Nuclear Instruments and Methods in Physics Research B 539 (2023) 9–14

Contents lists available at ScienceDirect

Nuclear Inst. and Methods in Physics Research, B

journal homepage: www.elsevier.com/locate/nimb

Angular distribution of photons emitted in collision of low-energy electrons with noble gases

A.I. Milstein [a,b], S.G. Salnikov [a,b,*], M.G. Kozlov [c,d]

[a] Budker Institute of Nuclear Physics of SB RAS, 630090 Novosibirsk, Russia
[b] Novosibirsk State University, 630090 Novosibirsk, Russia
[c] Petersburg Nuclear Physics Institute of NRC "Kurchatov Institute", 188300 Gatchina, Russia
[d] St. Petersburg Electrotechnical University "LETI", Prof. Popov Str. 5, 197376 St. Petersburg, Russia

ARTICLE INFO

Keywords:
Bremsstrahlung
Noble gases
Polarization radiation
Angular distribution

ABSTRACT

The angular distribution of photons emitted at interaction of low-energy electrons with atoms of noble gases is investigated. The angular asymmetry of the bremsstrahlung cross section is significant and strongly depends on the electron and photon energies. The effect of polarization radiation is considered. It is shown that this effect is noticeable even below the electroluminescence threshold, in contrast to the generally accepted point of view. In the case of argon, the account for the polarization radiation improves the agreement between the predictions and the available experimental data for the yield of bremsstrahlung photons.

1. Introduction

The investigation of low-energy electron interaction with noble gases is very important from both experimental and theoretical points of view. From the theoretical point of view, elastic scattering and bremsstrahlung are very sensitive to the details of the interaction, namely, to the static and polarization potentials. Due to the polarization potential, such an interesting phenomenon as the Ramsauer effect becomes pronounced in noble gases [1]. This effect leads to a nontrivial energy dependence of the electron-atom scattering cross section. From the experimental point of view, the characteristics of photon radiation due to electron-atom scattering are important for the development of highly sensitive dark matter detectors [2–6].

In our recent paper [7] we have investigated the spectrum of radiated photons and clarified many theoretical aspects that were under discussion during a long time. As a result, in the cases of argon and xenon atoms, good agreement was achieved between our predictions and experimental data for the photon yield [2–6]. In addition to the photon yield, the angular distribution of emitted photons is also very important, because it gives us an additional possibility to verify our understanding of the bremsstrahlung physics in noble gases. Therefore, the results concerning the angular distributions will be very useful to experimentalists developing dark matter detectors.

In the present paper we investigate the differential cross section for bremsstrahlung at the interaction of low-energy electrons with noble gases below the electroluminescence threshold. It is generally accepted that below this threshold such an effect as polarization radiation is negligible (see, e.g., Ref. [8]). In this work we have included the contribution of polarization radiation into consideration and have shown that this effect is noticeable even below the electroluminescence threshold. In our paper the amplitude of radiation is a sum of the amplitude corresponding to photon emission by an incident electron and the amplitude of photon emission by atomic electrons. The effect of polarization radiation appears mainly due to the interference of these two amplitudes. The formalism developed here is applicable for all noble gases. All numerical results and the comparison with the experiment are presented for the case of argon.

2. Scattering and radiation of nonrelativistic electrons

For electron wave functions, we use the well known partial wave expansions (see, e.g., Ref. [9])

$$\psi_{\boldsymbol{p}}^{(\pm)}(\boldsymbol{r}) = \frac{1}{2p}\sum_{l=0}^{\infty}(2l+1)\,i^l e^{\pm i\delta_l} R_l(p,r) P_l(\cos\theta)\,, \tag{1}$$

where $p=\sqrt{2m\varepsilon}$ is the electron momentum, m and ε are the electron mass and energy, respectively, δ_l are the scattering phases, P_l are the Legendre polynomials, θ is the angle between the momentum $\boldsymbol{p}$ and vector $\boldsymbol{r}$, and $R_l(p,r)$ are the radial wave functions having the following asymptotics at large distances

$$R_l(p,r)\xrightarrow{r\to\infty}\frac{2}{r}\sin\left(pr-\frac{\pi l}{2}+\delta_l\right). \tag{2}$$

* Corresponding author at: Budker Institute of Nuclear Physics of SB RAS, 630090 Novosibirsk, Russia.
E-mail addresses: A.I.Milstein@inp.nsk.su (A.I. Milstein), S.G.Salnikov@inp.nsk.su (S.G. Salnikov), kozlov_mg@pnpi.nrcki.ru (M.G. Kozlov).

https://doi.org/10.1016/j.nimb.2023.03.013
Received 19 January 2023; Received in revised form 28 February 2023; Accepted 15 March 2023
Available online 23 March 2023
0168-583X/

The function $\psi^{(+)}(\boldsymbol{r})$ contains at large distances the plane wave and a divergent spherical wave, while $\psi^{(-)}(\boldsymbol{r})$ contains the plane wave and a convergent spherical wave.

