Resonant amplification of a relativistic electron bremsstrahlung on an atom in the undulator field

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A R T I C L E   I N F O

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It is shown that the influence of the undulator field on the motion of a relativistic electron leads to a significant increase of the bremsstrahlung cross section on an atom near the end of the spectrum of undulator radiation. Observation of this effect is quite a realistic task.

1. Introduction

At present, the processes of e^+e^- pair production by a photon in an atomic field and electron bremsstrahlung in this field have been studied in detail using various approaches both at high and low particle energies. At high energies, using the quasiclassical approximation, calculations can be performed exactly in the parameters of an atomic field. A detailed review of the results obtained within this approximation can be found in [1].

The processes of radiation and e^+e^- pair production in an atomic field are described in terms of cross sections, since the atomic field is localized. Another type of problems includes these processes in macroscopic fields, for example, in laser fields of different spectral composition and polarization. Such processes are described either in terms of probability per unit time or in terms of total probability [2].

Of great interest is also the study of an influence of macroscopic fields on the processes of quantum electrodynamics in an atomic field. The peculiarity of these problems is that a photon can be emitted by an electron in a macroscopic field without any atomic field. As a result, a singularity arises in the amplitude of the process, which is related to the presence of a cascade process. This singularity cannot be eliminated by introducing a pole into the electron propagator, since it is in the continuous spectrum. Thus, the probability of the process depends on the conditions of the experiment and cannot be described in terms of the cross section.

Various results related to the study of an influence of laser fields on bremsstrahlung and pair production processes in an atomic field can be found in Refs. [3–10]. Some effects related to the influence of constant macroscopic fields on processes in atomic fields are described in the book [11]. As a rule, the influence of laser fields becomes noticeable at a very high intensity, which complicates the experiments.

In this paper, we study the bremsstrahlung of a relativistic electron on an atom during the motion of an electron beam in an undulator field. It is shown that the influence of the undulator field will be noticeable at moderate values of the undulator magnetic field and not very high electron energies. In this case, it is possible to choose the region of frequencies and emission angles of the emitted photons in such a way that it will not overlap with the region available for radiation only in the undulator field. As a result, the process can be described in terms of a cross section. However, if the energy and direction of the emitted photon momentum is close to the accessible region of radiation in the undulator field then a large increase in radiation (resonant amplification) takes place. We show that the observation of this effect is a realistic task. All calculations in our work are carried out within the framework of the quasiclassical approximation exactly in the parameters of the atomic field.

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2. Cross section of the process

If a beam of relativistic electrons moves in an undulator, then the cross section $d\sigma$ of a photon radiation by an electron in an atomic field can be represented as the sum $d\sigma = d\sigma_a + d\sigma_f$, where $d\sigma_a$ is the cross section of radiation in the absence of the undulator field, and $d\sigma_f$ is the contribution related to the undulator field. The bremsstrahlung spectrum $d\sigma_a/d\omega$ and the cross section $d\sigma_a/d\omega d\Omega_{\boldsymbol{k}}$ differential in the photon momentum $\boldsymbol{k}$ have the form [12]

$$\frac{d\sigma_a}{d\omega} = \frac{4\alpha\eta^2}{m^2\omega}\left[\left(1 + \frac{\varepsilon'^2}{\varepsilon^2} - \frac{2\varepsilon'}{3\varepsilon}\right)L + \frac{1}{9}\frac{\varepsilon'}{\varepsilon}\right],$$

$$\frac{d\sigma_a}{d\omega d\Omega_{\boldsymbol{k}}} = \frac{4\alpha\eta^2\varepsilon^2}{\pi m^4\omega(1+b^2)^2}\left\{[L + \ln(1+b^2)]\left[1 + \frac{\varepsilon'^2}{\varepsilon^2} - \frac{\varepsilon'}{\varepsilon}\frac{4b^2}{(1+b^2)^2}\right] - \left[1 + \frac{\varepsilon'^2}{\varepsilon^2} + \frac{(1+b^2)^2 - 10b^2}{(1+b^2)^2}\frac{\varepsilon'}{\varepsilon}\right]\right\},$$

$$b = \gamma\theta_k, \quad \eta = Z\alpha, \quad L = \log 183 Z^{-1/3} - \mathrm{Re}\,\psi(1+i\eta) - C, \tag{1}$$

