LOW EMITTANCE LATTICE CELL WITH LARGE DYNAMIC APERTURE*

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Abstract

Compact low emittance lattice cell providing large dynamic aperture is essential for development of extremely low (pm range) emittance storage rings. As it is well known, a pair of identical sextupoles connected by a minus-identity matrix transformer in ideal case of kick-like magnets provides infinite dynamic aperture. Though the finite sextupole length degrades the aperture, it is still large enough, and in this report we discuss development of the low emittance cell providing the –*I* condition for both horizontal and vertical chromatic sextupoles.

INTRODUCTION

For an extremely low emittance storage ring with emittance in the pm × rad range one of the main constraints is very low dynamic aperture (DA) [1]. Explanation of this fact is simple: emittance minimization gives very low dispersion outside of the dipoles and consequently strong chromatic sextupoles shrink DA.

Arrangement of the chromatic sextupoles in pairs of identical magnets connected by –*I* transformer [2] eliminates all nonlinear aberrations, and one can achieve large DA almost independent of emittance and other lattice details. For the finite length sextupoles only the second order aberrations are cancelled exactly while others remain, however even in this case DA is still large enough. Additionally, the third order aberrations (octupole-like) can be mitigated by correction scheme described in [3], allowing dynamic aperture increase.

Below, we discuss a compact low emittance optical cell providing –*I* condition in both planes. The cell can be used as a block to build storage ring with desired energy, emittance, circumference, straight sections number and length, etc. Finally, we demonstrate our approach in a 3 GeV storage ring structure with horizontal emittance of $\varepsilon_x = 10$ pm which is a diffraction limit for 1 Å radiation). Details of this study can be found in [4].

CELL DESIGN PRINCIPLES

Task Definition

We start with the most appropriate for the low emittance cell - TME lattice [5]. In TME cell we cannot obtain the –*I* transform for both transverse coordinates, so we modify it slightly by splitting the magnet and introducing the quadrupole q1 between two magnet halves (Figure 1).

A Split Magnet TME (SM-TME) cell is still compact, provides low emittance but optically more flexible than original TME.

Practical constraints we use below for numerical example include: (1) the ring energy is 3 GeV, (2) the magnet length (one-half of the original TME magnet) is L = 0.5–2 m; the quadrupole and sextupole length are ≤ 0.3 m, (3) maximum quadrupole and sextupole strength are ~100 T/m and ~5000 T/m^2 respectively, which seems suitable for 25–30 mm poles bore diameter magnets.

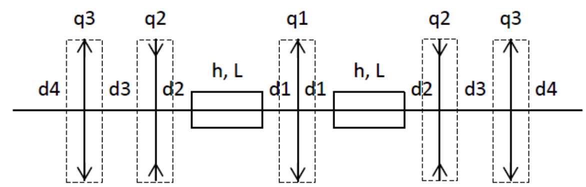

Figure 1: Split Magnet TME cell.

To build the whole ring we follow a traditional approach proposed by D.Einfeld [6], when several TME cells compose the MBA (Multiple Bend Achromat) super-cell, with dispersion suppressors at the both ends. In our case we decided to have 45×5BA super-cells with the following angle formula: 45×(5×1.46°+0.7°) = 360°, where the regular and the dispersion suppressor magnet bends are 1.46° and 0.7° respectively.

After development and detailed study of the Split Magnet TME structure we have found that similar cell was proposed earlier for KEKB arcs by H.Koiso and K.Oide [7] however they did not impose a low emittance condition combined with maximum cell compactness. Therefore we believe that systematic exploration of the cell attractive for diffraction limited storage rings is, nevertheless, interesting and useful.

Below, p_i denotes the inverse focal length of the quadrupole q_i: $p_i = (GL)_i / B\rho = -1/f_i$. At first we construct the $-I_{x,y}$ transformer in the horizontal and vertical planes by adjusting quadrupoles q_2 and q_3 for free (p_1, L) and fixed (d_3, d_4). As an option, we consider relaxed condition of only horizontal $-I_x$ transformer and impose the vertical stability conditions to this cell.

$-I_{x,y}$ Transformer

The simplest conditions for the –*I* transform include $\beta_{x,y1} = \beta_{x,y2}$, $\alpha_{x,y1} = \alpha_{x,y2}$, $\Delta\mu_{x,y1-2} = n_{x,y}\pi$, for $n_{x,y}$ odd. More sophisticated schemes can be found in [8].

Analysis shows that the –*I* solutions roughly correspond to the following simple estimation

$$L \cdot |p_1| \approx 2\phi \cdot \cot\phi \approx (\phi << 1) \approx 2 .$$

For the example we discuss below (2ϕ = 1.46°) the exact product is $L \cdot |p_1| = 1.999$. Shorter magnet requires stronger q_1 but, fortunately, even for rather short magnets

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ISBN 978-3-95450-132-8

($L < 0.5$ m) $|p_1|$ is still technically acceptable. Below we shall explore the following parameters $2\phi = 1.46°$, $d_3 = 1.13$ m, $d_4 = 0.5$ m with the solutions providing $-I_{x,y}$ over the SM-TME cell equal to $L = 1.824$ m and $p_1 = \pm 0.79$ m^{-1}. For such inverse focal length the central quadrupole parameters are $l = 0.1$ m, $G = 84$ T/m at 3 GeV which satisfy the modern small-aperture magnet technology. Our analysis shows that for original TME cell ($p_1 = 0$) there are no solutions for $-I$ transformer in both planes; however for one plane only (either horizontal or vertical) these solutions exist.

