DAMPING WIGGLER WITH TAPERED PERIOD*

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Abstract

Strong-field short-period wigglers installed in electron storage ring increase radiation damping integral I2 and also increase or decrease I5 integral responsible for quantum fluctuations. In case of I5 integral decreasee, beam emittance reduces. In the paper we discuss optimization of the wiggler lattice functions and additional reduction of I5 by applying longitudinal modulation of the wiggler period (tapering).

INTRODUCTION

Multipole wiggler – a periodic array of (strong) dipole magnets – is used successfully to control damping parameters of the beam in a circular accelerator by increase of the energy loss from synchrotron radiation. If wiggler parameters (peak field B_w and period length λ_w) together with lattice functions meet some specific requirements than such magnetic device can reduce equilibrium emittance beyond conventional limit. Damping wigglers are essential for synchrotron light sources or damping rings. As an example we mention PETRA III damping wiggler system [1], which allowed to decrease horizontal emittance by factor 4 and to achieve record emittance of 1 nm at 6 GeV beam energy [2].

Emittance decrease by wigglers depends on lattice functions behavior in the wiggler section, so one can optimize the functions in the same manner as for the dipole magnet.

Below we perform such optimization for FODO cell and for the lattice functions symmetrical with respect to the wiggler midpoint (analog to the TME magnet cell).

Another possibility for a more effective emittance minimization by the damping wiggler is compensation of the H(s) function growth along the wiggler by modulation of the wiggler period. To some extent this approach is similar to the longitudinal field decay in bending magnet proposed by Albin Wrulich [3]. The difference is that in the wiggler we can vary not only the peak field but also the period length. The latter seems easier than manipulation with the field amplitude contributing to both the second and the fifth synchrotron radiation integral.

WIGGLER MODEL

As we shall see below, an effective emittance reduction requires short wiggler period and high magnetic field amplitude. Beam trajectory in such device is almost sinusoidal, so we can use the Halbach wiggler model [4] that gives the following expression for the vertical field component in the wiggler gap

$$B_y(z) = B_w \cosh(ky)\cos(kz),$$

where z indicates the distance along the wiggler axis and $k = 2\pi/\lambda_w$. We assume that planar wiggler has rather wide poles producing flat field distribution in the horizontal direction. The wiggler dispersion function and its derivative are given by [5]

$$\eta_w(z) = \frac{h_w}{k^2}(1-\cos kz) = \eta_{w0} - \frac{\theta_w}{k}\cos kz,$$

$$\eta'_w(z) = \frac{h_w}{k}\sin kz = \theta_w \sin kz,$$

where $h_w = \rho_w^{-1} = B_w / B\rho$ is peak field curvature and $\theta_w = \lambda_w / 2\pi\rho_w$ is maximum wiggler deflection angle. The term η_{w0} can also include ring residue dispersion in the wiggler section. Here and below we assume that contribution of the end poles is negligible as compared to the regular part of long multipole wiggler.

Horizontal emittance is given by

$$\varepsilon_{x0} = C_q \frac{\gamma^2}{J_x}\frac{I_5}{I_2},$$

where $C_q = 3.832\times10^{-13}$ m, γ is the relativistic factor, J_x is the horizontal damping partition number and two synchrotron radiation integrals represent the radiation damping and quantum excitation, respectively [6]

$$I_2 = \oint_M \frac{ds}{\rho^2(s)}, \quad I_5 = \oint_M \frac{H(s)ds}{|\rho(s)|^3}$$

$$H(s) = \gamma_x\eta_x^2 + 2\alpha_x\eta_x\eta'_x + \beta_x\eta'^2_x.$$

where α_x, β_x and γ_x are the Twiss parameters. The integrals can be split to those over the bare lattice and over the wigglers: $I_2 = I_{20} + i_2$ and $I_5 = I_{50} + i_5$, thus, the wiggler resulting emittance refers to the initial one as

$$\frac{\varepsilon_w}{\varepsilon_{x0}} \approx \frac{1 + i_5 / I_{50}}{1 + i_2 / I_{20}}. \quad (1)$$

For a sinusoidal wiggler and average $\bar{\beta}_x$ in the wiggler section, the wiggler synchrotron radiation integrals are given by

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$$i_2 = \frac{L_w h_w^2}{2} \text{ and } i_5 \approx \frac{8}{15} N\theta_w h_w^2 \left(5\frac{\eta_{0x}^2}{\bar{\beta}_x} + \bar{\beta}_x \theta_w^2 \right) \quad (2)$$

where $L_w = N\lambda_w$ is total wiggler length, N is number of wiggler periods.

