Bogolyubov Intitute for Theoretical Physics, National Academy of Sciences of Ukraine
Joint Institute for Nuclear Research (Dubna)

NEW TRENDS IN HIGH-ENERGY PHYSICS

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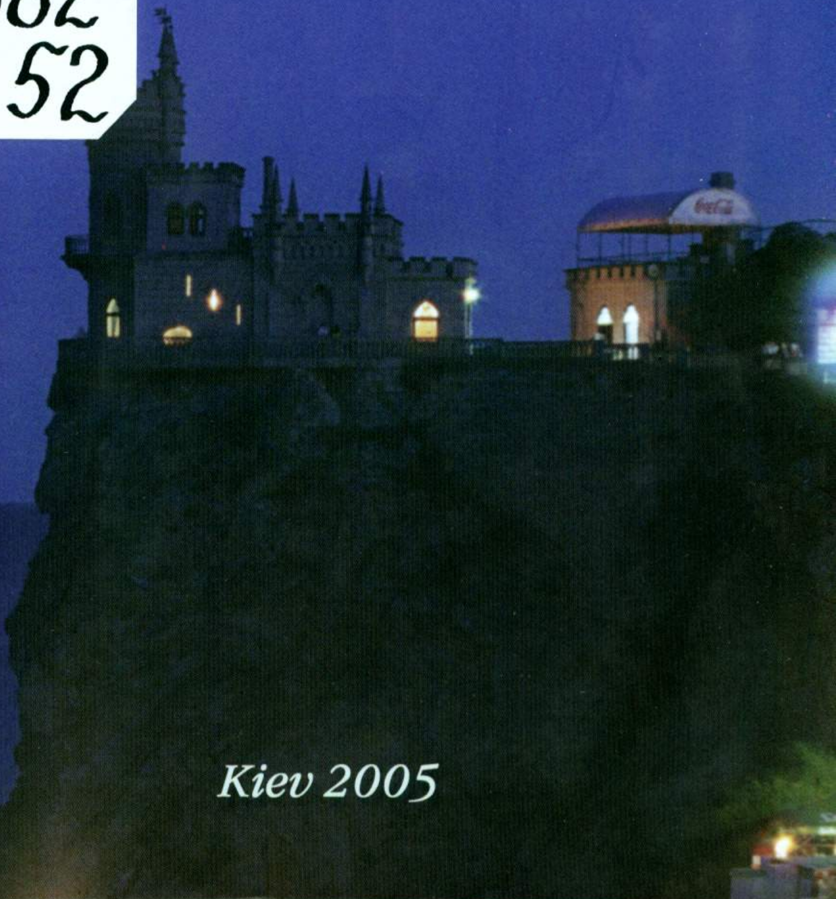

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P.N. Bogolyubov, P.O. Fedosenko,
L.L. Jenkovszky, and Yu.A. Karpenko

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DOUBLE CHARMONIUM PRODUCTION at B-FACTORIES

V.L. Chernyak

Budker Institute of Nuclear Physics, Novosibirsk, Russia

It is shown that long standing difficulties in reconciling the values of the cross section $\sigma(e^+e^- \to J/\psi + \eta_c)$ measured by BELLE and BABAR and calculated within non-relativistic QCD (NRQCD) are caused not by some misinterpretation of the data or other exotic explanations, but by poor applicability of NRQCD to such processes.

Using general theory of hard exclusive processes in QCD together with more realistic models of charmonium wave functions, it is shown that the experimental results can be naturally explained.

1. Introduction. A surprisingly large rate for hard exclusive processes of the type $e^+e^- \to J/\psi + \eta_c$ has been observed by the BELLE Collaboration in 2002, [1]. In this experiment the process $e^+e^- \to J/\psi + X$ was studied. The separate cross-sections of various exclusive double charmonium modes: $e^+e^- \to J/\psi + \eta_c$, $J/\psi + \chi_{co}$, etc. were extracted then from the number of events in the η_c or χ_{co} peaks in the mass spectrum of the system recoiling against the reconstructed J/ψ, see table and Fig.1.

The BELLE Collaboration also performed simultaneous fits to the production and helicity angle distributions. The measured angular and helicity distributions for $J/\psi + \eta_c$ have the general form $(1 + \alpha \cos^2 \theta)$ and are consistent with the expectations for production of this final state via a single virtual photon: $\alpha_{prod} = \alpha_{hel} = +1$, [2].

