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New Intuitive Regularizing Approaches for Deconvolution Problems

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Abstract. We propose two intuitive simple approaches that would help to make the solving scheme of deconvolution problem so as to minimize strongly the systematic component of the error of the right part of convolution equation. Specific changes are included to the block Toeplitz matrix that are defined according to the main purpose which is to minimize the influence of either discretization errors, or those originated by bound effects, or both. Wherein the problem does not lose its linear features, remaining a system of linear equations but with changed matrix (convolution operator). Two different modified solving schemes have been adapted and successfully tested in one- and two-dimensional cases with model signal distributions, respectively. The final gain of precision in the first case was 10 to 30 % by the RMS value of restored solutions' error whereas in the second case only the approach we propose yielded a satisfactory result.

INTRODUCTION

Solving one- (Eq. 1) or two-dimensional (Eq. 2) discrete deconvolution problems is a part of a great number of important practical challenges including those in optics, spectroscopy, etc. They usually occur when it is necessary to minimize the spatial resolution of detected image to subfocus values for defining the smallest features, or to improve image quality in general, i.e. to eliminate a blurring effect and stochastic noises. There are many successful examples of using results of this problem in the fluorescence and confocal optical microscopy (Refs. 1- 2), the transmitting electron microscopy (e.g., in analyzing the relative disposition of single atoms in crystal lattice structure [3], the processing of medical [4, 5] and astronomic [6, 7] images as well as common photographs and flow video [8].

$$S_i = \sum_k P_i(k) = \sum_k K_{i-k} I_k \quad , \qquad S = A * I \tag{1}$$

$$S_{i,j} = \sum_{k,l} P_{i,j}(k,l) = \sum_{k,l} K_{i-k,j-l} I_{k,l} \quad , \qquad vec(S) = A * vec(I) \tag{2}$$

where S is a one- (left) or two-dimensional (right) image corrupted by blurring and noises that are detected experimentally and characterized by a step of retrieving information, I is an image to be restored, K is a spread function, A is a convolution operator that is block Toeplitz matrix (tensor), *vec*() – an operation of vectorization [13]

This problem is related to ill-posed inverse of a high ill-conditioning level (Refs. 9, p. 3-4, 32-36). Such problems can be solved only approximately with mathematical methods of specific kind called regularizing algorithms (Refs. 9, p. 47-48). A large number of such methods have been developed that are adapted to the approximate solution of deconvolution problems. For example, at present a wide range of ordinary algorithms (Refs. 10) are used for solving typical two-dimensional deconvolution problems in deblurring photographic images, some of the algorithms being included to generally accessible software (RestoreTools for Matlab® (Refs. 11), DeconvolutionLab for ImageJ® (Refs. 12)). These methods are also successfully applied to processing astronomic and microscopic images in a

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range of cases. However, for the tasks with stricter requirements to the detailing and precision of image restoring the application of these ordinal regularizing methods may lead to some potential troubles.

Firstly, the solution of all regularizing algorithms is principally approximate (Refs. 9, p. 19) at all times. For each such method only a convergence of some kind (i.e., by the norm of some space within it or its subspace/subset) may be guaranteed (Refs. 9, p. 47-48) and only provided that it is used and set correctly. Considering this last condition it should be noted that these algorithms have only one main parameter of regularization, and the sensibility to it being selected correctly grows as the ill-conditioning level of inverse problem increases. The optimal setting of this parameter provides a good balance between the influence of the level of the right part of the equation to an obtained solution and its penalty systematic error (Refs. 13, p. 77-79). The latter is embedded to the solution by itself in accordance with a corresponding regularizing rule (its full value distribution on the signal domain remains unknown to the user of the algorithm at all times). However, despite of the existence of many reliable recommendations (Refs. 9, p. 69-72; 13, p. 79-82) the parameter of regularization can be selected only by inexact empiric means because of the principal lack of information (e.g., the distribution of signal information along its spectrum, the value of its norm (Refs. 6, p. 173-174; 14, p. 52-55), etc.) about the signal to be restored for a typical (not simulated) deconvolution problem.

