Diamagnetic Plasma Confinement in Linear Traps

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Abstract:
Near the β-limit of plasma equilibrium the magnetic flux tubes typically become quite flexible, allowing the ballooning instability, and inflatable, allowing the mirror instability. Inflating a flux tube in a mirror trap means that its volume increases, while the cross-section of the magnetic nozzles at its ends remains essentially constant. This should lead to an increase in the axial confinement time. The "diamagnetic" mode of plasma confinement corresponds to the ultimate inflated state, when the main volume of a flux tube resembles a low-field "bubble", while the axial plasma confinement is significantly enhanced. This type of equilibrium occurs above the threshold of the mirror instability and corresponds to its saturated state (saturation being due to the finite number of particles in the trap). If the vacuum magnetic field of the trap has a uniform stretch near its minimum, the roughly cylindrical "bubble" with non-paraxial ends will occupy just this stretch. Due to diamagnetic reduction of the internal field the cross-field transport inside the "bubble" increases, just as the axial losses are stifled. The radial structure of the equilibrium "bubble" is then determined by the balance of particle and energy fluxes. It has a rather sharp boundary, of the order of one ion larmor radius, so that the finite-larmor-radius (FLR) effects are quite strong. The confinement time can be found from solution of the system of equilibrium and transport equations and is shown to be $\tau_E \approx \sqrt{\tau_\parallel \tau_\perp}$. This means that the diamagnetic confinement could allow construction of relatively short linear traps as fusion reactors, provided that the ballooning instability can be somehow suppressed. A variant of the conducting-shell stabilization of the boundary in conjunction with FLR suppression of localized modes can do the trick.

1 Introduction

The linear mirror traps are mostly written off by the fusion community and the funding authorities as possible contenders for prototyping a fusion reactor. The reasons for this can be found in student textbooks and even a few years ago seemed quite convincing and valid: the electron heat flux along open field lines in a simple mirror is extremely high, while the technologies of ambipolar plugs and thermal barriers, developed in tandem mirrors, are too costly and complicated, and, besides, never worked as well as intended. The tandem-mirror program in US was completely terminated in 1987. The only surviving tandem mirror is GAMMA-10 in Japan [1] that is today engaged in the plasma-material-interaction (PMI) research. However, two specialized linear traps, the gas-dynamic trap, GDT [2], and the multiple-mirror trap, GOL-3 [3], in the Budker Institute in Novosibirsk were able to obtain new important results that may change the fate of linear traps for fusion. Both these traps bear legacies of theta-pinches, namely, the highly-collisional regimes of axial losses, and thus are naturally compatible with high-density plasmas. Besides, an enhanced scattering due to ever-present plasma turbulence has little (GDT) or even a stifling effect (GOL-3) on axial losses. In the case of GDT, the gas-dynamic outflow rate is already the upper limit of axial ion losses, since the loss-cone is full. For the multiple-mirror confinement an enhanced ion scattering in parallel velocity is even desirable, since the optimum period of the field corrugation should be close to the mean free path, which is too long at fusion conditions without turbulence.

Currently, the main drawback of linear gas-dynamic traps as fusion reactors is in the geometry: while it is easy to construct an axially symmetric tube-like reactor, it has to be very long and thin. The reasons are as follows: the fusion power is proportional to the plasma volume and squared density, n, while the lost power is proportional to the plasma cross-section in mirror throats and density. The plasma occupies a magnetic flux-tube, so that cross-sections of the mirror throats and of the active zone are related as the ratio of the magnetic fields, i.e., the mirror ratio $R = B_m/B_0$. As a result, $Q_{DT} \propto nRL$. Now the maximum density as well as the mirror ratio are related to the maximum attainable confining magnetic field. Indeed, in the paraxial approximation the equilibrium is limited by β: $n \propto \beta B_v^2$, where B_v is the confining (vacuum) field in the active zone, while the magnetic field within the plasma is reduced as $B_0 = B_v\sqrt{1-\beta}$. Thus

$$Q_{DT} \propto LB_vB_m\frac{\beta}{\sqrt{1-\beta}}. \qquad (1)$$

It follows that both the mirror field, B_m, and the confining field, B_v, should be chosen as high as technically possible, while the plasma radius can be made small as long as transverse losses stay less than axial. This last requirement is actually determining the length-to-radius ratio of the optimized reactor, $L/a \gg 1$, and its fusion power, W_F. The pure gas-dynamic scheme leads thus to $L > 5km$ and $W_F > 10GW$, which is clearly unacceptable. Still, the successful GDT design with sloshing ions may be used for a neutron driver of nuclear waste burner or a hybrid reactor [4].