This year, the BABAR Collaboration also presented results of similar measurements, in overall agreement with the BELLE results (see table).

The problem arisen soon after the BELLE publication, in the close of 2002, when cross sections of these processes have been calculated by E. Braaten and J. Lee within the NRQCD (Non-Relativistic QCD) approach [4], and much smaller values (typically one order smaller) were obtained (see table).

J/ψ +	$\eta_c(1S)$	χ_{co}	$\eta_c(2S)$
BELLE	$25.6 \pm 2.8 \pm 3.4$	$6.4 \pm 1.7 \pm 1.0$	$16.5 \pm 3.0 \pm 2.4$
BABAR	$17.6 \pm 2.8 \pm 1.8$	$10.3 \pm 2.5 \pm 1.6$	$16.4 \pm 3.7 \pm 2.7$
NRQCD	$2.3 \pm 1.$	$2.3 \pm 1.$	$1. \pm 0.5$

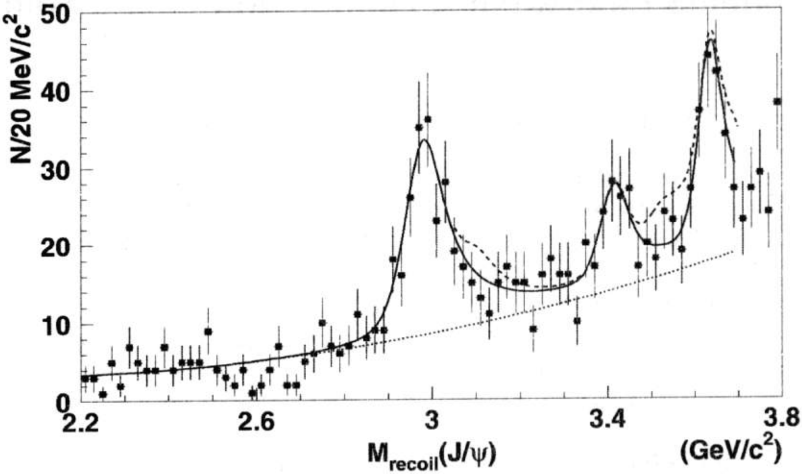

Figure 1: **BELLE data for** $e^+e^- \to J/\psi + X$

This large discrepancy initiated further studies, both experimental and theoretical.

Various explanations were proposed, e.g. that the two-photon production of $(J/\psi + J/\psi)$ can be significant and can imitate those of $J/\psi + \eta_c$ (G. Bodwin, E. Braaten and J. Lee, [5]). Later, this possibility was excluded by the BELLE Collaboration, [3].

Even more exotic variants of explanation were proposed. For instance, S. Brodsky, A.Goldhaber and J. Lee [6] proposed that the scalar gluonium (with its mass miraculously coinciding with the charmonium energy levels) has been produced together with J/ψ instead of second charmonium, etc.

Two main purposes of our paper [7] were:

a) to show that the real origin of these large discrepancies is due to a poor accuracy of NRQCD when applied to such processes. {The main reason is that the charm quark is not sufficiently heavy and, as a result, the charmonium wave functions describing the distributions of quarks inside the charmonium in momentum fractions are not sufficiently narrow for a reasonable application of NRQCD to the description of charmonium production. And, as usual, hard exclusive processes are particularly sensitive to the widths of hadron wave functions.}

b) to describe more adequate approach to calculation of such processes.

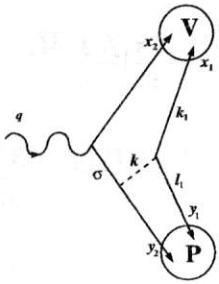

Figure 2: One of the four similar diagrams for the form factor $F_{J/\psi\,\eta_c}$.

