PARTICLE MOTION FEATURES IN THE STORAGE RING WITH THE LONGITUDINAL MAGNETIC FIELD

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Abstract

The Courant-Snyder parametrization is most convenient way for describing particle-uncoupled motion in a storage ring. In the case of the weak-coupling this parametrization can be extended with the Edward-Teng [1] or Lebedev [2] description ways. The eigenvectors and eigenvalues of the betatron oscillation matrix slightly modified but the its physics meaning is kept. Such parametrizations work for the strong coupling yet but the physics interpretation obtained results are complicate enough. This article proposed another way for the description of the particle motion in the storage ring with the strong longitudinal magnetic field.

INTRODUCTION

At the presence of the strong longitudinal (or drive) magnetic field the particle motion can be decompose on the fast Larmour rotation around the magnetic force line and slow drift of Larmour center. The center of the Larmour rotation moves along the magnetic field force line and drifts slowly in the plane (*x*,*y*). This drift motion is induced by the small transverse component of the magnetic and electrical forces, non-homogenuity of the longitudinal magnetic field. Thus the particle motion can be decomposed to cyclotron (fast) and drift (slow) motion modes. The equations describing these motion modes are weak coupled in the strong longitudinal magnetic field. In the limit of the infinite magnetic field this motion modes may be considered as uncoupling. Further the coupling property of the motion in the new variables can be taken into account with a perturbation method. The describing approximation is good at the strong value of the magnetic field because at a low value of the longitudinal magnetic field the coupling becomes very essential. Thus, this situation is opposite to the classical case when the initial uncoupling vertical and horizontal motion is coupled by a weak magnetic field. In our case the insufficient value of the longitudinal magnetic field produces the coupling between drift and cyclotron modes.

MOTION EQUATION

The two-dimensional motion of a particle in a focusing lattice structure can be described by the following set of the equation [4]

$$\frac{dp_s}{ds} + h \cdot p_x = \frac{e}{c}\frac{1+hx}{p_s}\left(p_x B_y - p_y B_x\right),$$

$$\frac{dp_x}{ds} - h \cdot p_s = \frac{e}{c}\frac{1+hx}{p_s}\left(p_y B_s - p_s B_y\right),$$

$$\frac{dp_y}{ds} = \frac{e}{c}\frac{1+hx}{p_s}\left(p_s B_x - p_x B_s\right).$$

Here *h* is curvature of a planar curve that is the base curve of the storage ring. The particle momenta

$$\gamma m \frac{d\vec{r}}{dt} = \vec{p}\,, \qquad \vec{p} = p_s\vec{\tau} + p_x\vec{n} + p_y\vec{b}\,,$$

have the components

$$p_x = \gamma m \frac{dx}{dt}, \quad p_y = \gamma m \frac{dy}{dt}, \quad p_s = \gamma m (1+hx)\frac{ds}{dt},$$

and corresponds to a radius-vector (*x*,*y*,*s*) written in the standard form [4]

$$\vec{r} = \vec{r}_0(s) + x \cdot \vec{n}(s) + y \cdot \vec{b}(s)\,.$$

Making following change of variables and linearization procedure

$$\left[\begin{aligned} P_1 &= \frac{p_x}{p_{s0}} - K_s\, y \\ Q_1 &= \frac{1}{K_s}\frac{p_y}{p_{s0}} + x \end{aligned}\right., \qquad \left[\begin{aligned} P_2 &= \frac{p_y}{p_{s0}} \\ Q_2 &= \frac{1}{K_s}\frac{p_x}{p_{s0}} \end{aligned}\right..$$

the new equations set can be obtained. Here the pair (P_1,Q_1) is the drift mode and the pair (P_2,Q_2) is cyclotron mode.

