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# Influence of Coulomb fields on formation of emittance in photoguns

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**Abstract.** A theoretical analysis of the formation of longitudinal and transverse emittance of bunches of charged particles in photoguns under the action of repulsive Coulomb fields is carried out. Numerical calculations have been performed to compare the characteristics of cylindrical and arbitrary-shape bunches with a uniform and Gaussian distribution of the charge density over the bunch volume. The problem of transverse dynamics of non-axisymmetric bunches is solved analytically. Comparison of the efficiency of the proposed numerical-analytical model with calculations using the Astra code is carried out. The obtained theoretical and numerical results can be useful in designing photoguns and choosing the optimal values of the parameters that determine the main characteristics of photoinjectors.
**Keywords.** photogun, electron bunch, emittance, Coulomb field.

## 1. Introduction

The modern colliders require intensive high-quality electron beams. The beam parameters for such projects, as SKEKB, FCC or Super C-Tau Factory need the beam charge of a few nC (1–6.5 nC), the energy spread less than 1%, the normalized transverse emittance less than 20 mm-mrad. The DC electron guns with thermionic cathode allow to achieve the beams with such charges, but the low emittance and the energy spread are difficult to obtain with such guns. The RF guns with metallic photo cathodes look like promising electron sources. These photo cathodes allow to achieve the beam length equals to the laser wave length. The modern lasers can have the wavelength less than 1 ps. The high accelerating gradient of the RF guns provides the relativistic energies of the beam at the end of the guns. All these leads to high quality of the beam emitted in the RF guns.

In spite of the advantages of the RF guns, there are a lot of challenges, which should be resolved. One of these is the high charge of the beam, and especially the effect of them on the beam emittance. Since 1985 such sources have become more and more popular. Photoguns are promising sources of electron bunches with a small transverse emittance [1–4]. In addition, they allow the extraction of much higher current densities comparing to traditional thermionic guns. This is achieved by the fact that the photoelectrons initiated by the laser pulse immediately enter the strong accelerating field of the first cavity adjacent directly to the photocathode. Such sources are required to have a maximum bunch charge of several nC at a sufficiently low transverse emittance of several mm-mrad. These requirements are, to a certain extent, contradictory, in connection with which a detailed study of the main parameters of the gun for the formation of the emittance seems to be urgent. For a charge of several nC, the Coulomb field of the bunch begins to dominate. In this case, the use of analytical techniques is possible only in the ultra relativistic energy range using additional assumptions about the shape and size of the bunch, and at the stage of accelerating the bunch from the photocathode, only a numerical study of the dynamics is possible.

## 2. Taking into account the Coulomb field of a bunch

The repulsive effect of the Coulomb fields of an electron bunch is usually considered in a coordinate system moving with the bunch. In this case, it is assumed that all electrons in the cross section move with the same velocity $v$, determined by the accelerating field with the amplitude $E_z$. In this system, the electromagnetic field $\left(E_x^{'}, E_y^{'}, E_z^{'}\right)$ turns out to be purely electrostatic. After analysis, the fields in the laboratory system are determined by the inverse Lorentz transformation:

$$E_x = \gamma E_x^{'},\ B_y = \gamma \frac{v}{c^2} E_x^{'},\ E_z = E_z^{'}.$$

In further calculations, for simplicity, we will omit the prime referring to the moving coordinate system. We take the $\gamma(z)$ dependence from the analysis of the longitudinal dynamics of the bunch, which is the purpose of a separate publication. The potential $\varphi(u)$ of a bunch is determined by integrating its charge density $\rho(s)$ over the bunch volume $V$:

$$\varphi(\mathrm{u}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(s)}{R_{us}} dV, s \in V \tag{1}$$

where $R_{us}$ is the distance between the observation point $u$ and the integration point $s$, $\varepsilon_0$ is the dielectric constant of vacuum.

*2.1. Cylindrical bunch with uniform charge distribution*

If one assumes that the bunch is uniformly charged, then the charge density $\rho$ can be taken outside the integral sign and replaced by the total charge of the bunch $Q/V$, where $V$ is the bunch volume. Next, you should make some assumptions about the shape of the bunch. Consider a uniformly charged cylinder of radius $R_b$ and of length $L_b$, whose center is located at the origin, $V = \pi(R_b)^2 L_b$. Integration over the azimuthal variable in the cylindrical coordinate system ($r$, $z$, $\theta$), we obtain an expression for the potential [5]:

$$\varphi_b(r,z) = \frac{\mathrm{Q}}{\pi V \varepsilon_0} \int_0^{R_b} r' dr' \int_{-L_b/2}^{+L_b/2} dz' \frac{K(t)}{A},\ A^2 = (r+r')^2 + (z-z')^2,\ t = 2\frac{\sqrt{rr'}}{A}, \tag{2}$$