2. Main formulae. The cross section of the process $e^+e^- \to J/\psi_\perp(p_1) + \eta_c(p_2)$ has the standard form:

$$\sigma\Big(e^+e^- \to \gamma^* \to J/\psi_\perp(p_1) + \eta_c(p_2)\Big) = \frac{\pi\alpha^2}{6}\left(\frac{|\vec{p}|}{E}\right)^3 Q_c^2\,|F_{VP}(s)|^2\,, \qquad (1)$$

where $(|\vec{p}|/E)^3$ is the P-wave phase space factor and $Q_c = 2/3$ is the charm quark charge, while the form factor $F_{VP}(s)$ is defined as:

$$\langle \psi_\perp(p_1), \eta_c(p_2)|J_\mu(0)|0\rangle = \epsilon_{\mu\nu\rho\sigma}\, e_\perp^\nu\, p_1^\rho\, p_2^\sigma\; F_{VP}(s)\,. \qquad (2)$$

Since there is one form factor only, the angular distribution is pure kinematical: $\sim (1+\cos^2\theta)$. Only the asymptotic form of $F_{VP}(s)$ will be needed because $s = M^2_{\Upsilon(4S)} \simeq 112\,\mathrm{GeV}^2$ in these experiments.

General theory of hard exclusive processes in QCD has been developed in 1977 in papers [8],[9] (see [10] for a review). It was shown in [8] that at large $s = (p_1+p_2)^2$ the leading power term of the general two-hadron form factor has the following behavior (up to logarithmic corrections):

$$\langle H_1(p_1, s_1, \lambda_1;\, H_2(p_2, s_2, \lambda_2)|\; J_\lambda\; |0\rangle \sim \Big(1/\sqrt{s}\Big)^{|\lambda_1+\lambda_2|+2n_{min}-3}. \qquad (3)$$

It is seen that the asymptotic behavior is independent of hadron spins s_1 and s_2, but depends essentially on their helicities λ_1 and λ_2. For the process considered, $e^+e^- \to \gamma^* \to J/\psi_\perp(p_1) + \eta_c(p_2)$, the J/ψ-meson is transversely polarized, i.e. has helicities $|\lambda_1| = 1$ only. So, the matrix element in eq.(3) behaves as $\sim 1/s$. Since in eq. (2) $e_\perp \sim 1$, while $p_1 \sim p_2 \sim \sqrt{s}$, all that results in $F_{VP}(s) \sim 1/s^2$.

The leading term of $F_{VP}(s)$ is given by four diagrams, one of which is shown in Fig.2, and the result looks as:

$$|F_{VP}(s)| = \frac{32\pi}{9} |\frac{f_V f_P \overline{M}}{q_o^4}| \, I_o ,$$

$$I_o = \int_0^1 dx_1 \int_0^1 dy_1 \alpha_s(k^2) \Big\{ \frac{Z_t Z_p V_T(x) P_P(y)}{d(x,y)\, s(x)} - \frac{\overline{M}_Q^2}{\overline{M}^2} \frac{Z_m^\sigma Z_t V_T(x) P_A(y)}{d(x,y)\, s(x)} +$$

$$+ \frac{1}{2} \frac{V_L(x)\, P_A(y)}{d(x,y)} + \frac{1}{2} \frac{(1-2y_1)}{s(y)} \frac{V_\perp(x)\, P_A(y)}{d(x,y)} + \tag{4}$$

$$+ \frac{1}{8} \Big(1 - Z_t Z_m^k \frac{4\overline{M}_Q^2}{\overline{M}^2}\Big) \frac{(1+y_1) V_A(x) P_A(y)}{d^2(x,y)} \Big\}.$$

Here: $s = 4E^2$, $q_o^2 = (|\vec{p}| + E)^2 \simeq (s - 2\overline{M}^2)$, $k = (k_1 + l_1)$ is the gluon momentum in Fig.1, $d(x,y)$ and $s(x)$ originate from the gluon and quark propagators, Z_t and Z_p are the renormalization factors of the local tensor and pseudoscalar currents:

$$d(x,y) = \frac{k^2}{q_o^2} = \Big(x_1 + \frac{\delta}{y_1}\Big)\Big(y_1 + \frac{\delta}{x_1}\Big), \quad \delta = \Big(Z_m^k \frac{\overline{M}_Q}{q_o}\Big)^2,$$

$$s(x) = \Big(x_1 + \frac{(Z_m^\sigma \overline{M}_Q)^2}{y_1 y_2\, q_o^2}\Big), \quad s(y) = \Big(y_1 + \frac{(Z_m^\sigma \overline{M}_Q)^2}{x_1 x_2\, q_o^2}\Big),$$