Improvement of efficiency of mirror plugs may be in theory sufficient for construction of a gas-dynamic fusion reactor with reasonable length and power. This approach can be based on the multiple-mirror scheme as in the GDMT project [5], or on the active-helical-mirror scheme that will be tested by SMOLA device [6]. Still, in order to pretend to burn advanced fuels with low reactivity, the open traps need to increase the volume of reacting plasma without increasing axial losses, and this opportunity is offered by the very interesting β-dependent factor in Eq.(1). As $\beta \to 1$ the effective mirror ratio of the trap starts to grow rapidly due to diamagnetic radial expansion of the flux-tubes (see Fig.1). The increase in fusion efficiency due to β can be translated into a corresponding decrease in L, i.e., a compact fusion reactor based on a linear trap may become possible.

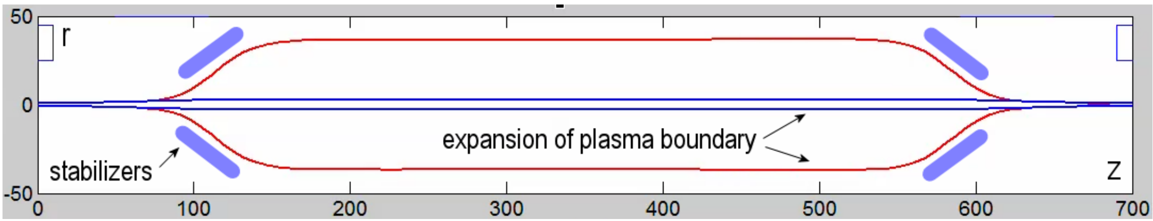

FIG. 1: Expansion of flux tubes at high β leads to corresponding increase in the effective mirror ratio of a linear trap. If there is a quasi-uniform patch of the vacuum field at the bottom of the magnetic well, the resulting "bubble" will be roughly cylindrical. The plasma boundary at cylinder ends needs stabilization.

The idea that equilibria with high β may have improved axial confinement in configurations with magnetic mirrors is rather old. For spindle cusps it is around for 60 years [7], and just a decade less for theta pinches [8]. Since the magnetic structure of classical linear theta-pinches is essentially identical to axisymmetric mirror traps, the scaling $Q_{DT} \propto 1/\sqrt{1-\beta}$, derived by Taylor and Wesson, applies to all mirrors. Furthermore, since the enhancement of confinement is essentially due to geometric factor, the diamagnetic confinement mode is half-way between the field-reversed configuration (FRC) and the linear gas-dynamic trap. It certainly improves on the standard GDT confinement by offering a greatly reduced reactor size at the cost of a more tricky MHD stabilization. The energy confinement in the diamagnetic "bubbles" will be probably worse than that in FRCs, but initial estimates suggest that a less stringent maintenance of equilibrium and stability will be required.

For realization of the $\beta \to 1$ limit we should address some difficult questions. How large the $1/\sqrt{1-\beta}$-factor can be made in realistic equilibria? Even if it is large, it may not be as beneficial as expected. While the magnetic flux is expelled from the plasma core, the transverse transport is also bound to increase up to infinity as the ions become unmagnetized. Will the gain in axial confinement be sufficient to justify the increased radial diffusion? The existence of an equilibrium state does not guarantee that it can be realized in experiment. It should be made stable at least to the ideal MHD modes. This is also very tricky at high β, as the predicted limit of ballooning stability in linear traps may be significantly lower than 1 [9].

2 Equilibrium

Our aims here are very specific: to study the high-β limit of equilibrium in long and thin axially symmetric traps. It appears that this particular limit typically results in significant growth of the initially thin plasma in radius. That expansion is axially localized around the minimum of the vacuum magnetic field.

The transverse pressure of plasma in a linear trap can be approximated as $p_\perp \approx p_\perp(\psi, B)$, where ψ is the flux function labeling magnetic surfaces, and B is the local magnetic field strength. Using the minimum of the magnetic field on a field line $B_0(\psi)$ for normalization of field, it can also be rewritten as $p_\perp = p_\perp(\psi, R)$, where $R(\psi, \ell) = B/B_0$ is the local mirror ratio of the magnetic field. The equation of paraxial equilibrium looks like $B_v^2 = B^2 + 8\pi p_\perp$ where $B_v(\vec{r})$ is the confining vacuum magnetic field. Let's divide it by the square of the minimum vacuum magnetic field on a field line, B_{v0}^2, and normalize the pressure by its value at the field minimum. We get