Here $K(t)$ is a complete elliptic integral of the first kind. Differentiation of the potential gives the components of the electric field of the bunch:

$$E_{r,b}(r,z) = \frac{Q}{2\pi r V \varepsilon_0} \int_0^{R_b} r' dr' \int_{-L_b/2}^{+L_b/2} \frac{1}{A} \left[ K(t) - E(t) \frac{(r')^2 - r^2 + (z-z')^2}{(r-r')^2 + (z-z')^2} \right] dz', \tag{3}$$

$$E_{z,b}(r,z) = \frac{Q}{\pi V \varepsilon_0} \int_0^{R_b} r' dr' \int_{-L_b/2}^{+L_b/2} dz' \frac{(z-z')E(t)}{A\left[(r-r')^2 + (z-z')^2\right]} \tag{4}$$

where $E(t)$ is a complete elliptic integral of the second kind. These integrals can be approximated by the polynomial formulas

In fact, the potential and components of the electric field depend on time. This dependence is determined by the fact that the ultra relativistic bunch flies along the $z$-axis with the speed of light; therefore, within the integration limits, the value of $L_b$ should be replaced by $L_b + ct$.

The radial component of the electric field will be the sum of the high-frequency field and the field of the bunch $E_r + E_{r,b}$. An expression for the azimuthal component of the intrinsic magnetic field of an ultra relativistic bunch can be obtained in a similar way. This component generates a radial force that partially compensates for the repulsive effect of the Coulomb particle field

$$B_{\theta,b}(\mathrm{u}) = \frac{\mu_0}{4\pi} \int_V \frac{\left[\vec{j_z}(s) \times \vec{R}_{us}\right]}{R_{us}^3} dV, s \in V \tag{5}$$

where $\mu_0$ is the magnetic constant of vacuum, $j_z$ is axial component of the current density of the bunch in the volume $V$, $R_{us}$ is the distance between the observation point $u$ and the source point $s$.

Expressions (1)–(3) for the potential and components of the electric field can be easily generalized to the case of an arbitrary charge distribution and an arbitrary shape of an axisymmetric bunch of particles. To do this, it is only necessary to introduce the distribution of the space charge density under the integral sign and replace the limits of integration with respect to the variables *r* and *z* so that they change within the volume occupied by the bunch. In the case of a cylindrical bunch, one can write out simplified formulas reflecting the limiting cases.

The maximum effect of Coulomb fields on the transverse dynamics of particles is expected near the photocathode, where the magnetic field of the resonator and the electric field of the bunch make a contribution to the transverse force. Strictly speaking, to calculate the self-consistent electrostatic field, including the field of the electrodes and the Coulomb field of the bunch, it is necessary to solve the boundary-value problem for the Poisson equation with boundary conditions specified on the surfaces of all electrodes. In fact, such a rigor in taking into account the boundary conditions for a self-consistent field is needed only if the distance from the center of the bunch to the electrode boundary is comparable to the size of the bunch itself. At large distances, it is sufficient to consider the field as a simple sum of the field of the electrodes and the field of the bunch in empty space. In our case, we will use a simpler model. Since the photocathode is a flat conducting surface, the radial component of the total electric field on it must be equal to zero; therefore, the boundary condition on the photocathode is automatically satisfied if the field of a specularly reflected bunch with an opposite charge, which is located on the other side of the photocathode. As the bunch moves away from the photocathode, the radial component of the bunch field will first increase, but with an increase in energy, the relativistic factor will decrease it. The most difficult to analyze the dynamics of the formation of the emittance is the region of acceleration of the bunch from the photocathode to relativistic energies. In addition, the bunch formed by the laser is not long at all; therefore, in the non relativistic energy range, it is possible to investigate the effect of Coulomb fields only numerically. In further calculations, we present data for the radial fields of the bunch in the reference system, that is, taking into account the coefficient *1 / γ2 (z)* because of the beam magnetic field.