$$Z_p = \Big[\frac{\alpha_s(k^2)}{\alpha_s(\overline{M}_Q^2)}\Big]^{\frac{-3C_F}{b_o}} ; Z_t = \Big[\frac{\alpha_s(k^2)}{\alpha_s(\overline{M}_Q^2)}\Big]^{\frac{C_F}{b_o}} ; Z_m(\mu^2) = \Big[\frac{\alpha_s(\mu^2)}{\alpha_s(\overline{M}_Q^2)}\Big]^{\frac{3C_F}{b_o}} ; \tag{5}$$

$$\overline{M}_Q(\mu^2) = Z_m(\mu^2)\, \overline{M}_Q , \qquad Z_m^k = Z_m(\mu^2 = k^2), \quad Z_m^\sigma = Z_m(\mu^2 = \sigma^2),$$

where $\overline{M}_Q(\mu^2)$ is the running $\overline{MS}$-mass, $C_F = 4/3$, $b_o = 25/3$.

Entering the above equations, the non-perturbative twist 2 and twist 3 wave functions of the vector V-meson with the helicity λ determine the distribution of two quarks inside the quarkonium bound state in longitudinal momentum fractions x_1 and x_2, and are defined as:

$$\langle V_\lambda(p) | \overline{Q}_\beta(z)\, Q_\alpha(-z) | 0 \rangle_{\mu^*} =$$

$$\frac{f_V \overline{M}}{4}\int_o^1 dx_1\, e^{i(pz)(x_1-x_2)}\Big\{\hat{e}_\lambda V_\perp(x) + \hat{p}\,\frac{(e_\lambda z)}{(pz)}\,[V_L(x) - V_\perp(x)] \tag{6}$$

$$+ f_v^t(\mu^*)\frac{(\sigma_{\mu\nu} e_\lambda^\mu p^\nu)}{\overline{M}}\, V_T(x) + f_v^a(\mu^*)(\epsilon_{\mu\nu\alpha\beta}\gamma_\mu\gamma_5\, e_\lambda^\nu p^\alpha z^\beta)\, V_A(x)\Big\}_{\alpha\beta}.$$

For the pseudoscalar P-meson:

$$\langle P(p)|\overline{Q}_\beta(z)\, Q_\alpha(-z)|0\rangle_{\mu^*} = i\frac{f_P\overline{M}}{4}\int_0^1 dx_1 e^{i(pz)(y_1-y_2)}\Big\{\frac{\hat{p}\,\gamma_5}{\overline{M}}\, P_A(y) - \tag{7}$$

$$- f_p^p(\mu^*)\,\gamma_5\, P_P(y) + f_p^t(\mu^*)\,(i\sigma_{\mu\nu}\gamma_5\, p^\mu z^\nu)\, P_T(y)\Big\}_{\alpha\beta}.$$

The values of dimensionless constants in the above formulae follow directly from the exact QCD equations of motion: $i\hat{D}Q = M_Q Q$:

$$f_p^p(k^2) = \frac{\overline{M}}{2\overline{M}_Q} Z_p;\;\; f_v^t(k^2) = \frac{2\overline{M}_Q}{\overline{M}} Z_t;\;\; f_v^a(k^2) = \frac{1}{2}\Big(1 - Z_t Z_m^k \frac{4\overline{M}_Q^2}{\overline{M}^2}\Big). \tag{8}$$

3. Wave functions. For heavy quarkonium the light-front $1S$-Coulomb wave function can be taken as:

$$\Psi(x,\, \vec{k}_\perp) \sim \left(\frac{\vec{k}_\perp^2 + (1-4x_1x_2)M_Q^{*\,2}}{4x_1x_2} + q_B^2\right)^{-2},$$

$$\phi(x) \sim \int d^2\vec{k}_\perp\, \Psi(x,\, \vec{k}_\perp) \sim x_1x_2\Big\{\frac{x_1x_2}{[1-4x_1x_2(1-\bar{v}^2)]}\Big\}. \tag{9}$$

Here: q_B is the Bohr momentum and $\bar{v} = q_B/M_Q^* \ll 1$ is the mean heavy quark velocity.

