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# Direct computation of the growth rate for the instability of a warm relativistic electron beam in a cold magnetized plasma

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The instability of an electron beam propagating through a plasma plays an essential role in a great variety of problems. By now, main regimes of the instability are well understood, though even for the case of an isotropic plasma direct computation of the growth rate has been realized only recently [1]. In this paper we numerically solve the dispersion relation to yield the growth rates for the case of a warm relativistic electron beam, magnetized cold plasma, and arbitrary direction of **k**. Of special interest is the effect of the magnetic field on the instability of oblique waves. According to the existing notion [2] which is based on the hydrodynamic approximation, increase of the magnetic field results in monotonic decrease of oblique wave's growth rates and makes the instability almost one dimensional at high fields. Here we use the kinetic approach to trace the modification of unstable modes as the magnetic field changes from low to high values, and show that the existing notion is not fully correct. Eigenfrequencies $\omega$ of the beam-plasma system are determined by the equation

$$\left| k_i k_j - k^2 \delta_{ij} + \frac{\omega^2}{c^2} \varepsilon_{ij} \right| = 0, \tag{1}$$

where the dielectric tensor $\varepsilon_{ij}$ for the cold plasma of the density $n_0$ and the beam propagating in $z$-direction along the magnetic field **B** has the following components for high $\omega$:

$$\varepsilon_{xx} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} + \frac{4\pi e^2}{\omega} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} A_n J_n^2 \frac{n^2 \omega_c^2}{\gamma^2 k_\perp^2},$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = \frac{i\omega_c}{\omega} \frac{\omega_p^2}{\omega^2 - \omega_c^2} - \frac{4\pi i e^2}{\omega} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} A_n J_n J_n' \frac{n\omega_c v \sin\theta}{\gamma k_\perp},$$

$$\varepsilon_{yy} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} + \frac{4\pi e^2}{\omega} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} A_n {J_n'}^2 v^2 \sin^2\theta,$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \frac{4\pi e^2}{\omega} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} B_n J_n^2 \frac{n\omega_c v \cos\theta}{\gamma k_\perp},$$

$$\varepsilon_{yz} = -\varepsilon_{zy} = \frac{4\pi i e^2}{\omega} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} B_n J_n J_n' v^2 \sin\theta \cos\theta,$$

$$\varepsilon_{zz} = 1 - \frac{\omega_p^2}{\omega^2} + \frac{4\pi e^2}{\omega} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} B_n J_n^2 v^2 \cos^2\theta,$$

where $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the plasma frequency, $\omega_c = eB/(mc)$ is the cyclotron frequency

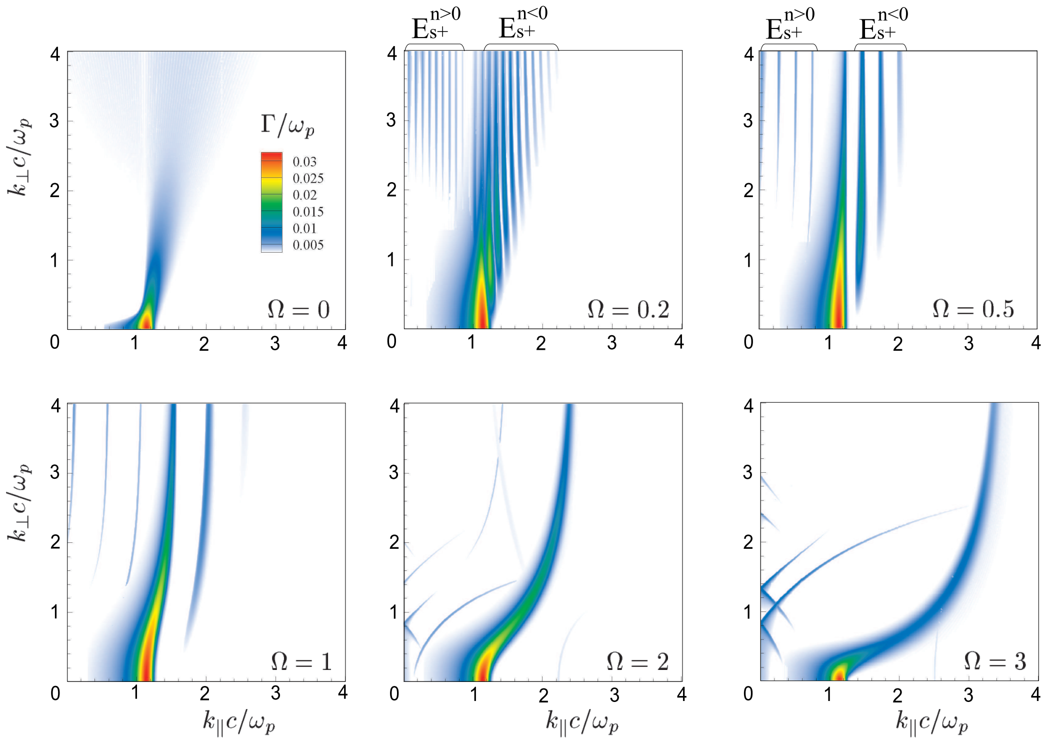


Figure 1: Growth rate maps in the **k**-space for magnetic fields of various strength.