Emittance Minimization

A requirement for the $-I_x$ transformer in the horizontal plane unambiguously defines the dispersion function in the midpoint of the central quadrupole q_1:

$$\eta_1 = L(1-\cos\phi)/\phi \quad (1)$$

An intrinsic feature of the $-I$ cell is optical transparency: periodic lattice functions exist for any initial betas. Therefore horizontal beta profile can be chosen from emittance minimization requirement only. Given horizontal beta β_{x1} in the centre of q_1 the following minimum emittance was found

$$\varepsilon_{x\min} = \varepsilon_{TME} \cdot \frac{3}{2\sqrt{30}} \sqrt{5 + 3(Lp_1 - 5)^2}\,, \quad (2)$$

with

$$\beta_{x1\min} = \frac{4\sqrt{2}L}{80 - 30Lp_1 + 3L^2 p_1^2}\,,$$

where ε_{TME} is minimum emittance for TME cell with 2ϕ bending angle.

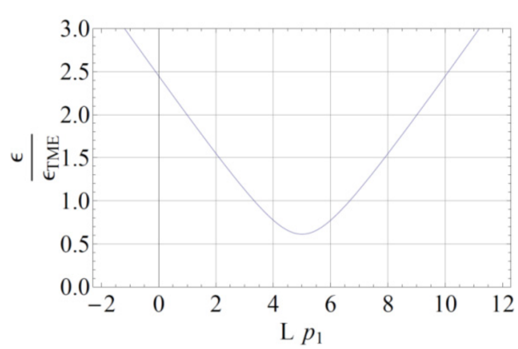

Figure 2: Ratio $\varepsilon_{SM}/\varepsilon_{TME}$ vs. $L \cdot p_1$.

Figure 2 demonstrates the plot of equation (2) providing the following conclusions: (1) for $L \cdot p_1 \approx 2$, corresponding to $-I$ transformer in both planes, the SM-TME emittance is just ~50% larger than the TME minimum. Keeping in mind that the exact TME conditions require very strong optics that can hardly be met in real life, the result is not so bad; (2) another point $L \cdot p_1 \approx -2$, which also provides $-I$ transformation in both planes, gives much larger emittance and therefore is unacceptable; (3) emittance reduction below TME minimum $\varepsilon_{SM}/\varepsilon_{TME} \approx 0.6$ is possible for SM-TME cell but requires rather strong quadrupole q_1 for reasonable magnet length $L \cdot p_1 = 5$. $-I$ transformation does not exist in this case; (4) the point $L \cdot p_1 = 0$ corresponds to TME cell with $-I_x$ superimposed and has emittance 2.5 times larger than pure TME.

Sometimes large dynamic aperture is needed in one direction only (usually in horizontal). In this case the optical matching conditions are more relaxed, and we can consider, as before, $-I_x$ with the emittance minimization, but instead of $-I_y$ we only require the vertical stability, which has usual meaning $-1 < \cos\mu_y < +1$.

TEST LATTICE

In this section we apply described approach to construct the lattice cell with 10 pm emittance and $-I_{x,y}$ transformation.

Lattice Cell

Emittance minimization together with $-I$ transformation in both directions defines dispersion function in the cell in the unique way (1). Cell initial beta functions are defined by two factors: minimum emittance (horizontal beta) and minimum strength of chromatic sextupoles (both betas). To compensate the natural chromaticity we installed sextupoles close to q_3 (horizontal) and q_2 (vertical) and found their integrated strengths as functions of the initial betas values. Horizontal sextupole strength is small in the range of the beta values providing minimum emittance, while vertical sextupole strength is low for very small β_{y0}. In our design we fixed the following parameters for the chromatic sextupoles (h and v): 0.15 m, 4450 T/m^2 and 0.3 m, -5350 T/m^2. The gradients seem not too large for the aperture 20÷30 mm. Figure 3 shows the lattice cell magnets and optical functions.

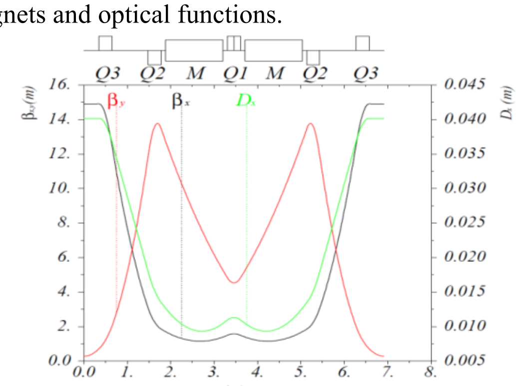

Figure 3: SM-TME cell for ultimate storage ring.