It originates from the Schrödinger equal-time wave function $\Psi_{Sch}(r) \sim \exp(-q_B r) \to \Psi_{Sch}(\vec{k}) \sim (|\vec{k}|^2 + q_B^2)^{-2}$, supplemented with the proposed by M. Terent'ev [11] and commonly used substitution ansatz: $\vec{k}_\perp \to \vec{k}_\perp,\ k_z \to (x_1 - x_2)M_0/2,\ M_0^2 = (M_Q^{*\,2} + \vec{k}_\perp^2)/x_1x_2$.

To improve for the above pure Coulomb form of the charmonium wave functions, the modified simple model form was used ($\int_0^1 dx_1\phi_i(x, v^2) = 1$):

$$\phi_i(x, v^2) = c_i(v^2)\,\phi_i^{asy}(x)\Big\{\frac{x_1x_2}{[1-4x_1x_2(1-v^2)]}\Big\}^{1-v^2}, \tag{10}$$

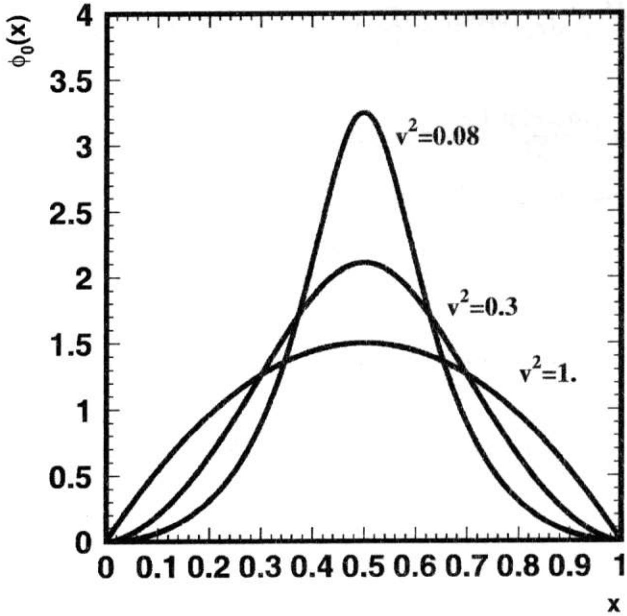

Figure 3: The shape of the leading twist wave function $\phi(x, v^2)$ for: $v^2 = 1$ - light quarks (asymptotic); $v^2 = 0.30$ - (charmonium); $v^2 = 0.08$ - (bottomonium).

where v is now a parameter which has the meaning of the characteristic quark velocity in the bound state, $\phi_i^{asy}(x)$ is the appropriate asymptotic wave function of light quarks which is known exactly, and $c_i(v^2)$ is the normalization constant.

These wave functions $\phi_i(x, v^2)$ interpolate in the simplest way between two exactly known extreme cases: very heavy quark with $v \to 0$, and very light quark with $v \to 1$.

In the non-relativistic case of very small $v \ll 1$ the wave functions $\phi_i(x, v^2)$ are strongly peaked around the point $x_1 = x_2 = 1/2$, so that $\phi_i(x, v^2 \to 0) \to \delta(x_1 - \frac{1}{2})$. This is the wave function corresponding to NRQCD. And clearly, a decreasing quark mass leads to larger v and to wider $\phi_i(x, v^2)$, see Fig.3 . [1]

4. Numerical results. Using the above described wave functions one

[1] It is seen that even for $\Upsilon(1S)$ the wave function $\phi(x, v^2 = 0.08)$ is still far from the δ - function form of NRQCD.

obtains from eqs. (1, 2, 4):

$$I_o \simeq 5.5\,, \quad F_{VP}(s = M^2_{\Upsilon(4S)}) \simeq 3.5 \cdot 10^{-3}\,\mathrm{GeV}^{-1},$$

$$\sigma\Big(e^+e^- \to \gamma^* \to J/\psi_\perp + \eta_c\Big) \simeq 33\,\mathrm{fb}. \tag{11}$$

This agrees with the BELLE result { assuming that $Br(\eta_c > 2\,\mathrm{charged})$ is not significantly less than unity }.