$$\frac{dQ_1}{ds} = \frac{K_y}{K_s} + \frac{1}{K_s}\left(N + h\cdot K_y\right)Q_1 - \frac{K}{K_s^{\,2}}P_1 + \frac{K}{K_s}Q_2 - \left(N_s + N + h\cdot K_y\right)\frac{P_2}{K_s}$$

$$\frac{dP_2}{ds} = K_y - \left(K_s^{\,2} - K\right)Q_2 - \frac{1}{K_s}\left(N + h\cdot K_y\right)P_2 + \left(N + hK_y\right)Q_1 - \frac{K}{K_s}P_1$$

$$\frac{dP_1}{ds} = \left(h - K_x\right) - \left(K + hK_x\right)Q_1 - \frac{1}{K_s}\left(N + hK_y\right)P_1 + \frac{1}{K_s}\left(K + hK_x\right)P_2 + \left(N + hK_y\right)Q_2$$

$$\frac{dQ_2}{ds} = \frac{h - K_x}{K_s} + \left(1 + \frac{K}{K_s^{\,2}} + \frac{hK_x}{K_s^{\,2}}\right)P_2 + \frac{\left(N + hK_y\right)}{K_s}Q_2 - \frac{\left(K + hK_x\right)}{K_s}Q_1 - \frac{\left(N + hK_y + N_s\right)}{K_s^{\,2}}P_1$$

The following designation of magnetic field coefficients are used here: $K_x = eB_y/p_{s0}c$, $K_y = eB_x/p_{s0}c$, $K_s = eB_s/p_{s0}c$, $K = (e/p_{s0}c)(\partial B_y/\partial x)$ is a normal component of magnetic field gradient, $N = (e/p_{s0}c)(\partial B_x/\partial x)$ is a skew-quadrupole component of magnetic field gradient, $N_s = (e/p_{s0}\,c)(\partial B_s/\partial s)$ is the longitudinal gradient of longitudinal magnetic field.

One can make sure that these equations have Hamiltonian form [5]. The corresponding Hamiltonian looks in the following way

$$H = \frac{h-K_x}{K_s}P_2 - K_y Q_2 + \frac{1}{2}\left(1+\frac{K}{{K_s}^2}+\frac{hK_x}{{K_s}^2}\right)P_2^2 + \frac{N+hK_y}{K_s}P_2 Q_2 + \frac{1}{2}\left({K_s}^2 - K\right)Q_2^2 +$$
$$+\frac{K_y}{K_s}P_1 - (h-K_x)Q_1 - \frac{1}{2}\frac{K}{{K_s}^2}P_1^2 + \frac{N+hK_y}{K_s}P_1 Q_1 + \frac{1}{2}(K+hK_x)Q_1^2 -$$
$$-(N+hK_y)Q_1 Q_2 + \frac{K}{K_s}P_1 Q_2 - \frac{1}{K_s}(K+hK_x)Q_1 P_2 - \frac{1}{{K_s}^2}(N+hK_y+N_s)P_1 P_2 \,.$$

The non-quadratic terms relate to the difference between the base curve of the storage ring and the equilibrium orbit. Really the equilibrium orbit in such system can be non-planar but the planar geometry is very convenient with technical point of view.

ACTION-ANGLE FRAME

In general case the two-dimensional linearly coupled motion can be written as [3]

$$X = \begin{vmatrix} Q_1 \\ P_1 \\ Q_2 \\ P_2 \end{vmatrix} = P \begin{vmatrix} \sqrt{2J_1}\cos\Phi_1 \\ -\sqrt{2J_1}\sin\Phi_1 \\ \sqrt{2J_2}\cos\Phi_2 \\ -\sqrt{2J_2}\sin\Phi_2 \end{vmatrix} \quad P = \begin{vmatrix} p_{11} & 0 & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & 0 \\ p_{41} & p_{42} & p_{43} & p_{44} \end{vmatrix}$$

where J_1 and J_2 are the actions, $\Phi_1=2\cdot\pi\cdot\mu_1$ and $\Phi_2=2\cdot\pi\cdot\mu_2$ are the angle of the action, $\cdot\mu_1$ and $\cdot\mu_2$ are tune shift of the two eigenmodes. The 4x4 matrix ***P*** transfers the coordinates from the action-angle to the laboratory frame. Because of an ambiguity in the definition of the eigenmodes phase it is possible to choose the elements p_{12} and p_{34} in ***P*** are both zero.

