Revision of results on $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ massesA.G. Shamov^{a,*}, O.L. Rezanova^{a,b}^a Budker Institute of Nuclear Physics, 11, akademika Lavrentieva prospect, Novosibirsk, 630090, Russia^b Novosibirsk State University, 1, Pirogova street, Novosibirsk, 630090, Russia

ARTICLE INFO

Article history:

Received 24 November 2022

Received in revised form 7 February 2023

Accepted 8 February 2023

Available online 10 February 2023

Editor: M. Doser

ABSTRACT

We have reconsidered the results on the masses of the narrow bottomonium states $\Upsilon(1S)$ – $\Upsilon(3S)$ obtained in 1982–1986 at CESR, DORIS and VEPP-4 colliders in order to fix shortcomings of the mass determination procedures. For experiments at CESR and DORIS this includes the incorrect accounting of the radiative corrections and usage of the electron mass value revised in 1986. In analyses of all experiments the interference of the resonance production and the nonresonant process was ignored. The corrected mass values for five experiments are suggested. The corrections vary from 0.1 to 0.4 MeV. The discrepancy between CESR and VEPP-4 results on $\Upsilon(1S)$ mass has been reduced from 3.3 to 1.8 standard deviations.

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1. Introduction

The new experiment on the high precision measurement of the Υ -meson mass has been planned at the VEPP-4M collider [1] with the KEDR detector [2]. The resonant depolarization method [3,4] will be used for the beam energy determination. At moment the laser polarimeter is under development [5] and the test scan of the $\Upsilon(1S)$ has been performed [6]. In this context it is important to overcome known drawbacks in the analyses of the preceding experiments and correct its results.

The mass of $\Upsilon(1S)$ was measured by the MD-1 detector at VEPP-4 [7,8] and CUSB detector at the collider CESR with the accuracy of about 0.1 MeV [9]. With lower accuracy of 0.4–0.5 MeV the mass of $\Upsilon(2S)$ was measured by ARGUS and Crystal Ball detectors at DORIS [10] and by MD-1 [11]. The mass of $\Upsilon(3S)$ was determined with 0.5 MeV uncertainty by MD-1 only [11].

In all these experiments the mass values were obtained by fitting the inclusive hadronic cross section as function of the c.m. energy. The energy was determined using resonant depolarization method thus the results are proportional to the value of electron mass. In 1986 the -8.5 ppm shift of the electron mass value has occurred [12]. The results of the mass measurements at VEPP-4 have been corrected in 2000 [12]. The results from CESR and DORIS stayed intact.

Another problem to solve is accounting of the radiative correction in experiments [9] and [10] according to the work [14]

containing the mistake as was stated in Ref. [15]. The impact on the mass values is of order of 0.1 MeV. Despite to the existence of correct studies of the narrow resonance production since 1975, the incorrect resonance shape from Ref. [14] was employed for determination of leptonic widths and masses of ψ - and Υ -states in many experiments. Concerning leptonic widths the problem was solved in Ref. [16], the corrected values were included in PDG tables. However, the masses of Υ states [9] and [10] were not corrected neither in Ref. [16] nor in Ref. [12] where radiative corrections were fixed up for J/ψ mass measurement by the OLYA detector in 1980 [17]. The MD-1 experiments on masses of upilon states [7,8,11,18] were performed with proper radiative correction accounting.

Besides, there is a mistake in the calculation of the resonance curve in Ref. [9], that will be discussed in details below.

The common drawback of all measurements of ψ - and Υ - state masses mentioned above is ignoring of the interference between resonant and nonresonant contributions to the hadron production. First time it was accounted in the J/ψ - and $\psi(2S)$ - mass measurement in the experiment [19] and was discussed in details in Ref. [20].

In all experiments under discussion except [9], the dependence of the hadronic cross section on c.m. energy was not published. In Refs. [12,16], in order to correct the resonance leptonic width [16] and mass [12], the equidistant data points were simulated using the published values of the resonance curve parameters. Then two fits were performed with the correct fitting function and that of the published paper. The variation of the resonance parameter was added to its published value. This method uses published values of

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parameters, biased by the incorrect fit, and does not account for the specific layout of energy points.

To overcome these drawbacks in this work we obtained coordinates of data points from the plots in electronic versions of publications using the graphical editor and converted them to the physical quantities employing measured coordinates of the axes' ticks. Such data were not absolutely reliable thus we shifted published values as described above. In Ref. [9] the measured values of cross section and the energy were published thus we could just properly refit the CUSB data.

In the next sections we describe the necessary corrections, discuss data published by CUSB [9], and then obtain corrected mass values for three resonances from five experiments. Shifts of mass values exceed the quoted values of systematic uncertainties by the factors of 2-3 and sometimes reach the total uncertainties of results.

Precise measurements require some corrections to mass values due to accelerator-related effects. We kept them according to original papers. The systematic error estimates were not changed. The addition uncertainties related to the correction procedure are negligible.