Secondly, as the ill-conditioning level of an inverse problem grows, the sensibility to a systematic error also increases (as for a stochastic one). The latter may originate from the restrictions of the precision for the discretization of a convolution equation (in other words, the step of detecting signal cannot be infinitely small due to apparatus restrictions), as well as the influence of the bound conditions, and the finite accuracy of the real spread function of applied apparatus. All these factors introduce a final summary error with nonstochastic specific to the right part of inverse problem. At the same time, as mentioned above the regularization is an approach for solving ill-posed inverse problems based on the controllable purposive implantation of a penalty systematic error to the solution. However, this method exhibits simplicity and required accuracy only in cases of stochastic errors (such as apparatus noise). The detail exploring of the mode of operation (Refs. 6, p. 173-174; 14, p. 52-55) for one method (Tikhonov regularizing algorithm (Refs. 9, p. 66-68, 118-119; 13, p. 72) with a regularizing term of null derivative (Refs. 9, p. 73-74; 13, p. 68, 90-91) as solution norm without additional a priori restrictions) can serve as an example for a conclusion that the existence of components of the final summary errors of the right part of the problem that are characterized by complex nonstochastic nature can break the rules of its correct application under its prevailing value. It would impose the impossibility to obtain an acceptable approximate solution. In addition, after exploring some attempts (Refs. 9, p. 69-73, 118-123) to embed a quite simplified account of this factor into regularizing rules we conclude that these approaches lose their accuracy in comparison with cases of absence of any systematic error with significant value.

METHODS AND EQUIPMENT

In order to minimize the negative influence to two specific systematic components of the error of the right part of convolution equation in solving inverse deconvolution problem we proposed and tested two approaches that modify specifically the ordinary solving schemes for the problem. The schemes linearity is retained as for any system of linear equation.

The first approach aims at minimizing the systematic error originated from inaccurate discretization. For this approach we propose defining the signal to be restored and the spread function at intermediate (between the pixels) spatial positions by certain rules. The discretizing step for the problem is left unchanged. In the example of the one-dimensional deconvolution problem it is equivalent to the modification (Eq. 3) for the values of the function Pi from (Eq. 1). In general, the following actions consist in changing the ordinary solving scheme for the problem (Eq. 1) by a similar one but with modified matrix K_{new} instead of K. The definition of intermediate spatial positions of the function I is considered to be made by a Lagrange polynomial of third or higher kinds (Refs. 15, c. 391). At the same time the intermediate values of the spread function (designated by $K_{i^*-k}(m)$) are to be defined by the values obtained experimentally or by using approximation with sufficient accuracy. In the latter case the accuracy of approximation should be controlled taking into account possible non-Gaussian behavior (Refs. 16) of the spread function.

$$\sum_{m=1}^{2n+1} K_{i-k}^{*}(m) \sum_{r=1}^{2w+1} I_{k+(-w+r-1)} \prod_{u=1\,(u\neq r)}^{2w+1} \frac{(-n+m)*\frac{1}{2n+1}-(-w+u-1)}{(-w+r-1)-(-w+u-1)} * \frac{1}{2n+1} \quad , \qquad (3)\ *$$

where $2n+1$ is the odd value of the level of effective imitation of decreasing discretizing step in (Eq. 1); $2w+1$ is the preset odd value of the kind of a Lagrange polynomial approximating I at intermediate spatial positions within each pixel; $K_{i^*-k}(m)$ – the values of the more detailed spread function in the positions m between $j-k-1$ and $j-k+1$.

The result of this modification for (Eq. 1) should be perfectly similar to a solving scheme resulted by n time decreasing discretizing step in (Eq. 1). The latter procedure is rather time-consuming (for both the experiment and solving) and may deem impossible due to a limited total experimental time and a fixed smallest spatial step of detecting signal. The relative final accuracy of I and K_{new} may be approximately evaluated by the relationship between the expected RMS and norm for these functions. Roughly speaking, these values should be smaller than the condition number (Refs. 9, p. 32-37; 13, p. 59-60) of the regularized inverse problem. In our opinion, an acceptable level of the accuracy improving can be expected for relatively smooth signals for which restoring of the smallest attributes with expected relative changes of single percent along the detecting space is rather interesting.