where $\gamma = \varepsilon/m$, Z is the atomic charge number, α is the fine-structure constant, $\varepsilon, \varepsilon', \omega$ are the energies of the initial electron, final electron, and photon, respectively, m is the electron mass, θ_k is the angle between the photon momentum $\boldsymbol{k}$ and the electron momentum $\boldsymbol{p}$, $\psi(x) = d\Gamma(x)/dx$, $\Gamma(x)$ is the Euler gamma function, $C = 0.577\dots$ is the Euler constant, $\hbar = c = 1$. Recall that the argument of logarithm in L is proportional to r_{scr}/λ_C, where $r_{scr} \sim a_B Z^{-1/3} = \lambda_C Z^{-1/3}/\alpha$ is the screening radius, λ_C is the Compton wave length. The proportionality coefficient in the argument is determined by the combination of the parameters in the Molière potential describing the atomic potential in the Thomas-Fermi model.

To calculate the contributions $d\sigma_f/d\omega$ and $d\sigma_f/d\omega d\Omega_{\boldsymbol{k}}$, we use the results of Ref. [8], where the effect of a laser wave on the bremsstrahlung process of relativistic electron in an atomic field has been studied. In Ref. [8], the laser wave propagated in the direction opposite to the electron momentum $\boldsymbol{p}$. In that case, the vector-potential of the laser field $\boldsymbol{A}_l(t+z)$ can be replaced by $\boldsymbol{A}_l(2z)$. The undulator field is described by a similar vector potential $\boldsymbol{A}_f(z)$. Therefore, the results for the undulator field can be obtained from the corresponding results of [8] for the laser field by replacing $\omega_l \to \omega_0/2$ (ω_l is the laser frequency) and the dimensionless parameter ξ by the undulator parameter [13] $K = eB/m\omega_0$, where e is the electron charge, $\boldsymbol{B}$ is the undulator magnetic field, $\omega_0 = 2\pi/\lambda_f$, λ_f is the spatial period of the undulator magnetic field. For a helical undulator, $\boldsymbol{B}$ has the form

$$\boldsymbol{B}(z) = B\left(\boldsymbol{e}_x\cos(\omega_0 z) + \boldsymbol{e}_y\sin(\omega_0 z)\right).$$

For $K \ll 1$ and $\omega > \omega^*$, where $\omega^* = 2\varepsilon^2\omega_0/m^2$ is the end of the spectrum of undulator radiation, the contribution to the bremsstrahlung spectrum due to the influence of an undulator field reads

$$\frac{d\sigma_f}{d\omega} = \frac{4\alpha\eta^2 K^2}{m^2\omega}\left\{\frac{\varepsilon^2 + \varepsilon'^2}{\varepsilon^2}\left\{L + \frac{1}{2} + \frac{6L(9y^2-7) - 3y^2 + 1}{18(y^2-1)^2} + y^2\left[\left(L - \frac{3}{2}\right)l_1\right.\right.\right.$$

$$\left.\left.- \frac{l_1^2 + l_2^2}{8} - \frac{2y(3y^2-2)l_2 + (5-7y^2)l_1}{12(y^2-1)^2} - \frac{1}{2}\mathrm{Li}_2(\frac{1}{y^2})\right]\right\}$$

$$- 8y^3\frac{\varepsilon'}{\varepsilon}\left[\frac{L(y^2-1)}{y^3} + \frac{(12L+1)(7y^2-5)}{72y^3(y^2-1)} + \frac{(yl_1 - l_2)}{12(y^2-1)}\right.$$

$$\left.\left. - \left(L - \frac{3}{2}\right)(yl_1 + l_2) + (y-1)\mathrm{Li}_2\left(\frac{1}{y}\right) + (y+1)\mathrm{Li}_2\left(-\frac{1}{y}\right)\right]\right\},$$

$$y = \omega/\omega^*, \quad l_1 = \log(1 - 1/y^2), \quad l_2 = \log[(y+1)/(y-1)]. \tag{2}$$

Here $Li_2(x)$ is the dilogarithm. Note that ω^* is twice less than the maximum radiation frequency in the laser field at $\xi \ll 1$ [8]. The term (2) contains a singularity at $\omega \to \omega^*$. This is due to the possibility of a cascade process, in which an electron first emits a photon in an undulator field and then elastically scatters in an atomic field, or first scatters in an atomic field and then radiates in an undulator field. Although the cascade process is impossible for $\omega > \omega^*$, closeness of the edge of the undulator radiation spectrum, $\omega - \omega^* \ll \omega$, leads to a significant increase of the bremsstrahlung cross section. At $1 \gg \omega/\omega^* - 1 \gg 1/(mr_{scr})^2$, where $r_{scr} \sim Z^{-1/3}/m\alpha$ is the screening radius, the contribution to the spectrum has the form

$$\frac{d\sigma_f}{d\omega} = \frac{4\alpha\eta^2 K^2\omega^*}{3m^2(\omega^*-\omega)^2}\left[L + \frac{1}{2}\log\left(\frac{\omega - \omega^*}{\omega^*}\right) - \frac{1}{6}\right]. \tag{3}$$