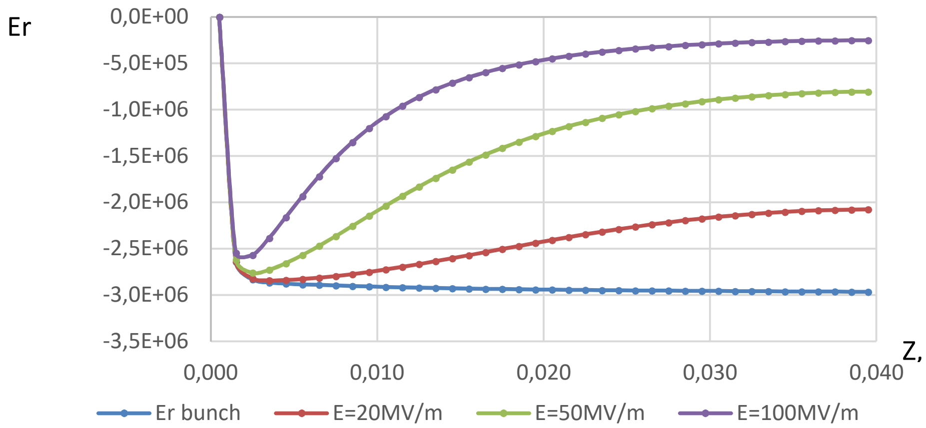


Fig.1. The radial component of the beam field of a cylindrical bunch 2.5 mm in diameter, 1 mm long, with a charge of 1.5 nC, uniform density without taking into account its own magnetic field (blue line) and taking into account the gamma factor for different values of the accelerating field *E*.

In the considered model, one more assumption is made. The direct and inverse Lorentz transformations are carried out in the same way for all particles of the bunch. This is possible if the characteristic size of the bunch is much smaller for the high-frequency field wavelength that determines the particle dynamics. Analysis of the data presented in Fig. 1 shows that at high gradients of accelerating fields, the effect of Coulomb fields on the dynamics of the formation of the transverse emittance is significant only in the nonrelativistic region of the bunch motion, that is, in the half of the first resonator adjacent to the photocathode.

*2.2. Cylindrical bunch with Gaussian charge distribution*

Let us also consider a variant of a bunch in which the charge density is described by the normal Gaussian distribution law

$$(x)=\frac{1}{(2\pi)^{1/2}\sigma}\exp\left[-\frac{1}{2}\left(\frac{x-M}{\sigma}\right)^2\right]$$

where σ is the variance and $M$ is the mean value of the distribution.

Given the characteristic dimensions of the bunch $\sigma_r$ and $\sigma_z$, in integrals (1)–(3) we replace the value $Q/V$ by the charge density $\rho(r, z)$, introducing it under the integral sign. Since 99% of the area of distribution (6) is in the range of values (–3σ, 3σ), for comparison with the previous results, the length of the bunch should be increased by 6 times, and the radius by 3 times, having assigned the previous sizes equal to the values $f\,\sigma_r$ and $\sigma_z$. In this case, the average value $M_r = 0$, and $M_z$ corresponds to the $z$ coordinate of the middle of the bunch. The values of the Coulomb fields for such a bunch are shown in Fig.2. Here, the drop in the radial field component by an order of magnitude in comparison with a bunch with a uniform charge density is explained by the fact that the curves in Figs.1–2 correspond to the fields acting on the outermost particle with the maximum radius, which is acted upon by the total charge of the bunch $Q$. For a Gaussian distribution, such a distance is equal to $3\sigma_r$, and the radial field falls off as the square of this distance.

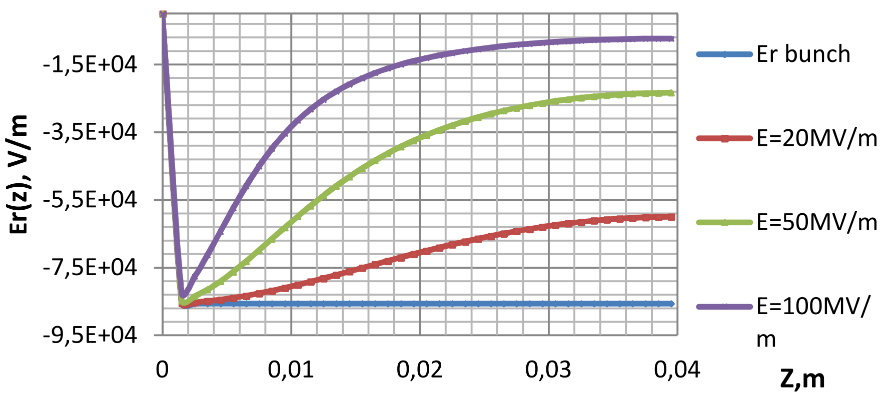


Fig.2. The radial component of the field of a cylindrical bunch with a Gaussian distribution of the charge density $Q$ = 1.5 nC for different values of the accelerating field $E$.