The differential cross section of the bremsstrahlung process has the form

$$d\sigma = \alpha\,\frac{p_f}{p_i}\,\frac{\omega\,d\omega\,d\Omega_{p_f}\,d\Omega_{\boldsymbol{k}}}{(2\pi)^4}\,|\mathcal{M}|^2, \qquad \mathcal{M} = \boldsymbol{e}\cdot\boldsymbol{J}\,, \tag{3}$$

where α is the fine-structure constant, $\omega = \varepsilon_i - \varepsilon_f$ is the emitted photon energy, ε_i and ε_f are the electron energies before and after collision, $\boldsymbol{e}$ is the photon polarization vector, Ω_{p_f} and $\Omega_{\boldsymbol{k}}$ are the solid angles of the final electron and the photon, respectively, $\hbar = c = 1$. The current $\boldsymbol{J}$ can be written as $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2$, where

$$\begin{aligned}
\boldsymbol{J}_1 &= \int d^3\boldsymbol{r}\,\psi_f^{(-)*}(\boldsymbol{r})\,\hat{\boldsymbol{p}}\,\psi_i^{(+)}(\boldsymbol{r})\,,\\
\boldsymbol{J}_2 &= \sum_{s,a}\int d^3\boldsymbol{r}\,\psi_f^{(-)*}(\boldsymbol{r})\,\frac{\alpha}{r^3}\\
&\times\left[\frac{\langle 0|\hat{\boldsymbol{p}}_a|s\rangle\langle s|\boldsymbol{r}\cdot\boldsymbol{r}_a|0\rangle}{E_0 - E_s + \omega} + \frac{\langle 0|\boldsymbol{r}\cdot\boldsymbol{r}_a|s\rangle\langle s|\hat{\boldsymbol{p}}_a|0\rangle}{E_0 - E_s - \omega}\right]\psi_i^{(+)}(\boldsymbol{r})\,.
\end{aligned} \tag{4}$$

The term $\boldsymbol{J}_1$ corresponds to emission of a photon by an incident electron, and the term $\boldsymbol{J}_2$ describes emission of a photon by atomic electrons in the second order of perturbation theory. Here $|0\rangle$ denotes the ground state of an atom, $|s\rangle$ is an intermediate atomic state, E_0 and E_s are corresponding atomic energies, $\boldsymbol{r}$ and $\hat{\boldsymbol{p}}$ are the coordinate and momentum operators of a scattered electron, respectively, and index a denotes atomic electrons.

Using the commutation relation

$$\left[\hat{\boldsymbol{p}}, H\right] = -i\,\frac{\partial U}{\partial \boldsymbol{r}} = -i\,\boldsymbol{n}\,\frac{\partial U}{\partial r}\,, \qquad \boldsymbol{n} = \frac{\boldsymbol{r}}{r}\,, \qquad H = \frac{\hat{\boldsymbol{p}}^2}{2m} + U(r)\,, \tag{5}$$

where $U(r)$ is the electron potential energy in an atomic field, we write $\boldsymbol{J}_1$ in the form

$$\boldsymbol{J}_1 = -\frac{i}{\omega}\int d^3\boldsymbol{r}\,\psi_f^{(-)*}(\boldsymbol{r})\,\boldsymbol{n}\,\frac{\partial U}{\partial r}\,\psi_i^{(+)}(\boldsymbol{r})\,. \tag{6}$$

By means of the commutation relation $\hat{\boldsymbol{p}}_a = im\left[H_a, \boldsymbol{r}_a\right]$, we write $\boldsymbol{J}_2$ as

$$\begin{aligned}
J_2^j &= im\omega\int \frac{d^3\boldsymbol{r}}{r^3}\,\psi_f^{(-)*}(\boldsymbol{r})\,\alpha_d^{jn}(\omega)\,r^n\,\psi_i^{(+)}(\boldsymbol{r})\,,\\
\alpha_d^{jn}(\omega) &= -\alpha\sum_{s,a}\left[\frac{\left\langle 0\left|r_a^j\right|s\right\rangle\left\langle s\left|r_a^n\right|0\right\rangle}{E_0 - E_s + \omega} + \frac{\left\langle 0\left|r_a^j\right|s\right\rangle\left\langle s\left|r_a^n\right|0\right\rangle}{E_0 - E_s - \omega}\right],
\end{aligned} \tag{7}$$

where $\alpha_d^{jn}(\omega)$ is the polarizability tensor. Due to spherical symmetry of the atomic potential we have

$$\alpha_d^{jn}(\omega) = \alpha_d(\omega)\,a_B^3\,\delta^{jn}\,, \tag{8}$$

where $\alpha_d(\omega)$ is the dimensionless dynamic polarizability of an atom, $a_B = 1/m\alpha$ is the Bohr radius. Finally we obtain

$$\boldsymbol{J}_2 = i\,\alpha_d(\omega)\,a_B^3 m\omega\int \frac{d^3\boldsymbol{r}}{r^2}\,\psi_f^{(-)*}(\boldsymbol{r})\,\boldsymbol{n}\,\psi_i^{(+)}(\boldsymbol{r})\,. \tag{9}$$

The term $\boldsymbol{J}_2$ corresponds to the contribution of the so-called polarization radiation (see [10,11]).