The second approach directed to suppressing the systematic error originated from the incorrect account of bound effects consists in modifying the ordinary solving scheme (Eq. 4) (Refs. 13, p. 37-38) of deconvolution problem (Eq. 2) by means of (Eq. 5) considered immediately in two-dimensional generalization by the example of a 3×3 matrix. The core of this modification is to change void values of block Toeplitz matrix by those of extended spread functions K^{**} (i.e., the values of spread functions K out of its bounds). In our opinion, this approach can be effective for signals fully localized near center of the spatial field of their detection by condition of its limitations (e.g., in the case of its range being three or four times smaller than the value of FWHM of spread function). For example, similar cases may be expected occasionally in processing micro- and nano-XRF images (Refs. 17) on exploring distribution of chemical elements within inclusions and micro- or nanoobjects.

$$\begin{pmatrix} \begin{bmatrix} K_{2,2} & K_{1,2} & 0 \\ K_{3,2} & K_{2,2} & K_{1,2} \\ 0 & K_{3,2} & K_{2,2} \end{bmatrix} & \begin{bmatrix} K_{2,1} & K_{1,1} & 0 \\ K_{3,1} & K_{2,1} & K_{1,1} \\ 0 & K_{3,1} & K_{2,1} \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} K_{2,3} & K_{1,3} & 0 \\ K_{3,3} & K_{2,3} & K_{1,3} \\ 0 & K_{3,3} & K_{2,3} \end{bmatrix} & \begin{bmatrix} K_{2,2} & K_{1,2} & 0 \\ K_{3,2} & K_{2,2} & K_{1,2} \\ 0 & K_{3,2} & K_{2,2} \end{bmatrix} & \begin{bmatrix} K_{2,1} & K_{1,1} & 0 \\ K_{3,1} & K_{2,1} & K_{1,1} \\ 0 & K_{3,1} & K_{2,1} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} K_{2,3} & K_{1,3} & 0 \\ K_{3,3} & K_{2,3} & K_{1,3} \\ 0 & K_{3,3} & K_{2,3} \end{bmatrix} & \begin{bmatrix} K_{2,2} & K_{1,2} & 0 \\ K_{3,2} & K_{2,2} & K_{1,2} \\ 0 & K_{3,2} & K_{2,2} \end{bmatrix} \end{pmatrix} * \begin{pmatrix} \begin{bmatrix} I_{1,1} \\ I_{2,1} \\ I_{3,1} \end{bmatrix} \\ \begin{bmatrix} I_{1,2} \\ I_{2,2} \\ I_{3,2} \end{bmatrix} \\ \begin{bmatrix} I_{1,3} \\ I_{2,3} \\ I_{3,3} \end{bmatrix} \end{pmatrix} = A * \begin{pmatrix} \begin{bmatrix} I_{1,1} \\ I_{2,1} \\ I_{3,1} \end{bmatrix} \\ \begin{bmatrix} I_{1,2} \\ I_{2,2} \\ I_{3,2} \end{bmatrix} \\ \begin{bmatrix} I_{1,3} \\ I_{2,3} \\ I_{3,3} \end{bmatrix} \end{pmatrix}, \quad (4)$$