The coordinates vector X in the laboratory frame in the points 1 and 2 can be connect with transfer matrix $\boldsymbol{T}_{1\to 2}$

$$X_2 = T_{1\to 2}\cdot X_1$$

or through the action frame

$$X_2 = P_2 \cdot R(\Delta\Phi_1, \Delta\Phi_2) P_1 \cdot X_1$$

where

$$R(\Delta\Phi_1,\Delta\Phi_2) = \begin{vmatrix} R(\Delta\Phi_1) & 0 \\ 0 & R(\Delta\Phi_2) \end{vmatrix}, R(\Delta\Phi_i) = \begin{vmatrix} \cos\Delta\Phi_i & \sin\Delta\Phi_i \\ -\sin\Delta\Phi_i & \cos\Delta\Phi_i \end{vmatrix}$$

is rotation matrix in the action-angle frame.

Matrix P can be expressed with Edwards-Teng parametrization parameters.

$$P = \begin{vmatrix} d\sqrt{\beta_1} & 0 & W_{11}\sqrt{\beta_2} - W_{12}\frac{\alpha_2}{\sqrt{\beta_2}} & -\frac{W_{12}}{\sqrt{\beta_2}} \\ -d\frac{\alpha_1}{\sqrt{\beta_1}} & \frac{d}{\sqrt{\beta_1}} & W_{21}\sqrt{\beta_2} - W_{22}\frac{\alpha_2}{\sqrt{\beta_2}} & \frac{W_{22}}{\sqrt{\beta_2}} \\ -W_{22}\sqrt{\beta_1} - W_{12}\frac{\alpha_1}{\sqrt{\beta_1}} & \frac{W_{12}}{\sqrt{\beta_1}} & d\sqrt{\beta_2} & 0 \\ W_{21}\sqrt{\beta_2} + W_{11}\frac{\alpha_2}{\sqrt{\beta_2}} & -\frac{W_{11}}{\sqrt{\beta_1}} & -d\frac{\alpha_2}{\sqrt{\beta_2}} & \frac{d}{\sqrt{\beta_2}} \end{vmatrix}$$

Twiss parameters can be calculated from matrix ***P***

$$d = \sqrt{p_{11}p_{22}} = \sqrt{p_{33}p_{44}}$$

$$\beta_1 = \frac{p_{11}}{p_{22}}, \alpha_1 = -\frac{p_{21}}{p_{22}}, \gamma_1 = \frac{p_{21}^2 + p_{22}^2}{p_{11}p_{22}}$$

$$\beta_1 = \frac{p_{33}}{p_{44}}, \alpha_1 = -\frac{p_{43}}{p_{44}}, \gamma_1 = \frac{p_{43}^2 + p_{44}^2}{p_{33}p_{44}}$$

So, the following procedure for description of the particle dynamics in the system with strong longitudinal magnetic field can be realized. The first, the one-turn transfer matrix $\boldsymbol{M}_1$ is calculated in a fixed point 1. The transfer matrices $\boldsymbol{T}_{1\to 2}(s)$ enables to recalculate the one-transfer matrix for the arbitrary point of storage ring

$$M_{ring}(s) = T_{1\to 2}(s) M_1 T_{1\to 2}(s)^{-1}$$

With set of the eigenvectors the matrix ***P*** is computed in the points 1 and s. After that the matrix ***R*** is calculated in the arbitrary point s of the ring. It enables to determine the phase incursion of the two eigenmodes. The Twiss parameters α, β, γ, r is calculated with standard procedure described in [6].

NUMERICAL EXAMPLE

As an example we consider the storage ring consists of four cells and has fourfold symmetry. Each cell consists of the bend magnet with the field index *n* and straight section with length *l*. The longitudinal magnetic field is imposed on this structure. The main parameters of the storage ring are shown in Table 1. The magnetic field is calculated in 3D program for real geometry of the coils and magnet yoke.

Table 1: The main parameters of the storage ring

Longitudinal magnetic field	25 kG
Bending magnetic field	3 kG
Curvature radius	60 cm
Field index, n	0.5
Length of the straight section in period	70 cm

The figure 1 shows the phase incursion of the drift and cyclotron modes. The straight solenoid doesn't induce the drift motion so the phase incursion of the drift mode in this region is zero. In the bending section the drift motion is observed induced by the action of the bending magnetic field with field index *n=0.5*. The cyclotron mode rotates with practically constant rate.