As a result, the total current $\boldsymbol{J} = \boldsymbol{J}_1 + \boldsymbol{J}_2$ reads

$$\boldsymbol{J} = -\frac{i}{\omega}\int d^3\boldsymbol{r}\,\psi_f^{(-)*}(\boldsymbol{r})\,\boldsymbol{n}\left[\frac{\partial U}{\partial r} - \frac{\alpha_d(\omega)\,a_B^3 m\omega^2}{r^2}\right]\psi_i^{(+)}(\boldsymbol{r})\,. \tag{10}$$

The integral in this formula is expressed via the unit vectors $\lambda_i = \boldsymbol{p}_i/p_i$ and $\lambda_f = \boldsymbol{p}_f/p_f$, so that

$$\begin{aligned}
\omega\boldsymbol{J} &= \frac{\lambda_i + \lambda_f}{1+x}\,A + \frac{\lambda_i - \lambda_f}{1-x}\,B\,, \qquad x = \lambda_i\cdot\lambda_f\,,\\
A &= \frac{1}{2}(\lambda_i + \lambda_f)\cdot\omega\boldsymbol{J}\,, \qquad B = \frac{1}{2}(\lambda_i - \lambda_f)\cdot\omega\boldsymbol{J}\,,
\end{aligned} \tag{11}$$

where A and B are the functions of x, ε_i and ε_f. The term proportional to the vector $\left[\lambda_i \times \lambda_f\right]$ does not contribute to $\boldsymbol{J}$ due to parity conservation. Using the recurrent relation for the Legendre polynomials,

$$x\,P_l(x) = \frac{(l+1)P_{l+1}(x) + l\,P_{l-1}(x)}{2l+1}\,, \tag{12}$$

and the orthogonality relation of the Legendre polynomials, we obtain

$$\begin{aligned}
A &= \frac{\pi}{2}\sum_{l=0}^{\infty}(l+1)\left[P_{l+1}(x) + P_l(x)\right]\left[M_{l+1,l} - M_{l,l+1}\right],\\
B &= \frac{\pi}{2}\sum_{l=0}^{\infty}(l+1)\left[P_{l+1}(x) - P_l(x)\right]\left[M_{l+1,l} + M_{l,l+1}\right].
\end{aligned} \tag{13}$$

Here the radial matrix elements are defined as

$$\begin{aligned}
M_{l',l}(p_f, p_i) &= \frac{\exp\left\{i\left[\delta_l(p_i) + \delta_{l'}(p_f)\right]\right\}}{p_i p_f}\\
&\times\int_0^\infty r^2 dr\,R_{l'}(p_f, r)\left[\frac{\partial U}{\partial r} - \frac{\alpha_d(\omega)\,a_B^3 m\omega^2}{r^2}\right]R_l(p_i, r)\,.
\end{aligned} \tag{14}$$

Summing up Eq. (3) over photon polarizations and integrating over $\Omega_{\boldsymbol{k}}$, we obtain the angular distribution of the final electrons

$$\frac{d\sigma_e}{d\Omega_{p_f}} = \frac{\alpha}{3\pi^3}\,\frac{p_i}{p_f}\,\frac{d\omega}{\omega}\left[\frac{|A|^2}{1+x} + \frac{|B|^2}{1-x}\right]. \tag{15}$$

To obtain the photon angular distribution, it is convenient to use the representation

$$\begin{aligned}
&\omega^2\int d\Omega_{p_f}\,J^j J^{n*} = a\,\delta^{jn} + b\left(\delta^{jn} - 3\lambda_i^j\lambda_i^n\right),\\
&a = \frac{\omega^2}{3}\int d\Omega_{p_f}\,|\boldsymbol{J}|^2\,, \qquad b = \frac{a}{2} - \frac{\omega^2}{2}\int d\Omega_{p_f}\left|\lambda_i\cdot\boldsymbol{J}\right|^2.
\end{aligned} \tag{16}$$