$$\begin{pmatrix} \begin{bmatrix} K_{2,2} & K_{1,2} & K^{**}_{0,2} \\ K_{3,2} & K_{2,2} & K_{1,2} \\ K^{**}_{4,2} & K_{3,2} & K_{2,2} \end{bmatrix} & \begin{bmatrix} K_{2,1} & K_{1,1} & K^{**}_{0,1} \\ K_{3,1} & K_{2,1} & K_{1,1} \\ K^{**}_{4,1} & K_{3,1} & K_{2,1} \end{bmatrix} & \begin{bmatrix} K^{**}_{2,0} & K^{**}_{1,0} & K^{**}_{0,0} \\ K^{**}_{3,0} & K^{**}_{2,0} & K^{**}_{1,0} \\ K^{**}_{4,0} & K^{**}_{3,0} & K^{**}_{2,0} \end{bmatrix} \\ \begin{bmatrix} K_{2,3} & K_{1,3} & K^{**}_{0,3} \\ K_{3,3} & K_{2,3} & K_{1,3} \\ K^{**}_{4,3} & K_{3,3} & K_{2,3} \end{bmatrix} & \begin{bmatrix} K_{2,2} & K_{1,2} & K^{**}_{0,2} \\ K_{3,2} & K_{2,2} & K_{1,2} \\ K^{**}_{4,2} & K_{3,2} & K_{2,2} \end{bmatrix} & \begin{bmatrix} K_{2,1} & K_{1,1} & K^{**}_{0,1} \\ K_{3,1} & K_{2,1} & K_{1,1} \\ K^{**}_{4,1} & K_{3,1} & K_{2,1} \end{bmatrix} \\ \begin{bmatrix} K_{2,4} & K_{1,4} & K^{**}_{0,4} \\ K_{3,4} & K_{2,4} & K_{1,4} \\ K^{**}_{4,4} & K_{3,4} & K_{2,4} \end{bmatrix} & \begin{bmatrix} K_{2,3} & K_{1,3} & K^{**}_{0,3} \\ K_{3,3} & K_{2,3} & K_{1,3} \\ K^{**}_{4,3} & K_{3,3} & K_{2,3} \end{bmatrix} & \begin{bmatrix} K_{2,2} & K_{1,2} & K^{**}_{0,2} \\ K_{3,2} & K_{2,2} & K_{1,2} \\ K^{**}_{4,2} & K_{3,2} & K_{2,2} \end{bmatrix} \end{pmatrix} * \begin{pmatrix} \begin{bmatrix} I_{1,1} \\ I_{2,1} \\ I_{3,1} \end{bmatrix} \\ \begin{bmatrix} I_{1,2} \\ I_{2,2} \\ I_{3,2} \end{bmatrix} \\ \begin{bmatrix} I_{1,3} \\ I_{2,3} \\ I_{3,3} \end{bmatrix} \end{pmatrix} = A * \begin{pmatrix} \begin{bmatrix} I_{1,1} \\ I_{2,1} \\ I_{3,1} \end{bmatrix} \\ \begin{bmatrix} I_{1,2} \\ I_{2,2} \\ I_{3,2} \end{bmatrix} \\ \begin{bmatrix} I_{1,3} \\ I_{2,3} \\ I_{3,3} \end{bmatrix} \end{pmatrix}, \quad (5)$$

where K^{**} is extended spread function quantitatively describing the values of K out of its borders.

In this work the TSVD and Tikhonov algorithms in spectral and variation forms with regularizing terms of null and second derivatives (Refs. 9, p. 98, 143-144; 13, p. 90-95) are used in all the cases of applying the ordinary solving scheme for the deconvolution problem and the ones we propose. The use of TSVD or Tikhonov algorithms in spectral forms (Refs. 6, p. 73; 13, p. 71-76) with any regularizing term mentioned above allows the inverse problem to be reduced to analytical form that is similar to a direct problem. It allows one to solve it much faster than in any alternative case. In addition, the use of TSVD or Tikhonov algorithms in spectral forms with a regularizing term of null derivative in the form of solution's norm finding a result has a clarified physical meaning of using the specific linear spectral filters the bound frequencies of which are set by regularizing parameters (Refs. 13, p. 71-76). In contrast, the use of the variation form of the Tikhonov algorithm reduces the problem to that of nonlinear optimization which can be solved much slower (by ten folds according to our observations) than the one in analytical form. However, it allows one to embed various additional a priori restrictions (e.g., non-negativity, convexity, roughness, missing of any breaks and other limitations (Refs. 9, p. 86-87, 125, 158-177)) for the solution into the solving problem if necessary.