$$R_v^2 = (1-\beta)\, R^2 + \beta P\,(R)\,, \tag{2}$$

where $R_v(\psi, \ell) = B_v/B_{v0}$ is the mirror ratio of the vacuum field, $\beta(\psi) = 8\pi p_\perp(\psi, 1)/B_{v0}^2$ is the β-value at the minimum of the field on a given field line, while $P(\psi, R) = p_\perp(\psi, R)/p_\perp(\psi, 1)$ is the normalized profile of pressure along the field line. This equation should be solved for $R(\psi, \ell)$ with restriction $R > 1$, while we are particularly interested in the case $1 - \beta \ll 1$. Let's assume that we are dealing with a typical mirror with a monotonically growing field from its center. Then the left-hand side grows with ℓ, $\partial R_v^2/\partial \ell \geq 0$, and to find a solution for all ℓ we should have a growing right-hand side too, then the solubility condition becomes

$$\partial P\,(R)/\partial R^2 \geqslant -(1-\beta)/\beta. \tag{3}$$

In linear traps there are always some areas, where the pressure derivative along the field line is negative, since the pressure should be higher inside of the trap than in the mirror throats. Looking at Eq.(3), one can see that such decreases of pressure are restricted, and at $\beta \to 1$ they are entirely prohibited by the paraxial equilibrium. According to Kotelnikov [10] the equilibrium solutions of the paraxial equilibrium equations can be piecewise continuous, while the points of discontinuity can be interpreted as non-paraxial areas. At high β the function $R(\ell)$ becomes discontinuous. It is comprised of two (or more) continuous intervals: the "bubble" branch, where condition (3) is satisfied at low $R \sim 1$, and the outer branch, where the same condition is satisfied at large R only.

In the most typical quasi-isotropic case the function $P(R)$ is monotonously decreasing from 1 to 0, and one can approximate the pressure profile by parabola: $P(R) \approx 1 - \delta^2 \left(R^2 - 1\right)^2$. Then the equilibrium equation becomes quadratic and has a positive solution for $R^2 - 1$ only if $D = (1-\beta)^2 - 4\delta^2\beta\left(R_v^2(\ell) - 1\right) \geqslant 0$. It follows that the solution is discontinuous, and the length to discontinuity is defined by

$$R_v^2(\ell_d) = 1 + \frac{(1-\beta)^2}{4\delta^2\beta}. \tag{4}$$

One can see that $R_v(\ell_d) \to 1$ with $\beta \to 1$, i.e., the "bubble" branch of solution collapses to the bottom of the magnetic well. This behavior is different in presence of sloshing ions. Then the "bubble" branch is finite-length, $R_v^2(\ell_d) \to P_r > 1$, where P_r is the maximum of the curve $P(R)$ [11]. However, formation of non-paraxial ends of a "bubble" will soon cause the pressure anisotropy to relax, so that $P_r \to 1$ and the quasi-isotropic limit will be restored.

The function $R_v(\ell)$ is extremely important for shaping the equilibrium. It describes the form of the magnetic well of the vacuum field of the trap along field lines. In particular, one can design a linear trap with a finite-length patch of uniform field at the well bottom. Then the branch of equilibrium that exists only at $R_v = 1$ becomes extended into a cylinder. The "bubble" length can thus be prescribed via the form of the vacuum field. The bubble edges at high β will coincide with the ends of the uniform-field patch, so that we will be able to place there some equipment for MHD stabilization. Indeed, in the cylindrical case the interchange source term is finite only at the ends of the cylinder, while in the middle uniform patch the plasma is marginally stable. Thus, by increasing length we can add plasma without worsening stability. Furthermore, by placing localized stabilizers directly at the ends of the cylinder it should be theoretically possible to suppress even the edge ballooning modes.

3 Radial structure and transport

Let's try to describe an axisymmetric steady-state equilibrium taking into account diffusion of the external magnetic field into the cylindrical "bubble". If everything is stationary, this means that there is a steady flux of plasma through the magnetic field from the inside, $F_\perp$, and its radial divergence is closed by axial plasma losses through mirrors, $F_\parallel$. The flux continuity equation is

$$[rF_\perp]'_r + rF_\parallel = 0. \tag{5}$$

The transverse plasma flux is due to diffusion of magnetic field,

$$F_\perp = \frac{c^2}{4\pi\sigma}\frac{\partial n}{\partial r} \equiv D_\perp n'_r, \tag{6}$$

where σ is the effective plasma resistivity, $n(r)$ is the number density. The parallel flux density, $F_\parallel$, depends on the confinement regime. If it is gas-dynamic, then $F_\parallel \approx nC_s/RL$, where C_s is the sound speed, R is the effective mirror ratio, and L is the length of the "bubble".