Then we use the relations for the Legendre polynomials

$$\begin{aligned}
&\int_{-1}^{1} dx\,\frac{\left[P_{l+1}(x) + P_l(x)\right]\left[P_{l'+1}(x) + P_{l'}(x)\right]}{1+x} = \frac{2\delta_{ll'}}{l+1}\,,\\
&\int_{-1}^{1} dx\,\frac{\left[P_{l+1}(x) - P_l(x)\right]\left[P_{l'+1}(x) - P_{l'}(x)\right]}{1-x} = \frac{2\delta_{ll'}}{l+1}\,,
\end{aligned} \tag{17}$$

which can easily be proven. Finally, integrating Eq. (3) over Ω_{p_f} we obtain the angular distribution of emitted photons

$$\begin{aligned}
&\frac{d\sigma_\gamma}{d\Omega_{\boldsymbol{k}}} = \frac{d\sigma}{d\omega}\cdot\frac{d\omega}{4\pi}\left[1 + \beta\,P_2(\cos\theta_{\boldsymbol{k}})\right],\\
&P_2(y) = \frac{3y^2 - 1}{2}\,, \qquad \frac{d\sigma}{d\omega} = \frac{\alpha}{2\pi^3\omega}\,\frac{p_f}{p_i}\,a\,, \qquad \beta = \frac{b}{a}\,,
\end{aligned} \tag{18}$$

where $\cos\theta_{\boldsymbol{k}} = \lambda_i\cdot\boldsymbol{n}_{\boldsymbol{k}}$, $\boldsymbol{n}_{\boldsymbol{k}} = \boldsymbol{k}/k$, and $\boldsymbol{k}$ is the photon momentum. For the coefficients a and b we have

$$\begin{aligned}
a &= \frac{4\pi^3}{3}\sum_{l=0}^{\infty}(l+1)\left[\left|M_{l,l+1}\right|^2 + \left|M_{l+1,l}\right|^2\right],\\
b &= \frac{2\pi^3}{3}\sum_{l=0}^{\infty}(l+1)\left\{\frac{6(l+2)}{2l+3}\,\mathrm{Re}\left[M_{l+1,l}M^*_{l+1,l+2}\right] - \frac{l+2}{2l+1}\left|M_{l,l+1}\right|^2\right.\\
&\left.- \frac{l}{2l+3}\left|M_{l+1,l}\right|^2\right\},
\end{aligned} \tag{19}$$

In Eq. (18) the angular asymmetry β is a function of ε_i and ω. The bremsstrahlung spectrum $d\sigma/d\omega$ in Eq. (18) coincides with that from Refs. [7,12] when neglecting the contribution of polarization radiation.

It was shown in Ref. [7] that the matrix elements $M_{l,l+1}$ in the limit $\omega \to 0$ have the form

$$M_{l,l+1}(p, p) = \frac{2}{m}\sin\left(\delta_l - \delta_{l+1}\right). \tag{20}$$

Therefore, in this limit the bremsstrahlung spectrum reads

$$\frac{d\sigma}{d\omega} = \frac{16\alpha}{3\omega m^2}\sum_{l=0}^{\infty}(l+1)\sin^2\left(\delta_l - \delta_{l+1}\right). \tag{21}$$

The angular asymmetry β at $\omega \to 0$ is expressed via the scattering

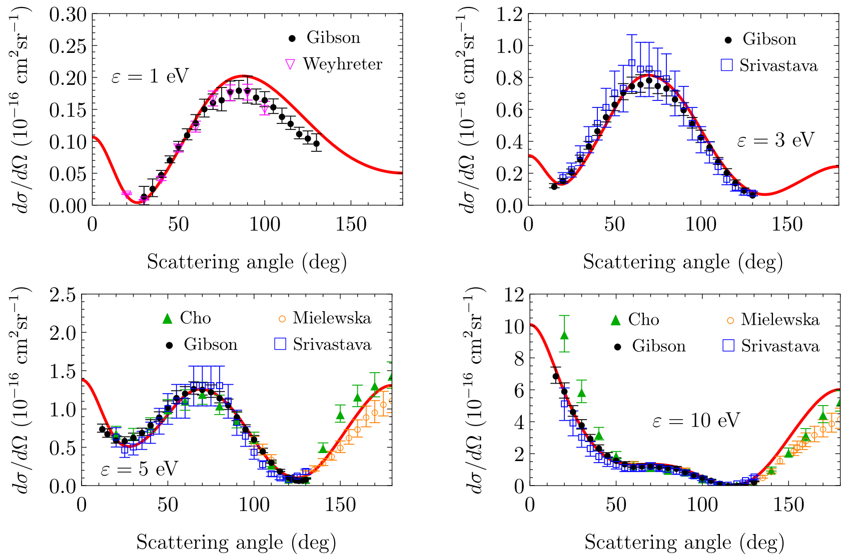

Fig. 1. The angular dependence of the elastic scattering cross section of an electron on argon for several electron energies ε. Experimental data are from Refs. [13–17].