*2.3. An arbitrary 3D-shape bunch*

Since the laser shoots at an angle to the axis of symmetry, and the center of the laser pulse spot can also deviate from the center of the photocathode, the resulting bunch can have a non-axisymmetric shape. The transverse dynamics of such a bunch has not been studied before. Under certain assumptions, the characteristics of a bunch field of an arbitrary 3D shape and with an arbitrary charge distribution can be calculated analytically. Such algorithm was proposed in the book [6]. Let us consider the non uniform parallelepiped mesh in Cartesian coordinates $\{x_i\}\times\{y_j\}\times(z_k)$ with linear approximation for the space charge:

$$\begin{cases} \rho(x,y,z)=\Big\{\Big[\big(\rho_{i+1,j,k}(x-x_i)+\rho_{i,j,k}(x_{i+1}-x)\big)\big(y_{j+1}-y\big)+\big(\rho_{i+1,j,k}(x-x_i)+ \\ +\rho_{i,j,k}(x_{i+1}-x)\big)\big(y-y_j\big)\Big]\Big\}\big(z_{k+1}-z\big)+\Big[\big(\rho_{i+1,j,k}(x-x_i)+\rho_{i,j,k}(x_{i+1}-x)\big)\big(y_{j+1}-y\big)+ \\ +\big(\rho_{i+1,j,k}(x-x_i)+\rho_{i,j,k}(x_{i+1}-x)\big)\big(y-y_j\big)\Big](z-z_k)/\Big[(x_{i+1}-x_i)(y_{j+1}-y_j)(z_{k+1}-z_k)\Big] \end{cases} \tag{6}$$

In order to evaluate the field potential at the point $(x_0, y_0, z_0)$:

$$\varphi(x_0,y_0,z_0)=\frac{1}{4\pi\varepsilon_0}\sum_{i,j,k}\int_{x_i}^{x_{i+1}}\int_{y_j}^{y_{j+1}}\int_{z_k}^{z_{k+1}}\frac{\rho(x,y,z)}{R}dxdydz\,, \tag{7}$$

one need to calculate the four moments of space charge

$$J_1=\iiint\frac{x}{R}dxdydz\,,\ J_2=\iiint\frac{xy}{R}dxdydz\,, J_3=\iiint\frac{xyz}{R}dxdydz\,, J_4=\iiint\frac{1}{R}dxdydz\,, \tag{8}$$

where, $R = [(x-x_0)^2 + (y-y_0)^2 + (z-z_0)2]^{1/2}$. The other integrals are obtained by cyclic change of variables $x$, $y$, $z$. After changing variables $\tilde{x}=x_0-x$, $\tilde{y}=y_0-y$, $\tilde{z}=z_0-z$ and $r^2 = x^2 + y^2 + z^2$, all integrals can be evaluated analytically:

$$\begin{cases} J_1=\frac{y}{4}\Big[yz+\big(x^2+y^2\big)ln|z+r|\Big]+\frac{z^3}{6}ln|y+r|+ \\ +\frac{x^2}{2}\left[zln|y+r|+yln|z+r|-z+x\tan^{-1}\left(\frac{z}{x}\right)-x\tan^{-1}\left(\frac{zy}{xr}\right)\right]+ \\ +\frac{1}{36}\left\{6x^2z-2x^3+6x^3\left[\tan^{-1}\left(\frac{z}{x}\right)-\tan^{-1}\left(\frac{zy}{xr}\right)\right]-3y\left(y^2+3x^2ln|z+r|\right)\right\} \end{cases}$$

$$J_2=\frac{zr^3}{12}+\frac{3}{24}\big(x^2+y^2\big)\Big[zr+\big(x^2+y^2\big)ln|z+r|\Big],$$

$$J_3=\frac{r^5}{15},$$

$$J_4=xyln|z+r|+yzln|x+r|+zxln|y+r|-\frac{1}{2}\left[x\tan^{-1}\left(\frac{zy}{xr}\right)+y\tan^{-1}\left(\frac{zx}{yr}\right)+z\tan^{-1}\left(\frac{xy}{zr}\right)\right].$$

In order to get the field gradient, we should differentiate these integrals analytically, and pay attention for the singularity problem to correctly calculate the field at the boundary of the bunch [6].