The continuity equation, (5), should be supplemented by the equilibrium equation. In the cylindrical case it is, fortunately, quite simple:

$$B^2 + 8\pi p = B_v^2, \tag{7}$$

where p is the plasma pressure, B is the local magnetic field, and B_v is the external (confining) magnetic filed.

For the sake of simplicity let's assume that $p = 2nkT$, $T = const$, and that the outflow regime is gas-dynamic. Then our system can be rewritten in terms of $\beta(r) = 8\pi p/B_v^2$ as[11]:

$$\left[\frac{\beta\beta' r}{1-\beta}\right]' = \lambda^{-2} r\beta\sqrt{1-\beta}. \tag{8}$$

Here the characteristic linear scale $\lambda = \sqrt{D_\perp \tau_\parallel/2}$ can be interpreted as the skin depth of the magnetic field by the time of the axial plasma outflow from the vacuum field of the trap. For gas-dynamic traps it is normally very small. Qualitatively, solution for the

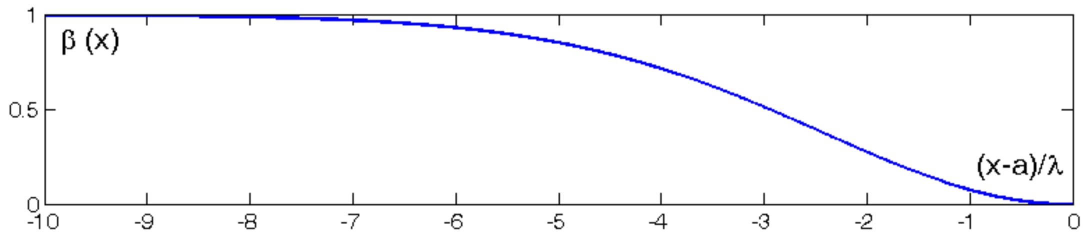

FIG. 2: The radial structure of the boundary layer of the "bubble" in the MHD slab-transport model. There is vacuum ($\beta = 0$) beyond the $r = a$ surface, and the low-field interior ($\beta \approx 1$) to the left.

"developed bubble" looks as follows: over most of the radius $\beta \approx 1$, while the transition layer from $\beta \approx 1$ to $\beta \approx 0$ (the boundary) has the characteristic radial scale $\lambda \ll r$. This structure can be successfully described in the slab approximation, i.e., we set $r \approx a = const$, $\lambda^{-1} y/a = f$, and introduce the normalized radial coordinate $x = (r - a)/\lambda$.

Equation (8) can be rewritten as a system

$$f'_x = \beta\sqrt{1-\beta}, \quad \beta'_x = f\,(1-\beta)/\beta, \tag{9}$$

with boundary conditions $\beta(\infty) = 0$, $f(-\infty) = f_0$, where f_0 is the normalized source of ions. It can be partially integrated:

$$f^2 = \frac{2}{15}\left[8 - \sqrt{1-\beta}\left(8 + 4\beta + 3\beta^2\right)\right], \tag{10}$$

so that $f_0 = 4/\sqrt{15} \approx 1.03$. Substituting Eq.(10) into the second line of Eq.(9), we get the radial structure as

$$\beta' = -\frac{4}{\sqrt{15}}\frac{1-\beta}{\beta}\sqrt{1 - \sqrt{1-\beta}\left(1 + \frac{\beta}{2} + \frac{3\beta^2}{8}\right)}. \tag{11}$$

The plasma boundary is quite "rigid", i.e., there is no pressure at all beyond $r = a$, see Fig.2.

The particle confinement time is the ion content in the "bubble" divided by the flux of particles that are lost from it in a stationary state. The flux can be found using f_0, so that

$$\tau_n = \frac{\pi a^2 L n}{\Phi} = \frac{a}{\lambda f_0}\tau_\parallel \approx \sqrt{2\tau_\perp \tau_\parallel}, \tag{12}$$

where $\tau_\parallel$ is the axial confinement time in the vacuum field, and $\tau_\perp$ is the diffusion time over the full "bubble" radius. If the axial electron recycling is limited by the properly designed expanders, the energy loss is proportional to the particle loss [12], so that $\tau_E \approx 3\tau_n/8 \approx \sqrt{\tau_\perp \tau_\parallel}$.

