10. Silvestrov G.I., "Problems of Intense Secondary Particle Beams Production," in *Proc. 13th Intern. Conf. on High Energy Accelerators*, Novosibirsk, 1986, v. 2, pp. 258-263.
11. Silvestrov G.I., "Lithium Lenses for Muon Colliders," in *Proc. 9th ICFA Beam Dynamics Workshop: Beam Dynamics and Technology Issues for* $\mu^+\mu^-$ *Colliders*, Montauk, NY, 1995, AIP Conf. Proc. **372**, pp. 168-177.
12. Skrinsky A.N., "Towards Ultimate Luminosity Polarized Muon Collider (problems and prospects)," in *Proceedings of the Symposium on Physics*

Potential and Development of $\mu^+\mu^-$ Colliders, San Francisco; APS Proc. **441**, 240-260 (1997).

13. Ankenbrandt C.M. *et al.*, (Muon Collider Collaboration), "Status of Neutrino Factory and Muon Collider – Research and Development and Future Plans," *Phys. Rev. ST Accel. Beams* **2**, 081001 (1999), http://publish.aps.org/ejnls/przfetch/abstract/PRZ/V2/E081001/.
14. Skrinsky A.N., "Remarks on High Energy Muon Collider," in Muon Colliders at 10 TeV to 100 TeV (HEMC'99 Workshop), Montauk 1999; AIP Conf. Proc. **530**, 311-315 (2000).