Note that $d\sigma_f/d\omega$ for ω close to ω^* can be noticeably larger than $d\sigma_a/d\omega$. It should be noted also that the shape of the spectrum at $0 < \omega/\omega^* - 1 \lesssim 1/(mr_{scr})^2$ is not universal and depends on the macroscopic parameters of the system. As is known, the cascade process depends on the macroscopic parameters of the setup and its contribution to the energy loss in proportion to T^2, where T is the time of passage through the undulator. However, this contribution is absent for $\omega > \omega^*$ because of the edge of the undulator radiation spectrum. Other restrictions related to the angular and energy spread in the beam will be considered in the next section.

There is also a singularity in the contribution $d\sigma_f/d\omega d\Omega_{\boldsymbol{k}}$ to the differential cross section. This feature is also related to the closeness to the allowed region of undulator radiation. The maximum radiation frequency in the undulator field depends on the photon emission angle:
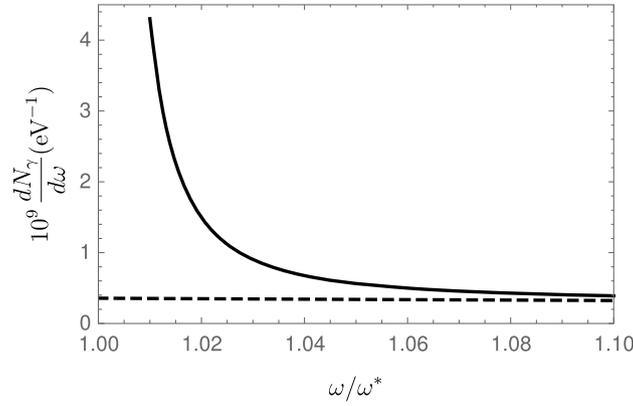
Fig. 1. Dependence of $dN_\gamma/d\omega = ns\, d\sigma/d\omega$ (solid line) and $N_\gamma^{(0)} = ns\, d\sigma_a/d\omega$ (dotted line) on ω/ω^* with the parameters specified in the text.
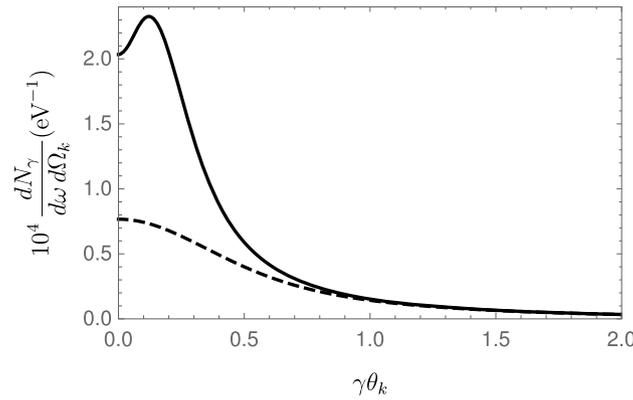
Fig. 2. The angular distribution $dN_\gamma/d\omega d\Omega_{\boldsymbol{k}}$ of photons as a function of $\gamma\theta_k$ at $\omega = 1.08\omega^* = 189.6\,\text{eV}$ (solid line). The dotted line is the angular distribution $dN_\gamma^{(0)}/d\omega d\Omega_{\boldsymbol{k}}$.

$$\omega_\theta^* = \frac{\omega^*}{1+b^2}, \quad b = \gamma\theta_k. \tag{4}$$

Here θ_k is the angle between the photon momentum and the undulator axis. At $\omega - \omega_\theta^* \ll \omega$ and $K \ll 1$ we have

$$\frac{d\sigma}{d\omega d\Omega_{\boldsymbol{k}}} = \frac{8\alpha\eta^2 K^2 \varepsilon^2 b^2 (1+b^4)(\omega_\theta^*)^3}{\pi m^4 (1+b^2)^6 (\omega - \omega_\theta^*)^4} \left[L - 5/2 + \log(b+1/b) + \log(\omega/\omega_\theta^* - 1) \right]. \tag{5}$$

The expression for $d\sigma_f/d\omega d\Omega_{\boldsymbol{k}}$ for arbitrary values of ω is cumbersome and we do not present it here.