The whole process can be approximately described as follows. Deep within the "bubble" the radial diffusion dominates, while the axial loss is vanishingly small. In fact the ions may not be magnetized inside of the "developed bubble" at all, having almost straight trajectories. All of the radial confinement is concentrated in the relatively thin boundary layer of width $\sim 6\lambda$. However, due to finite magnetic field within the boundary layer, the effective mirror ratio is also finite, so that the axial losses appear. In a unit of time the "bubble" looses particles from the layer of width λ and radius a by axial outflow, hence $\tau_n \sim a\tau_{\|}/\lambda$.

The simplified model of transport used in this section assumes that the axial outflow regime is gas-dynamic, which is justified inside of the "bubble" due to very large mirror ratio. However, the resulting width of the "bubble" boundary is unreasonably small, typically smaller than the ion Larmor radius. This is understandable, since the gas-dynamic approximation can break within the boundary, where the axial loss regime can be kinetic with value much smaller than the gas-dynamic estimate. In case of longer, kinetic $\tau_{\|}$, the boundary layer should become wider, at least of the order of the Larmor radius. This will lead to the corresponding increase of the diffusion time and the overall improvement of the confinement estimate.

A boundary layer with width comparable to the ion Larmor radius will experience strong stabilizing influence of the finite-larmor-radius (FLR) effects. This should help with suppression of short-scale ballooning modes having transverse scales less than the distance to the shell stabilizers [9]. In general, stability of the boundary layer should be very similar to that in FRCs, where the strong FLR effects are believed to suppress most short-scale modes [13].

4 Conclusion

A new scheme for confining high-β fusion plasmas in a linear trap is described [11]. It promises huge improvement of confinement quality as compared to the gas-dynamic scheme. A stable confinement of the $\beta \approx 1$ plasma cannot be easy, but there seems to be a straightforward way to use the conducting-shell stabilization method that is shown to work for FRCs [13]. Although there is still no detailed theory of stability and transport, it is probably worthwhile to attempt an experimental check of the predicted "bubble" formation and of the related improvement in confinement time. Such initial concept-exploration experiments are now in the planning stage in the Budker Institute of Nuclear Physics in Novosibirsk.

Acknowledgments

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References

[1] TAMANO, T., "Tandem mirror experiments in GAMMA 10", Physics of Plasmas 2 (1995) 2321.

[2] IVANOV, A. A., PRIKHODKO, V. V., "Gas-dynamic trap: an overview of the concept and experimental results", Plasma Phys. and Contr. Fusion 55 (2013) 063001.

[3] ARZHANNIKOV, A.V., et al., "Experimental Study of the Dynamics of Neutron Emission from the GOL-3 Multimirror Trap", Plasma Physics Reports 32 (2006) 94.

[4] ANIKEEV, A. V., et al., "Progress in mirror-based fusion neutron source development", Materials 8 (2015) 8452.

[5] BEKLEMISHEV, A., et al., "Novosibirsk project of gas-dynamic multiple-mirror trap", Fusion Science and Technology 63 1T (2013) 46.

[6] POSTUPAEV, V. V., et al., "Helical mirrors for active plasma flow suppression in linear magnetic traps", Fusion Eng. and Design 106 (2016) 29.

[7] BERKOWITZ, J., et al., "Cusped geometries", Theoretical and Experimental Aspects of Controlled Nuclear Fusion, (Proc. Second U.N. Intern. Conf. on Peaceful Uses of Atomic Energy, Geneva, 1958) Vol.31, United Nations, Geneva (1958) 171.

[8] TAYLOR, J. B., WESSON, J. A., "End losses from a theta pinch", Nucl. Fusion 5 (1965) 159.

[9] KAISER, T. B., PEARLSTEIN, L. D., "Finite Larmor radius and wall effects on the M=1 ballooning mode at arbitrary beta in axisymmetric tandem mirrors", Phys. Fluids 28 (1985) 1003.

[10] KOTELNIKOV, I. A., "Equilibrium of a high-β plasma with sloshing ions above the mirror instability threshold", Fusion Science and Technology 59 1T (2011) 47.

[11] BEKLEMISHEV, A. D., "Diamagnetic "bubble" equilibria in linear traps", Physics of Plasmas 23 (2016) 082506.

[12] RYUTOV, D. D., "Axial electron heat loss from mirror devices revisited", Fusion Science and Technology 47 (2005) 148.

[13] BINDERBAUER, M. W., et al., "A high performance field-reversed configuration", Physics of Plasmas 22 (2015) 056110.