The configuration of the computer used for solving problems in the framework of this work is Intel i7-3770 core, operational system Windows 10 and environment software Matlab® 2016 with the embedded tool Optimization ToolBox™. The regularizing parameter is selected by the discrepancy principle (Refs. 9, p. 69; 13, p. 80). In the case of using TSVD and Tikhonov algorithms with regularizing term of null derivative a Matlab® script developed by us was used. In the case of using the Tikhonov algorithms with regularizing term of second derivative the Matlab® function "deconvreg" was applied for restoring signals and images. For the latter case we introduce two additional special adjustable parameters because of a quite strong volatility of the optimal value and an interesting range of the regularizing parameter. Due to this in this case a special term "scaled regularizing parameter" will be used instead of main parameter for accommodation of the vision field in figures.

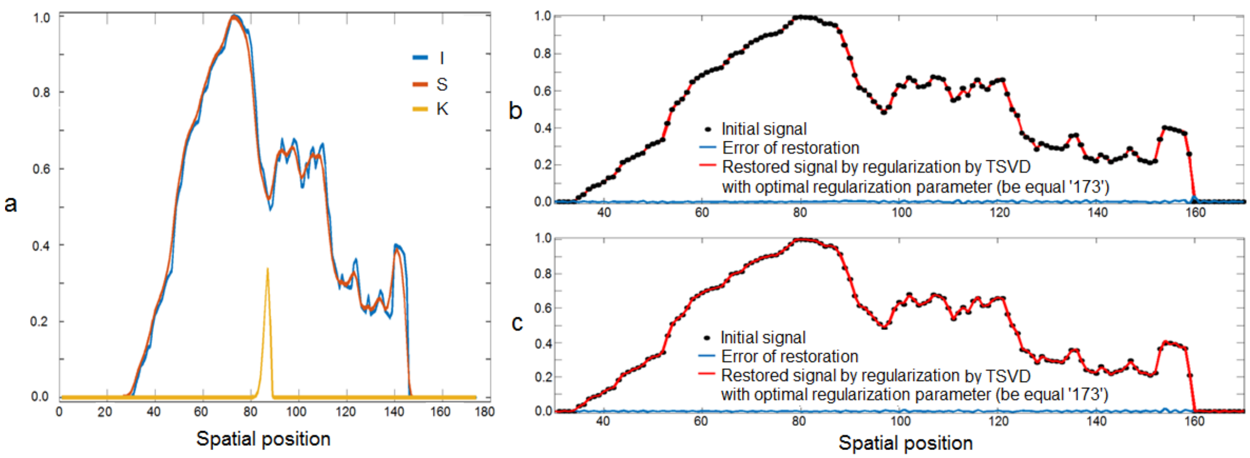

FIGURE 1. (a) All data for direct and inverse one-dimensional signal restoration problems (*I*, *K*, *S*); (b-c) Signal restored by TSVD with optimal value of regularization parameter by the ordinary (b) and proposed first regularization approaches (c).

RESULTS AND DISCUSSIONS

In order to examine how well the proposed approaches work to suppress systematic components of the total error of the right part of the deconvolution problems (Eqs. 1, 2) they were tested on two examples.

The first approach based on applying solving scheme (Eq. 3) was used on the example with a one-dimensional spread function and a signal (Fig. 1a) corrupted by blurring and stochastic noise (with RMS equal to 0 and 0.02 %) previously simulated in a direct convolution problem. The results of restoring the signal obtained by using TSVD with ordinal and modified by us solving schemes are shown in Fig. 1b-c. Analyzing the graphics (Fig. 2) of the solution RMS (its error of its divergence from initial signal simulated in the direct problem) leads to conclusion that the proposed first approach allows one to solve the inverse one-dimensional deconvolution problem with appreciable improvement in the solution precision of 15-20 %. A wider exploration demonstrated that the solution error improving gain was 10-30 % in accordance with the level of stochastic noise (the levels being defined as 0 to 0.06 %).