To obtain minimum emittance, the residue lattice dispersion function in the wiggler section should be set to zero. However, in reality, there is always some spurious dispersion, and (2) gives us an estimation of the tolerance for the residue dispersion suppression

$$\eta_{0x} << \bar{\beta}_x \theta_w / \sqrt{5}.$$

With this condition satisfied, the wiggler fifth integral takes the following form:

$$i_5 \approx \frac{8}{15} N\theta_w^3 h_w^2 \bar{\beta}_x = \frac{1}{15\pi^3} \lambda_w^2 h_w^5 L_w \bar{\beta}_x. \quad (3)$$

Insertion of i_2 and i_5 into (1) yields

$$\frac{\varepsilon_w}{\varepsilon_{x0}} \approx \frac{1 + h_w^5 \left[N\lambda_0^3 \bar{\beta}_x / (15\pi^3 I_5) \right]}{1 + h_w^2 \left[N\lambda_0 / 2I_2 \right]}. \quad (4)$$

Figure 1 shows emittance ratio (4) as a function of the wiggler field amplitude.

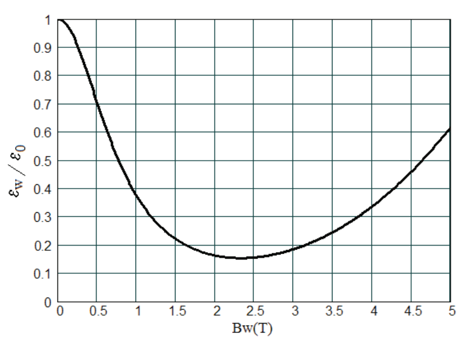

Figure 1: Resulting wiggler emittance vs. peak field.

For given wiggler period, the ratio $(\varepsilon_w / \varepsilon_0)$ has minimum at field $B_{w\,min}$. From (4) one can see that reduction of the emittance below the bare lattice limit for fixed wiggler length requires high magnetic field (i_2 increases) and small period length (i_5 decreases).

Here the second and the fifth wiggler integrals in the case of the piecewise (meander) field pattern are represented for reference:

$$i_{2M} = L_w h_w^2, \qquad i_{5M} \approx \frac{1}{48} \lambda_w^2 h_w^5 L_w \bar{\beta}_x. \quad (3)$$

The second integral is twice as much as that for the sine-like field profile with the same period and peak field, but the fifth meander integral is $15\pi^3/48 \approx 10$ times larger than that in (3).

OPTIMIZATION OF THE WIGGLER LATTICE FUNCTIONS

Wiggler fifth integral (3) depends on the average beta in the wiggler $\bar{\beta}_x$, hence it could be optimized in order to reach minimum of i_5. This procedure is similar to emittance minimization in the light sources. In this section we minimize i_5 for FODO like wiggler cell and for the cell with minimum of horizontal beta at the wiggler center (TME-like cell).

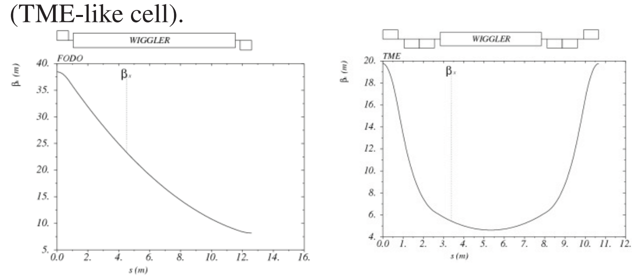

Figure 2: Two wiggler cells: FODO (left) and TME-like.

FODO Wiggler Cell

We start with FODO cell model (Figure 2, left), where the wiggler is placed between two quadrupoles. For a thin-quads model the maximum beta in the focusing quadrupole is given by [7]

$$\hat{\beta}_x = 2L_w \csc\mu_x \left(1 + \sin\frac{\mu_x}{2} \right),$$

where μ_x is FODO cell phase advance, and beta varies along the wiggler as

$$\beta_x(z) = \hat{\beta}_x - 2\sec\frac{\mu_x}{2}\left(1 + \sin\frac{\mu_x}{2} \right) z + 2\tan\frac{\mu_x}{2}\frac{z^2}{L_w}. \quad (5)$$

Inserting (5) in (3) we obtain, after averaging,

$$i_5 \approx \frac{8}{15} N\theta_w^3 h_w^2 L_w F(\mu_x),$$

$$F(\mu_x) = \frac{2}{3}\csc\mu_x \left(2 + \cos^2\frac{\mu_x}{2} \right).$$

Figure 3 shows the plot of $F(\mu_x)$ with minimum corresponding to $\sin(\mu_x/2) = \sqrt{3/5}$, $\mu_{x\,min} \approx 101.5^0$ and $F_{min}(\mu_{x\,min}) = 4/\sqrt{6} \approx 1.633$. In other words, the average beta in (3) providing minimum emittance is equal to $\bar{\beta}_x = L_w \cdot 4/\sqrt{6}$.

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It is worth noting that i_5 depends on μ_x similar to the main FODO lattice integral I_5 [7], however minimum for I_5 occurs at $\mu_{x\min} \approx 137^0$.