5. Some characteristic features. The mean value of the quark virtuality in Fig.2 diagram is: $\langle\sigma^2\rangle \simeq 33\,\mathrm{GeV}^2$, which is $\sim$ 1.5 times smaller than the typical value $\simeq q_o^2/2 \simeq 46\,\mathrm{GeV}^2$ for the narrow wave functions. The mean value of the gluon virtuality $\langle k^2\rangle$ can be inferred either from the mean value of $\langle Z_m^k\rangle \simeq 0.80$, or from the coupling $\alpha_s(k^2) : \langle\alpha_s(k^2)\rangle \simeq 0.263$. Both give the same result and show that the mean virtuality of the gluon in Fig.2 is: $\langle k^2\rangle \simeq 12\,\mathrm{GeV}^2$. This is $\sim$ 2 times smaller than a typical rough estimate: $\langle k^2\rangle \simeq q_o^2/4 \simeq 23\,\mathrm{GeV}^2$ for narrow δ- like wave functions.

The smaller values of $\langle k^2\rangle$ and $\langle\sigma^2\rangle$ is the main reason why the standard NRQCD-calculations underestimate the cross section considerably. In other words, the charm quark is not very heavy and its wave functions are not much like the δ-functions, although they are of course significantly narrower than similar wave functions of really light quarks (see Fig.3).

It is also of interest to make a comparison to the value of the cross section obtained in the limit which, in essence, corresponds to the approximations of NRQCD. For this, one has to replace in eq.(4): a) all wave functions by $V_i = P_i = \delta(x - 1/2)$; b) to omit the term with $V_A(x)$ (which is higher order correction $0(v^2/c^2)$ in NRQCD); c) to replace all renormalization factors Z_i by 1.

As a result:

$$I_o \simeq 1.6\,, \quad F_{VP} \simeq 1.1 \cdot 10^{-3}\,\mathrm{GeV}^{-1}\,, \quad \sigma \simeq 3.5\,\mathrm{fb}, \tag{12}$$

which is one order smaller than the data.

In the opposite limit when all $1S$ - charmonium wave functions are taken as the asymptotic ones of light quarks, the cross section will be $\sigma \simeq 70\,\mathrm{fb}$.

CONCLUSIONS

The difficulties with explaining the BELLE and BABAR results for $\sigma(e^+e^- \to J/\psi + \eta_c)$ **are not the real difficulties of QCD, but are rather due**

to a poor approximation of the dynamics of c-quarks by NRQCD. Within the approach described above, which we consider as more realistic, the experimental results look rather natural.

Further applications: the use of more realistic J/ψ - wave functions $V_i(x)$ given above instead of extreme form $\delta(x - 1/2)$ from NRQCD, will enhance also the calculated cross sections of inclusive direct production of J/ψ.

References

[1] K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. **89** (2002) 142001; hep-ex/0205104

[2] P. Pakhlov, "Further study of double charmonium prodution in e^+e^--annihilation at Belle", hep-ex/0412041.

[3] K. Abe et al. (Belle Collaboration), Phys. Rev. **D70** (2004) 071102; hep-ex/0407009

[4] E. Braaten, J. Lee, Phys. Rev. **D67** (2003) 054007; hep-ph/0211085

[5] G.T. Bodwin, J. Lee, E. Braaten, Phys. Rev. Lett. **90** (2003) 162001, hep-ph/0212181; Phys. Rev. **D67** (2003) 054023, hep-ph/0212352

[6] S.J. Brodsky, A.S. Goldhaber, J. Lee, "Hunting for glueballs in electron-positron annihilation", Phys. Rev. Lett. **91** (2003) 112001, hep-ph/0305269

[7] A.E. Bondar, V.L. Chernyak, Phys. Lett. **B612** (2005) 215, hep-ph/0412335

[8] V.L. Chernyak, A.R. Zhitnitsky, JETP Lett. **25** (1977) 510;
V.L. Chernyak, V.G. Serbo, A.R. Zhitnitsky, JETP Lett. **26**(1977) 594.

[9] V.L. Chernyak, A.R. Zhitnitsky, Sov. J. Nucl. Phys. **31** (1980) 544;
V.L. Chernyak, V.G. Serbo, A.R. Zhitnitsky, Sov. J. Nucl. Phys. **31** (1980) 552.

[10] V.L. Chernyak, A.R. Zhitnitsky, Phys. Rep. **112** (1984) 173.

[11] M.V. Terent'ev, Sov. J. Nucl. Phys., **24**(1976)106.

[12] V.L. Chernyak, Nucl. Phys. **B457** (1995) 96, hep-ph/9407353