Lattice Super-cell Structure

Described cell provides conditions for both horizontal and vertical $-I$ transformers but can accommodate only one sextupole pair. To correct chromaticity in both planes we need several cells with horizontal and vertical sextupole pairs respectively. Below we use 5BA super-cell with two chromatic correction sections in each direction as it is shown in Figure 4 without the dispersion suppressors, which are trivial.

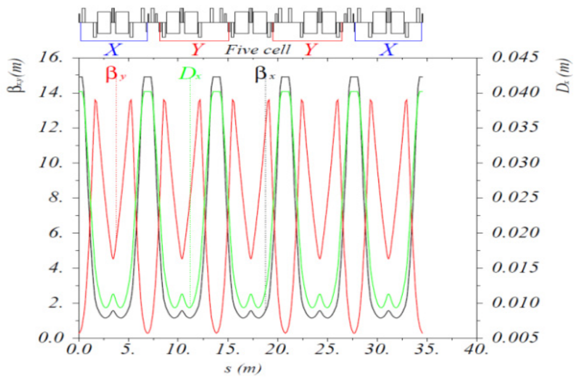

Figure 4 Five-cell superperiod with two horizontal sextupole pairs (denoted X) and two vertical ones (denoted Y).

To complete the ring design we need a number of straight sections connecting the super-cells. Instead of particular straight sections which parameters depend on type of equipment they will accommodate, we connect the super-cells by Twiss matrix with phase advances, which can be adjusted to get the maximum DA. Main parameters of the cell, 5BA and super-cell are listed in Table 2.

Table 2: Main Parameters of the Lattice at 3 GeV

Parameter	SM-TME	5BA	Super-cell
Length [m]	6.92	34.59	54.04
$\mu_{x,y}/2\pi$	0.5/0.5	2.5/2.5	3.463/3.418
ε_x[pm]	10.48	10.48	10.13
τ_x/τ_e[ms]	0.49/0.25	0.49/0.25	0.74/0.37
$10^4\times\sigma_E/E$	2.5	2.5	2.5
ξ_x/ξ_y	-1.05/-1.57	-5.25/-7.87	-7.05/9.75

Dynamic Aperture

For kick-like sextupoles arranged in the –*I* pairs all geometrical aberrations are cancelled exactly. For the real length magnets the second order is vanished but higher orders remain and cause dynamic aperture degradation. Additional mitigation for the next third order effects is possible (see, for instance, [7], where low strength sextupole correctors increase DA by ~30-50%), however here we intend to make a point of the bare –*I* chromatic section advantages and do not apply any correction schemes.

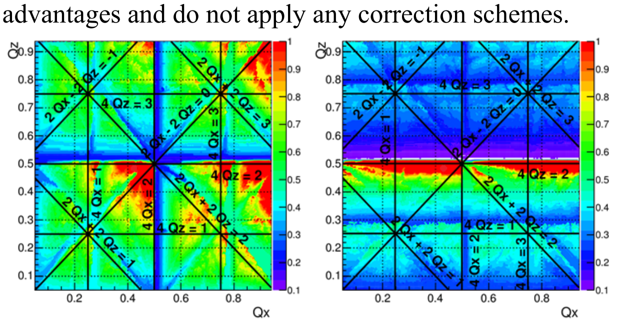

Figure 5: Horizontal (left) and vertical normalized DA as a function of the period fractional tunes.

With cell parameters fixed the only knobs affecting DA are tunes. We varied period phase advances by 2π by the matrix, connecting two adjacent super-cells and tracked DA in usual way for 1000 turns. The observation point betas were fixed at $\beta_{x,y} = 10$ m for the whole tune plane.

The tracking results are shown in Figure 5 and clearly demonstrate that all third order resonances are effectively suppressed by the sextupole pairs and only the fourth order resonances (octupole-like) are visible. The largest DA is in the corner below the half-integer resonances $\nu_{xp} = n/2$ and $\nu_{yp} = m/2$.

Figure 6 shows the transverse dynamic aperture for the chosen tune point; the aperture exceeds ±2 cm horizontally and 3 cm vertically without any additional optimization.

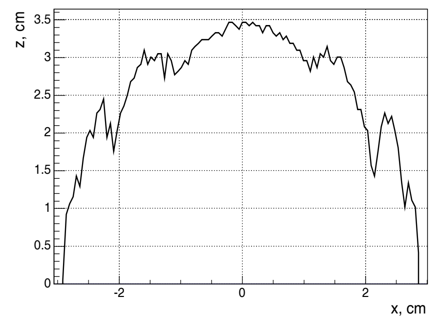

Figure 6: DA of the 10 pm emittance storage ring at $\beta_{x,y} = 10$ m

CONCLUSION

Compact Split Magnet TME cell with low emittance providing the –*I* transformation in both planes was studied in details.

The test storage ring based on the SM-TME cell was constructed with horizontal emittance of 10 pm. The –*I* sextupole pairs correcting chromaticity provided large dynamic aperture (more than ±20 mm horizontally and ±30 mm vertically at $\beta_{x,y} = 10$ m). Additional sextupole or/and octupole correctors can increase DA even more.

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ISBN 978-3-95450-132-8