phases as

$$\beta(\omega \ll \varepsilon_i) = \frac{1}{4}\left(\sum_{l=0}^{\infty}(l+1)\sin^2\left(\delta_l - \delta_{l+1}\right)\right)^{-1}$$
$$\times \sum_{l=0}^{\infty}(l+1)\sin\left(\delta_l - \delta_{l+1}\right)\left\{\frac{3(l+2)}{2l+3}\sin\left(\delta_l + \delta_{l+1} - 2\delta_{l+2}\right)\right.$$
$$\left. - \frac{5l+4}{2l+1}\sin\left(\delta_l - \delta_{l+1}\right)\right\}. \quad (22)$$

3. The influence of polarization radiation

In our recent paper [7] we have analyzed the spectrum of photons emitted by low-energy electrons at scattering on argon and xenon without account for polarization radiation. Here we consider the case of argon and take polarization radiation into account. We show that its contribution is noticeable, in contrast to the generally accepted opinion that it is negligibly small below the electroluminescence threshold.

For a quantitative description of the bremsstrahlung spectrum and the electron scattering cross section, we use the potential energy $U(r) = U_{st}(r) + U_{pol}(r)$, where $U_{st}(r)$ is a static potential determined by the charge distribution in the atom and $U_{pol}(r)$ is a polarization potential. In fact, the polarization potential is nothing but the van der Waals potential for interaction of electron with a neutral atom. For correct description of electron-atom scattering, it is necessary to take into account the polarization potential at all distances, but not only at $r \gg a_B$. Since the behavior of $U_{pol}(r)$ at $r \sim a_B$ is not well known, various parameterizations are exploited. In Ref. [7] and in the present work we use the parameterization

$$U_{pol}(r) = -\left[\frac{\alpha_d}{\left(\rho^2 + d^2\right)^2} + \frac{\alpha_q}{\left(\rho^3 + d^3\right)^2}\right]\mathrm{Ry}\,, \quad \rho = \frac{r}{a_B}\,, \quad \mathrm{Ry} = m\alpha^2/2\,. \quad (23)$$

In Ref. [7] the values of the dimensionless parameters α_d, α_q, and d of the polarization potential have been determined by comparison of the predictions for the total elastic scattering cross section of electron on an argon atom with the experimental data [18]. In order to improve theoretical predictions, here we also take into account the electron angular distributions in the elastic scattering of electrons on argon [13–17]. The best fit is obtained for $\alpha_d = 10.5$, $\alpha_q = 200.5$, $d = 2.38$. The static potential $U_{st}(r)$ was calculated using the Hartree–Fock-Dirac method [19]. The comparison of our predictions for the angular distributions of electrons scattered by argon is presented in Fig. 1 for a few electron energies. Some small deviation of our predictions from the experimental data could be a consequence of the specific parameterization of the polarization potential (23). It is possible to improve the agreement by introducing more complicated form of $U_{pol}(r)$. However, the obtained accuracy is sufficient for our purposes, because it is comparable with the accuracy of the experimental data on bremsstrahlung. We remind that the account for polarization potential is crucially important for correct description of the scattering cross section (see Fig. 2).

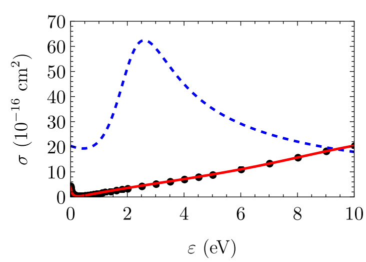

Fig. 2. The total elastic scattering cross section of an electron on argon. The solid line corresponds to the predictions obtained with account for polarization potential (23), and the dashed line represents the cross section calculated without polarization potential. Experimental data are from Ref. [18].

In Ref. [7] we have investigated the bremsstrahlung spectra without account for the polarization radiation. In order to include the polarization radiation in our consideration, we should modify Eq. (14) obtained under the assumption that the polarization radiation is determined by large distances. In fact, not only distances $r \gg a_B$ are important for the corresponding contribution, but also $r \sim a_B$. For a qualitative discussion, we replace a_B^2/r^2 in the polarization contribution in Eq. (14) by $1/(\rho^2 + d^2)$ in the same way as it is done in Eq. (23). Moreover, we replace the dynamic polarizability $\alpha_d(\omega)$ by the static polarizability α_d. We have checked that in the energy region under consideration the replacement $\alpha_d(\omega) \to \alpha_d$ does not change qualitatively the obtained results. We have also verified that our predictions are not very sensitive to a modification of polarization term in Eq. (14) at $r \sim a_B$. In the scattering of slow electrons by atoms, there is also an effect related to the identity of the electrons. This effect is usually taken into account in the polarization potential, which is described phenomenologically in our work. Therefore, we do not consider the effect of electron identity on the bremsstrahlung cross section. A comparison of our predictions for the bremsstrahlung spectra with and without polarization radiation is shown in Fig. 3 for several values of electron energies. The influence