for a nonrelativistic electron, $\gamma$ and $\mathbf{v}$ are relativistic factor and velocity corresponding to the electron momentum $\mathbf{p}$, $J_n \equiv J_n(k_\perp \gamma v \sin\theta/\omega_c)$ is the Bessel function of the order $n$, $J_n'$ is the derivative of Bessel function with respect to its argument, $\theta$ is the angle between the electron momentum and $z$-direction, $\mathbf{k} = (k_\perp, 0, k_\parallel)$,

$$A_n = \frac{1}{\omega - k_\parallel v\cos\theta - n\omega_c/\gamma}\left[\frac{1}{v}\frac{\partial f}{\partial p} + \frac{\omega\cos\theta - k_\parallel v}{\omega p v \sin\theta}\frac{\partial f}{\partial \theta}\right],$$

$$B_n = \frac{1}{\omega - k_\parallel v\cos\theta - n\omega_c/\gamma}\left[\frac{1}{v}\frac{\partial f}{\partial p} + \frac{1}{pv\cos\theta}\left(\frac{n\omega_c}{\gamma\omega\sin\theta} - \sin\theta\right)\frac{\partial f}{\partial \theta}\right],$$

and $f$ is the distribution function of the beam. We assume the beam to be monoenergetic with the momentum $p$:

$$f(p',\theta) = \frac{n_b}{2\pi p^2 G}\delta(p'-p)g(\theta), \quad g(\theta) = \exp(-\theta^2/\Delta\theta^2), \quad G = \int_0^{\pi/2} g(\theta)\sin\theta\, d\theta.$$

The main result of the paper is presented in Fig. 1 that shows transformation of the growth rate maps as the dimensionless magnetic field $\Omega = \omega_c/\omega_p$ changes from low to high values. The maps are plotted for $p = 2.29\,mc$, $n_b = 0.002\,n_0$, and $\Delta\theta = 0.2$. It is seen that, for any $\Omega$, the fastest growing mode is the one propagating along the magnetic field. This contrasts with the hydrodynamic approximation predicting that, due to relativistic anisotropy of the beam "mass", in weak magnetic fields ($\Omega \lesssim 1$) the dominant role is played by obliquely propagating waves. To

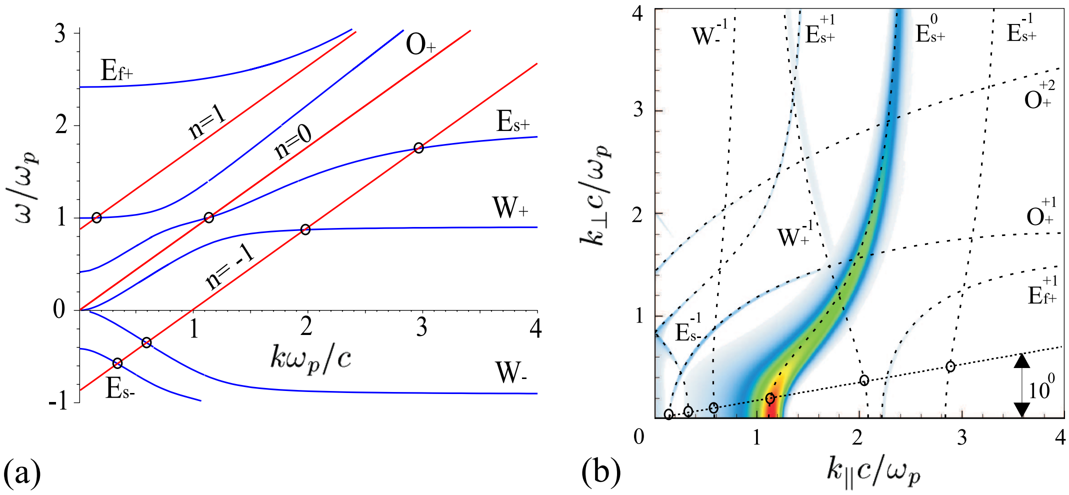


Figure 2: (a) Natural modes of the cold plasma and several beam modes ($n = 0, \pm 1$) for $\Omega = 2$ and oblique wave propagation (the angle $\phi$ between $\mathbf{k}$ and $\mathbf{B}$ is $10^\circ$); (b) lines of mode intersections in the $\mathbf{k}$-space (dash lines) superimposed on the instability map. Small circles at the two figures denote corresponding waves.