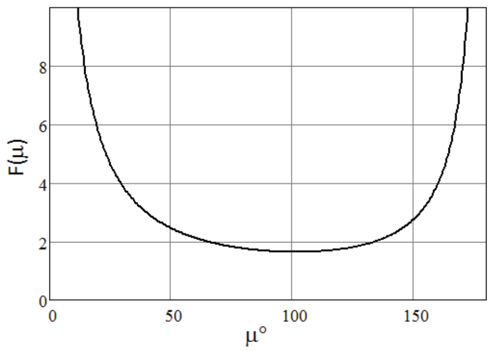

Figure 3: $F(\mu_x)$ for FODO like wiggler cell.

Symmetric (TME-like) Wiggler Cell

Figure 2 on the right shows schematically cell that reminds TME solution. Having β_{x0} as wiggler central beta, beta variation along the wiggler half is given by $\beta_x(z) = \beta_{x0} + z^2/\beta_{x0}$ and calculation of i_5 yields

$$i_5 \approx \frac{8}{15} N\theta_w^3 h_w^2 \left(\beta_{x0} + \frac{L_w^2}{12\beta_{x0}}\right).$$

The structural factor in the brackets can be minimized with respect to β_{x0}, giving $\beta_{x0\min} = L_w/\left(2\sqrt{3}\right)$, and

$$i_{5\min} \approx \frac{8}{15} N\theta_w^3 h_w^2 \frac{L_w}{\sqrt{3}}.$$

As it is seen, symmetric beta with minimum in the wiggler center gives minimum i_5 by factor $2\sqrt{2}$ smaller than one for FODO cell. However, FODO cell is more compact (contains fewer quadrupoles), so particular solution should take into account all these factors.

WIGGLER PERIOD TAPERING

Assuming negligible variation of the horizontal beta over wiggler period, the fifth wiggler integral (3) can be rewritten as

$$i_5 \approx \frac{1}{15\pi^3} \sum_{n=1}^{N} \lambda_{wn}^3 h_{wn}^5 \overline{\beta}_{xn}, \tag{6}$$

where beta function is averaged over wiggler period. Neglecting horizontal focusing, $\beta_x(z)$ grows inside the wiggler from some minimum like in the drift and, simultaneously, $\overline{\beta}_{xn}$ and i_{5n} grow in every next period. One can compensate this grows by introduction of longitudinal variation of the wiggler period in the way that provides

$$\lambda_{wn-1}^3 \overline{\beta}_{xn-1} = \lambda_{wn}^3 \overline{\beta}_{xn}. \tag{7}$$

Substituting the general expression

$$\overline{\beta}_{xn} = \frac{1}{\lambda_{wn}} \int_0^{\lambda_{wn}} \left(\beta_{xn}(0) - 2\alpha_{xn}(0)z + \gamma_{xn}(0)z^2\right) dz.$$

in (6) one can find for $\lambda_n = \lambda_{n-1} - \Delta\lambda_{n-1}$, $\Delta\lambda/\lambda << 1$ the following recurrence relation for a successive period length decay

$$\lambda_n \approx \lambda_{n-1} \cdot \left[1 - \frac{-6\alpha_{xn-1}d_{n-1} + \left(1+\alpha_{xn-1}^2\right)d_{n-1}^2}{9 - 30\alpha_{xn-1}d_{n-1} + 26\left(1+\alpha_{xn-1}^2\right)d_{n-1}^2}\right], \tag{8}$$

where $d_n = \lambda_n / \beta_{xn}$.

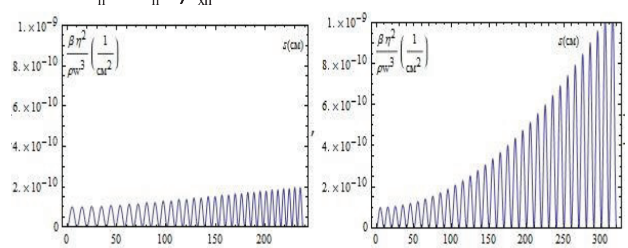

Figure 4: Fifth integral growth in the wiggler with (left) and without successive period length reduction.

We have applied the above relation to the wiggler with the following initial (without the period length tapering) parameters: $L_w = 3.2$ m, $\lambda_{w0} = 0.2$ m, $N_{w0} = 16$, $\alpha_{x0} = 0$, $\beta_{x0} = 1$ m and $\rho_w = 4$, and found that the fifth wiggler integral decreases by factor of ≈4 as compared to the fixed period case (see Figure 4).

CONCLUSION

Several issues were considered to minimize the damping wiggler contribution to the emittance excitation lattice integral (I_5 integral) including optimization of the optical functions and the wiggler period decay to compensate the horizontal beta growth along the wiggler length.

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