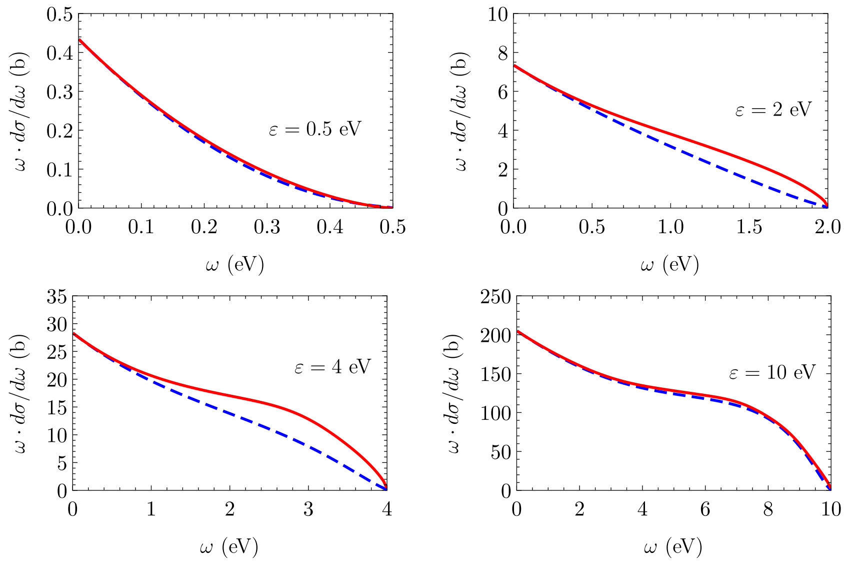

Fig. 3. The dependence of $\omega \cdot d\sigma/d\omega$ on ω given by Eq. (18) for a few values of electron energy ε. The case of argon is considered. The solid line corresponds to the spectrum with account for polarization radiation, and the dashed line corresponds to the spectrum without contribution of polarization radiation.

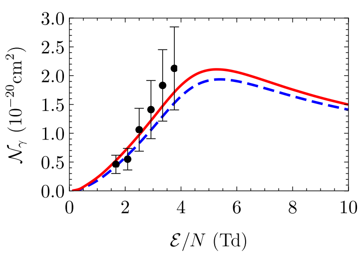

Fig. 4. The reduced yield of bremsstrahlung photons $\mathcal{N}_\gamma$ as a function of the reduced electric field $\mathcal{E}/N$, where $\mathcal{E}$ is the electric field and N is the concentration of atoms. Theoretical predictions are obtained by means of Eq. (24). The solid line corresponds to the spectrum with account for the polarization radiation, and the dashed line corresponds to the spectrum without contribution of polarization radiation. Experimental data are from Ref. [4].

of polarization radiation is noticeable for electron energies about a few electron-volts. Note that the effect of polarization radiation appears mainly due to the interference of the amplitude corresponding to photon emission by an incident electron and the amplitude of photon emission by atomic electrons (see Eq. (14)).

In Ref. [7] we have compared our predictions for the reduced yield of bremsstrahlung photons $\mathcal{N}_\gamma$ with the experimental data obtained in Refs. [2–5] for the case of argon. In these experiments, electrons were accelerated in an electric field and scattered on gaseous argon with emission of photons. The emitted photons in the wavelength region $\lambda = 0 \div 1000\,\mathrm{nm}$ were registered. The reduced yield, which is defined as the number of bremsstrahlung photons per electron per atomic concentration and per drift path, is given by

$$\mathcal{N}_\gamma = \int_{\lambda_{\min}}^{\lambda_{\max}} d\lambda \int_{\omega}^{\infty} d\varepsilon\, \frac{v_e}{v_d}\frac{d\sigma}{d\omega}\frac{d\omega}{d\lambda}\, f(\varepsilon)\,, \tag{24}$$

where $v_e = \sqrt{2\varepsilon/m}$ is the electron velocity, v_d is the drift velocity, $f(\varepsilon)$ is the electron distribution function. The drift velocity and the electron distribution function are determined by the magnitude of the electric field. For these quantities, we use the results obtained by means of EEDF program [20]. The dependence of $\mathcal{N}_\gamma$ on the ratio $\mathcal{E}/N$, where $\mathcal{E}$ is the electric field and N is the concentration of atoms, is shown in Fig. 4. For the reduced electric field $\mathcal{E}/N$, we use the conventional units $1\,\mathrm{Td} = 10^{-17}\mathrm{V\,cm^2}$. It is seen that account for polarization radiation leads to an increase in the predictions for the photon yield $\mathcal{N}_\gamma$ by about 15%. As a result, this predictions are closer to experimental data [4].