classify the observed modes, we note that unstable roots appear due to coupling between natural modes of the cold plasma $\omega_{pl}(\mathbf{k})$ and beam modes $\omega_b(\mathbf{k}) \approx k_\parallel v + n\Omega/\gamma$. Each unstable mode can thus be labelled by two "parent" waves from which it originates. The labelling is illustrated by the example of $\Omega = 2$ case (Fig. 2). We denote high-frequency modes of the cold plasma (Fig. 2a): ordinary electromagnetic wave (O), fast extraordinary wave ($E_f$), slow extraordinary wave ($E_s$), and the whistler wave (W). The waves can propagate back and forth with respect to the beam (subscripts '$-$' and '$+$') and couple to different beam modes (with $n$ denoted by superscripts). Intersections of the cold plasma modes with the beam modes define the lines $\omega_{pl}(\mathbf{k}) = \omega_b(\mathbf{k})$ or, equivalently, $k_\perp(k_\parallel)$ that are shown by dash lines in Fig. 2b. The crests of the growth rate closely follow these lines, though not every line is followed by the instability. As we see from Fig. 1, for any magnetic field the growth rate is highest for Cherenkov excitation of the slow extraordinary wave $E_{s+}^0$. This wave is also excited at cyclotron resonances. The exact solution offers a view of how accurate are the hydrodynamic and kinetic approximations commonly used for analytical estimates of the growth rates. For the beam parameters chosen, the kinetic approximation gives an order of magnitude overstated value for the maximum growth rate, so we compare the exact solution with the hydrodynamic approximation only. For the Cherenkov

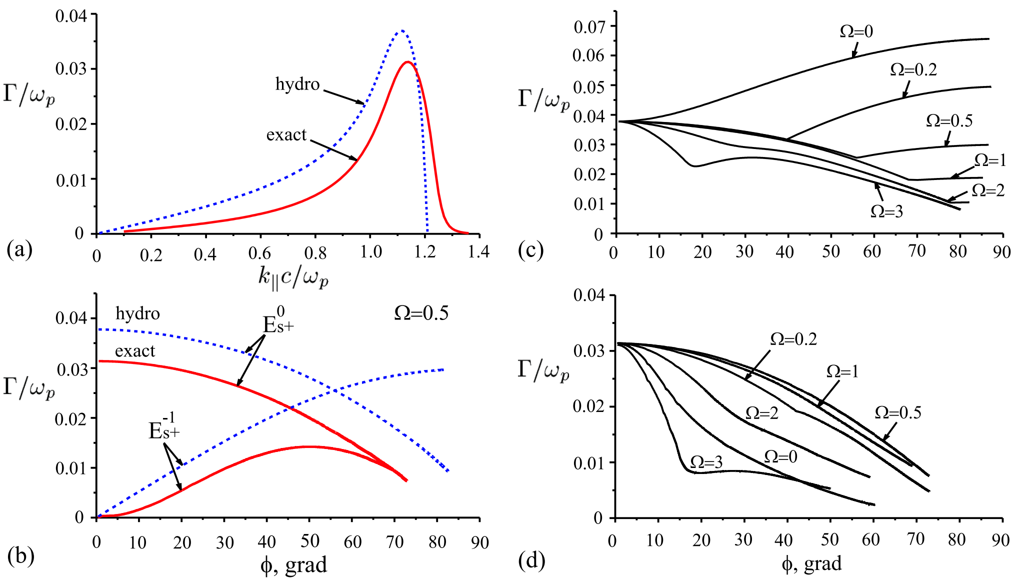


Figure 3: (a) Exact and hydrodynamic growth rates as functions of $k_{||}$ for purely longitudinal wave propagation. (b) Exact and hydrodynamic growth rates, maximized in $k$, as functions of the wave propagation angle $\phi$ for the modes $E^0_{s+}$ and $E^{-1}_{s+}$ at $\Omega = 0.5$. Effect of the magnetic field on the angular dependence of the maximum growth rate (determined by either $E^0_{s+}$ or $E^{-1}_{s+}$ mode): (c) hydrodynamic approximation, (d) exact solution of (1).

excitation of the purely longitudinal Langmuir wave the exact and approximate solutions are presented in Fig.3a. For oblique wave propagation, the most important role is played by the modes $E^0_{s+}$ and $E^{-1}_{s+}$. We compare exact and hydrodynamic results for them along the crests of the growth rate $k_\perp(k_{||})$. Fig. 3b shows that the hydrodynamic approximation adequately describes the angular dependence of the maximum growth rate for Cherenkov excitation of the mode $E^0_{s+}$, but fails to describe the cyclotron excitation of $E^{-1}_{s+}$ mode at large angles.

Now we check the effect of the magnetic field on the angular dependence of the maximum growth rate. The hydrodynamic approximation predicts that, as the magnetic field increases from zero to high values, the dominant role is transferred from oblique waves to the purely longitudinal mode (Fig. 3c). Both angular spread and magnetic field act to suppress the instability of oblique waves. The fully kinetic approach reveals a qualitatively different picture (Fig. 3d). A moderate magnetic field favors the instability: at $\Omega \lesssim 1$ the growth rate slower decreases with angle then with no magnetic field. The observed non-monotonic behavior of the growth rate at high propagation angles is a purely kinetic effect connected with overlapping of resonances.

## References


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