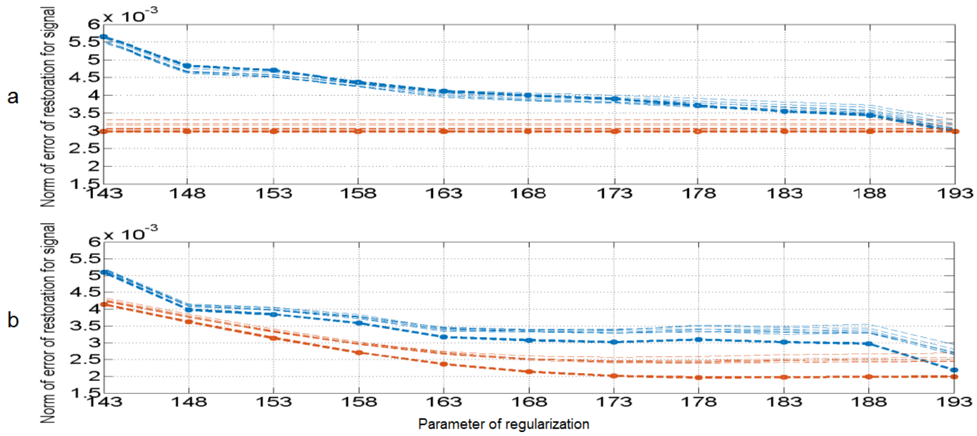

FIGURE 2. Relationship between values of restoration error and selected regularization parameter for the cases of using the ordinary (a) and first proposed regularization approaches (b). Results of TSVD are shown as red dot lines, results of Tikhonov algorithm, as blue dot lines. Results of various realizations of stochastic signal error at 0.02 % level are shown as thin dot lines, results of realizations of stochastic signal error at 0 % level, as bold dot lines with symbols «O».

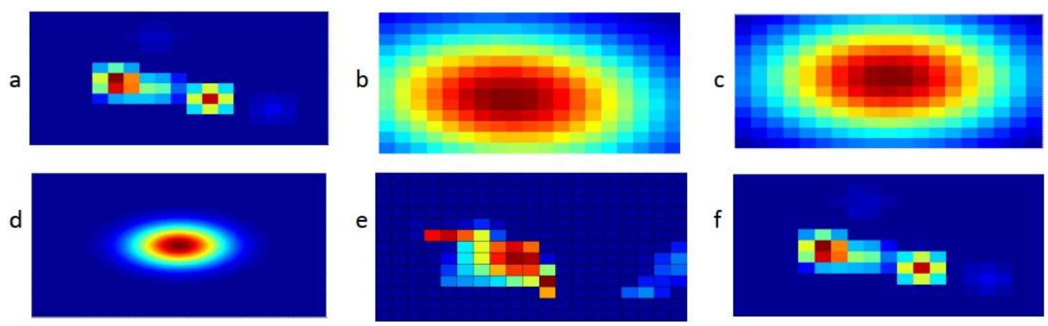

FIGURE 3. Initial (a) and blurred (b) images, spatially limited (c) and extended (d) spread functions K and K^{**} respectively, and the results without (e) and with (f) the second proposed regularization approach.

The second approach was tested on the example that is characterized by limited field of its detection (smaller than 3 time value of the spread function's FWHM). All initial data for solving the problem and the results are shown in Fig. 3. Qualitative analysis shows that after applying ordinary scheme of solving (Eq. 4) the two-dimensional deconvolution problem the results are completely unsatisfactory, but the use of the proposed modified scheme of solving (Eq. 5) provides a result with good precision (in comparison with initial signal in the direct convolution problem).

CONCLUSION

We have proposed and tested successfully two approaches for suppressing specific systematic components of the error of the right part of one- and two-dimensional deconvolution problems. These methods modify the ordinary solving schemes based on the use of Toeplitz matrices with keeping their linear feature. It is recommended to specify the signals to be restored with these methods. The first approach showed a sufficient improvement in the precision of its solution of 15-20 %, and as for the second approach its forceful superiority was demonstrated for the restored images.

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