In our paper [7] we have also compared our predictions for the photon yield $\mathcal{N}_\gamma$, obtained without account for the polarization radiation, with the experimental data for xenon. Good agreement have been achieved. On the other hand, we have found that the photon yield in the case of xenon is rather sensitive to the modification of the polarization term in Eq. (14) at $r \sim a_B$. Therefore, we do not discuss here the effect of polarization radiation for the case of low-energy electron scattering on xenon.

4. Photon angular distributions

Let us discuss now the angular distributions of bremsstrahlung photons given by Eq. (18). In the case of argon, our predictions for the asymmetry β of these distributions are shown in Fig. 5 for a few electron energies. The results are obtained with account for the polarization radiation and without such an account. The account for the polarization radiation results in noticeable modification of the asymmetry close to the end of the spectra.

It is seen that the asymmetry is negative for almost all photon and electron energies considered. In the limit $\omega \to \varepsilon_i$ the asymmetry tends to $\beta \to -1$, which can be explained as follows. The matrix element $\mathcal{M}$ of the process can be written as $\mathcal{M} = \boldsymbol{e}\cdot\boldsymbol{J}$, where the current $\boldsymbol{J}$ is expressed via the momenta $\boldsymbol{p}_i$ and $\boldsymbol{p}_f$ of initial and final electrons (see Eq. (3) and below). Summation over the photon polarizations $\boldsymbol{e}$ gives

$$\sum |\mathcal{M}|^2 = |\boldsymbol{J}|^2 - |\boldsymbol{n}_{\boldsymbol{k}}\cdot\boldsymbol{J}|^2\,, \qquad \boldsymbol{n}_{\boldsymbol{k}} = \boldsymbol{k}/k\,, \tag{25}$$

where $\boldsymbol{k}$ is the photon momentum. If $\omega = \varepsilon_i$ then $\boldsymbol{p}_f = 0$ and $\boldsymbol{J} \propto \boldsymbol{p}_i$. Therefore, $\sum |\mathcal{M}|^2 \propto 1 - \cos^2\theta_{\boldsymbol{k}}$ ($\theta_{\boldsymbol{k}}$ is the angle between $\boldsymbol{p}_i$ and $\boldsymbol{k}$), which corresponds to $\beta = -1$.

In Fig. 6 we show the photon angular distributions $d\sigma_\gamma/d\omega d\Omega_{\boldsymbol{k}}$ (see Eq. (18)) in polar coordinates for $\varepsilon_i = 4\,\mathrm{eV}$ and $\varepsilon_i = 10\,\mathrm{eV}$ and a few values of ω. It is seen that the angular asymmetry is well pronounced. Most of the bremsstrahlung photons are emitted in a direction perpendicular to the initial momentum of the electron.

5. Conclusion

We have investigated in detail the differential cross section of photon radiation due to interaction of low-energy electrons with atoms. In the case of argon, we have obtained numerical predictions for the

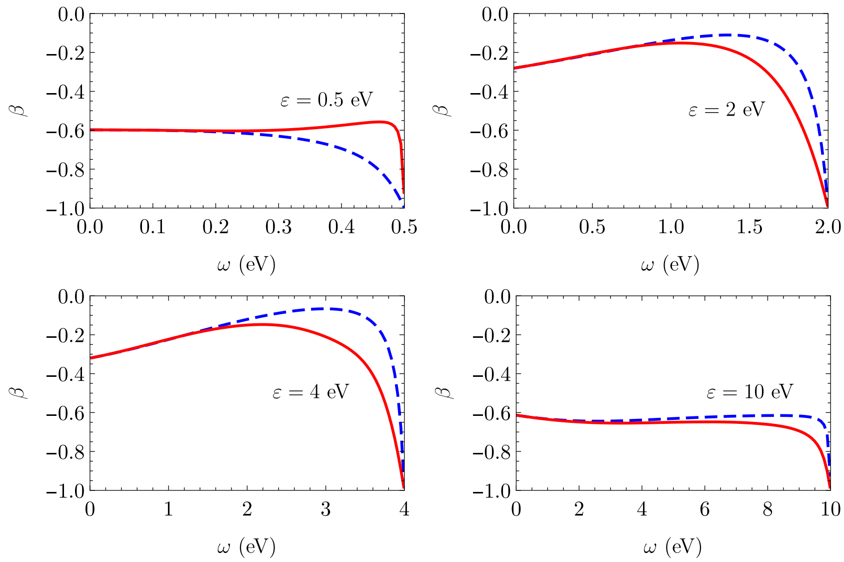

Fig. 5. The dependence of the asymmetry β on photon energy ω for bremsstrahlung on argon at a few electron energies. The solid line corresponds to the spectrum with account for polarization radiation, and the dashed line corresponds to the spectrum without contribution of polarization radiation.