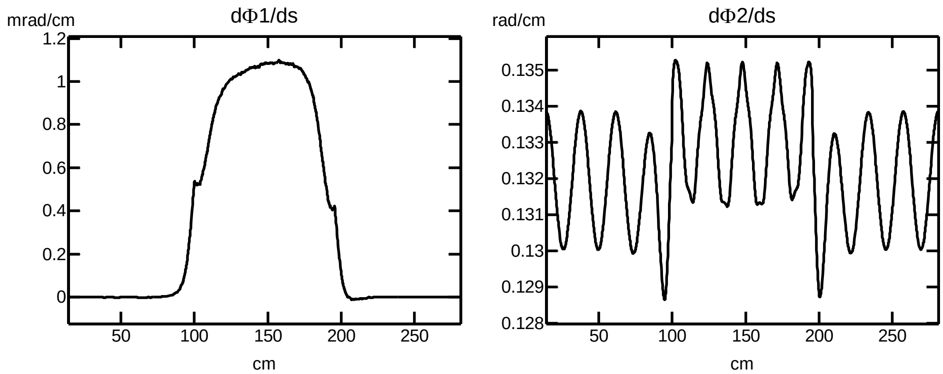

Figure 1: Rate of phase incursion for motion of drift and cyclotron modes.

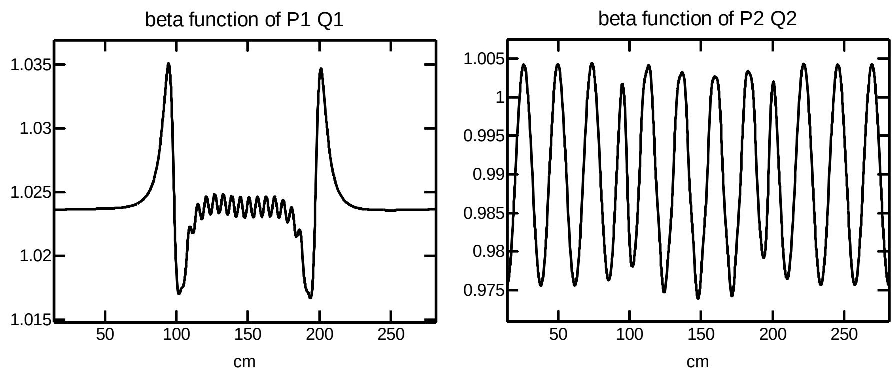

Figure 2: β-function of drift and cyclotron modes.

In our case the values of β-function are not related with phase incursion, but it describes only ratio between corresponding momentums and coordinates in their change during a revolution. Because the pair of Larmor circle position is new canonically conjugated variables in drift approximation (the other pair is the transversal velocities), so the most convenient presentation of β-function is a dimensionless form $\beta^{*}=K_s\beta$.

The dimensionless β*-function describes either a ratio between X and Y sizes of ellipse describing the beam shape (pair P_1, Q_1) or a ratio between v_x and v_y transversal velocities (pair P_2, Q_2). So, the figure 2 shows that the field index $n=0.5$ leads to the drift motion along circle. The ratio between transverse impulses is closed to 1.

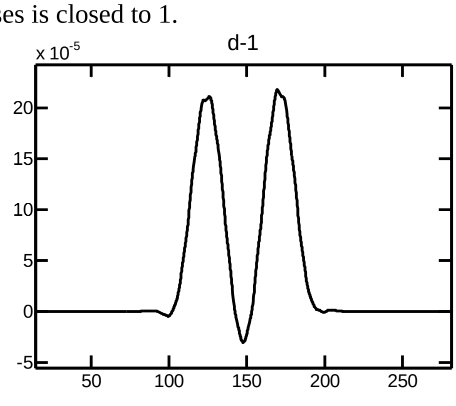

Figure 3: Coupling parameter in Edwards-Teng parametrization.

Figure 3 shows the coupling parameter *d* in Edwards-Teng parametrization. This value is very close to 1, so the chosen motion modes are practically independent.

CONCLUSION

The described above procedure enables to use the convenient Courant-Snyder parametrization for the drift and cyclotron modes at the presence of the strong longitudinal magnetic field. This results may be used at analysing the electron motion in the cooling device, the muon motion in the ionization cooler [7] or another system with strong solenoidal coupling [8].

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