### 3. Formation of the emittance of the bunch

In further considerations, we will use the ideas presented in the work of K.-J. Kim [2]. First, we introduce a scaling factor $A = \sigma_r/\sigma_z$, for a bunch with characteristic sizes $\sigma_r$, $\sigma_z$, allowing to separate a bunch in the form of a thin disk ($A > 1$) from long cigar-shaped bunch ($A < 1$). The magnitude of the normalized emittance is determined by the formula

$$\epsilon_s=\left(s^2p_s^2-sp_s^{\ 2}\right)^{1/2}$$

where the subscript $s = r$ corresponds to the transverse emittance, and $s = z$ – to the longitudinal one. Introducing the dimensionless parameter $\tau = eE_0/(2mc^2k)$, and the normalized space charge field $E_{sc}$ using the formula

$$E_{sc}(\mathrm{x},\mathrm{y},\mathrm{z}_0) = \frac{n_0}{4\pi\varepsilon_0}\tilde{E}(\mathrm{x},\mathrm{y},\mathrm{z}_0),$$

one can obtain the expression for the bunch emittance

$$\epsilon_s = \frac{\pi}{4\tau k \sin(\varphi_0)}\frac{I}{I_a}\mu_s(A)$$

Here $k$ is the wave vector, $\varphi_0$ is the bunch entry phase, $z_0$ is the axial coordinate of the bunch center, $n_0$ is the linear charge density of the bunch in the axial direction, $I$ is the peak current of the bunch, $I_a$ is the Alfven current, and a dimensionless function

$$\mu_s(A) = \left(\langle s\rangle^2\langle\tilde{E}_s\rangle^2 - \langle s\tilde{E}_s\rangle^2\right)^{1/2}$$

reflects the dependence of the emittance on the shape of the bunch and its field. For a bunch with a Gaussian charge density distribution

$$\rho(x,y,z_0) = \rho_0 \exp\left[-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma_x^2} + \frac{z_0^2}{\sigma_z^2}\right)\right],$$

where $\rho_0$ is the charge density at the center of the bunch, dimensionless functions of the form $\mu_s$, $\mu_z$ and normalized fields were evaluated analytically.

The results of our calculations are in good agreement with the data of papers [1, 2]. As an example of an essentially three-dimensional problem, consider a bunch in the form of a tri-axial ellipsoid with a Gaussian distribution of the charge density along all axes: $\sigma_x = 0.5$ cm, $\sigma_y = 1$ cm, $\sigma_z = 2$ cm, initial transverse emittance 1 mm-mrad. Longitudinal and transverse emittance for this bunch presented in Fig.3. Space charge was represented with cubic cells of 0.25 mm size. Taking into account the symmetries in $x$, $y$ and $z$ directions the total number of cells was 12 000 elements.

The limitation of the semi-analytical theory proposed by K.-J. Kim is in the fact that it considers only cylindrical bunches with a uniform or Gaussian law of space charge distribution, and this model does not take into account the fields of image charges in the vicinity of the emitter, that is, the normalized fields of bunches introduced by it does not take into account exactly the boundary condition on the cathode surface. Our proposed numerical-analytical model is free from these shortcomings.

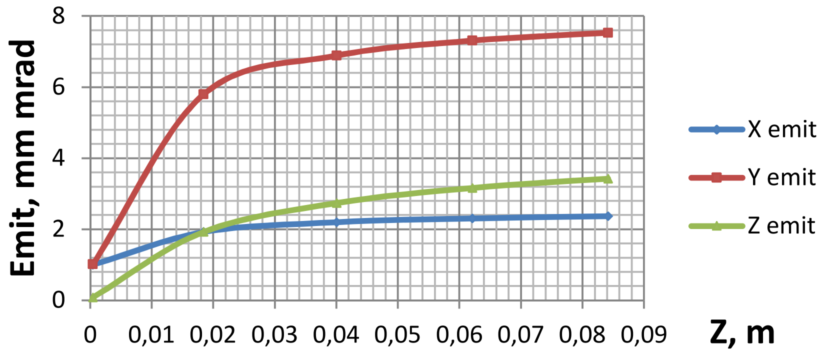


Fig.3. Vertical, horizontal and longitudinal emittance for 3D-shape bunch.

## 4. Conclusion

The performed theoretical analysis and numerical calculations give an idea of the emittance acquired by the bunch due to the influence of Coulomb fields when moving in the accelerating cavity of the photogun. The data obtained provide a relationship between the emittance not only with the geometric dimensions of the beam, but also with an arbitrary distribution of the space charge density in the bunch. It is shown that the output value of the bunch emittance is minimal for a uniform charge distribution and increases significantly when the charge is smeared for a normal distribution in the longitudinal and transverse directions. The results of numerical simulation will further serve as the basis for comparing the degree of influence of the high-frequency field of the resonators and the Coulomb fields of the bunch on the total emittance of the beam at the exit of the photogun.

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