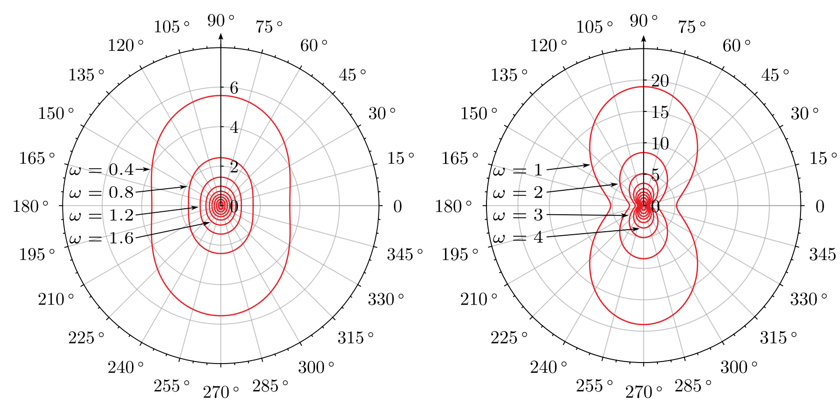

Fig. 6. The angular distributions $d\sigma_\gamma/d\omega d\Omega_k$ of emitted photons for a few values of photon energy ω pointed out in electron-volts. The left figure corresponds to electron energy $\varepsilon_i = 4\,$eV and the right figure corresponds to $\varepsilon_i = 10\,$eV. The asymmetry β is about −0.25 in the left figure and about −0.6 in the right one. The numbers along the vertical axis correspond to the values of cross sections in Barns/eV.

bremsstrahlung cross section. The angular asymmetry β of this cross section (see Eq. (18)) is significant and strongly depends on the electron and photon energies. It is shown that $\beta \to -1$ at $\omega \to \varepsilon_i$ for any value of initial energy of the electron.

The influence of the polarization radiation below the electroluminescence threshold is studied in detail. We have shown that, in contrast to the generally accepted point of view, the contribution of polarization radiation is noticeable even below this threshold. An account for the polarization radiation has allowed us to achieve better agreement between the predictions and the experimental data for the reduced yield of bremsstrahlung photons (see Fig. 4).

CRediT authorship contribution statement

A.I. Milstein: Investigation, Writing – original draft. **S.G. Salnikov:** Investigation, Writing – original draft. **M.G. Kozlov:** Investigation, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgment

We are grateful to A.F. Buzulutskov for valuable discussions.

References

[1] C. Ramsauer, Ann. Phys 371 (1922) 546.
[2] A. Buzulutskov, et al., Astropart. Phys. 103 (2018) 29.
[3] A. Bondar, et al., Nucl. Instrum. Methods Phys. Res. Sect. A Accel. Spectrometers, Detect. Assoc. Equip. 958 (2020) 162432.
[4] E. Borisova, A. Buzulutskov, Eur. Phys. J. C 81 (2021) 1128.
[5] E. Borisova, A. Buzulutskov, Europhys. Lett. 137 (2022) 24002.
[6] C.A.O. Henriques, et al., Phys. Rev. X 12 (2022) 021005.
[7] A.I. Milstein, S.G. Salnikov, M.G. Kozlov, Nucl. Instrum. Methods Phys. Res., Sect. B 530 (2022) 48.
[8] V. Kas'yanov, A. Starostin, JETP 21 (1965) 193.
[9] L.D. Landau, E.M. Lifshitz, Quantum Mechanics — Non-Relativistic Theory, third ed., Pergamon Press, Oxford, 1991.
[10] M.Y. Amusia, Phys. Rep. 162 (1988) 249.
[11] M.Y. Amusia, Bremsstrahlung (in Russian), Energoatomizdat, Moscow, 1990.
[12] L.G. D'yachkov, G.A. Kobzev, J. Phys. B At. Mol. Phys. 16 (1983) 1605.

[13] S.K. Srivastava, H. Tanaka, A. Chutjian, S. Trajmar, Phys. Rev. A 23 (1981) 2156.
[14] M. Weyhreter, B. Barzick, A. Mann, F. Linder, Z. Phys. D - Atoms 7 (1988) 333.
[15] J.C. Gibson, et al., J. Phys. B At. Mol. Opt. Phys. 29 (1996) 3177.
[16] B. Mielewska, I. Linert, G. King, M. Zubek, Phys. Rev. A 69 (2004) 062716.
[17] H. Cho, Y.S. Park, J. Korean Phys. Soc. 55 (2009) 459.
[18] M. Kurokawa, et al., Phys. Rev. A 84 (2011) 062717.
[19] V.F. Bratsev, G.B. Deyneka, I.I. Tupitsyn, Bull. Acad. Sci. USSR, Phys. Ser. 41 (1977) 173.
[20] N.A. Dyatko, I.V. Kochetov, A.P. Napartovich, A.G. Sukharev, https://nl.lxcat.net/download/EEDF.