3. Possibility of experimental observation of the effect

Let us now discuss the possibility to observe the effect of a strong increase of the bremsstrahlung cross section on an atom related to the motion in an undulator field. Consider an electron beam with energy $\varepsilon = 430\,\text{MeV}$. This is a typical value of energy for the injection complex of the BINP SB RAS [14]. Let us assume that this beam moves in an helical undulator with length $s = 100\,\text{cm}$, spatial period of the magnetic field $\lambda_f = 1\,\text{cm}$ and magnetic field $B = 0.1\,\text{T}$. With such undulator parameters, $\omega^* = 175.6\,\text{eV}$ and $K = 0.093$. The uncertainty ω^* associated with the finite length of the undulator is $\delta\omega^*/\omega^* \sim \delta\omega_0/\omega_0 = \lambda_f/s = 10^{-2}$. The value of $\delta\omega^*/\omega^*$ associated with the energy and angular spread of the electron beam is of the order of $\sim 10^{-3}$. Note that at $\omega > \omega^*$, there are practically no undulator photons, they appear only due to the angular and energy spread of the electron beam, the imperfection of the magnetic system, and the finite number of magnets in the undulator.

As a target, one can use gaseous helium with density $n = 10^{16}\,\text{cm}^{-3}$, which fills the undulator. In Fig. 1 the solid line shows the dependence of the photon spectrum $dN_\gamma/d\omega = ns\, d\sigma/d\omega$ on ω/ω^* at $1 \gg \omega/\omega^* - 1 \gg 1/(mr_{scr})^2$. For comparison, the dashed line shows $N_\gamma^{(0)} = ns\, d\sigma_a/d\omega$. It can be seen that the undulator field results in significant modification of the photon spectrum near ω^*. The number of photons N_γ emitted by one electron in the interval $1.05 > \omega/\omega^* > 1.01$ is $N_\gamma = 8.8 \times 10^{-9}$, and in the absence of the undulator $N_\gamma^{(0)} = 2.4 \times 10^{-9}$, the difference is almost four times. For $N = 10^{10}$ of electrons in the beam and the beam injection period $\tau = 1\,\text{sec}$, we will have 88 photons per second near the edge of the spectrum of undulator radiation.

Fig. 2 shows the angular distribution of bremsstrahlung photons $dN_\gamma/d\omega d\Omega_{\boldsymbol{k}}$ (solid line) near the edge of the spectrum of undulator radiation ω^* at $\omega = 1.08\omega^* = 189.6\,\text{eV}$. A large peak is seen compared to $dN_\gamma^{(0)}/d\omega d\Omega_{\boldsymbol{k}}$ (dotted line). Note that the angle of multiple scattering of electrons when passing through a gas target under these conditions is $10^{-2}/\gamma \ll 1/\gamma$, i.e., multiple scattering does not affect the applicability of the results obtained.

4. Conclusion

It is shown that the bremsstrahlung cross section of a relativistic electron moving in a superposition of atomic and undulator fields near the edge of the undulator radiation spectrum is much larger than in the case of the atomic field alone. Above the edge of the undulator spectrum, there are practically no undulator photons; they appear only due to the angular and energy spread of the electron beam, the imperfection of the magnetic system, and the finite number of magnets in the undulator. However, the virtuality of an electron emitting a photon near the edge of the undulator spectrum is very small (the bremsstrahlung photon formation length [15] is large). This is the origin of a significant enhancement of the bremsstrahlung cross section. Note that the use of an undulator instead of laser radiation has an important advantage. In a high-power laser beam, which is necessary to observe the bremsstrahlung amplification effect, the target is destroyed, in contrast to the case of an undulator field. In this paper we have considered the case of a helical undulator. For a planar undulator, the results for a spectrum and angular distribution averaged over the azimuth angle can be obtained by making the substitution $K^2 \to K^2/2$. This prescription is explained by the fact that the energy density of the magnetic field in a helical undulator is independent of z coordinate, and in a planar undulator the average in z value of the magnetic field energy is two times smaller than its maximum value.

We show that observation of the interesting coherent effect of bremsstrahlung cross section increasing due to influence of the undulator field is a realistic task.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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