2. Change of the electron mass value

As it was mentioned above, the experiments cited did employ the resonant depolarization method for the beam energy determination. In this method the measured ratio of the spin precession frequency Ω and the revolution frequency ω gives the Lorentz factor γ according to the relation

$$\Omega/\omega = 1 + \gamma \cdot \mu'/\mu_0, \quad (1)$$

where μ' and μ_0 are anomalous and normal parts of the electron magnetic moment [4]. To find the beam energy $E = \gamma m_e$, the value of the electron mass m_e is required. Before 1986 its accuracy was estimated to be 2.8 ppm [21] which corresponds to 26 keV uncertainty in the mass of $\Upsilon(1S)$. In 1986 the adjustment of fundamental physical constants [13] led to shift of the electron mass value by -8.5 ppm with increase of its accuracy to 0.8 ppm. The corresponding shifts of $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ masses are -81, -86, and -88 keV respectively [12]. For experiments [9,10] that was not accounted yet. Since that time the relative change of the electron mass value was about -0.2 ppm whereas the accuracy reached 0.3 ppb. The corresponding shift of the mass value is about -2 keV for all three narrow states of Υ .

3. Radiative corrections

Soon after the J/ψ -meson discovery a number of papers appeared on the radiative corrections for a narrow resonance production in e^+e^- -collisions. First of them is probably Ref. [22] which will be considered in the next section. However, the most popular theoretical work used for ψ - and Υ -data analysis until 1985 was Ref. [14]. It was directly addressed to experimentalists and published in "Nuclear Instruments and Methods". The calculations were performed in the approximation of zero resonance width. For the Gaussian collider energy spread distribution

$$G(W' - W) = \frac{1}{\sqrt{2\pi}\sigma_W} \exp\left(-\frac{(W' - W)^2}{2\sigma_W^2}\right) \quad (2)$$

the narrow resonance cross section in a final state f at the c.m. energy W was obtained in the form

$$\sigma^{R \rightarrow f}(W) = \frac{6\pi^2}{M^2} \frac{\Gamma_{ee}^{(0)} \Gamma_f}{\Gamma} \left(G_r(W - M) + \delta \cdot G(W - M) \right), \quad (3)$$

where

$$G_r(x) = \left(\frac{2\sigma_W}{M} \right)^\beta \frac{\Gamma(1 + \beta)}{\sqrt{2\pi}\sigma_W} \exp\left(-\frac{x^2}{4\sigma_W^2}\right) D_{-\beta}\left(-\frac{x}{\sigma_W}\right), \quad (4)$$

$$\delta = \frac{13}{12} \beta + \frac{2\alpha}{\pi} \left(\frac{\pi^2}{6} - \frac{17}{36} \right), \quad \beta = \frac{4\alpha}{\pi} \left(\ln \frac{W}{m_e} - \frac{1}{2} \right). \quad (5)$$

In the formulae above α is the fine structure constant, m_e is the electron mass, $\Gamma()$ is the gamma-function and $D_{-\beta}$ is the Weber function of parabolic cylinder for calculation of which the power series were specified. The electron partial widths $\Gamma_{ee}^{(0)}$ correspond to the lowest order of QED. The δ includes the vertex corrections and contribution of electron loops into the vacuum polarization, other contributions to it were not considered in Ref. [14].

The G_r function in (3) is so called "radiative Gaussian", which is a convolution of the collider energy spread with the probability of the energy loss due to soft photon radiation in e^+e^- -collision. It is known that the probability of the QED process which is not accompanied by such emission is zero, therefore the second term in eq. (3) is not correct. There must be $\delta \cdot G_r(W - M)$. The $G(x)$ -function unlike to $G_r(x)$ is symmetric, thus using of eq. (3) for data analysis increase the $\Upsilon(1S)$ -mass by about 0.1 MeV at the energy spread $\sigma_W \simeq 5$ MeV, as was noted in [8].

The energy spread distribution is not exactly Gaussian. To achieve the Υ mass accuracy better than 0.1 MeV, the pre exponential factor must be introduced in Eq. (2). In the first approximation it leads to some shift of the energy scale but does not change the resonance shape. The effect is discussed in detail in Ref. [23].

4. Interference effect

The interference effects in the inclusive hadronic cross section in the vicinity of a narrow resonance was considered already in Ref. [22]. With some up-today modifications the resonant and interference terms in the soft photon approximation can be written as [20]

$$\sigma^{r+i}(W) = \frac{12}{W^2} (1 + \delta) \times \left(\frac{\Gamma_{ee} \tilde{\Gamma}_h}{\Gamma M} \text{Im} f(W) - \frac{2\alpha \sqrt{R \Gamma_{ee} \tilde{\Gamma}_h}}{3W} \lambda \text{Re} \frac{f(W)^*}{1 - \Pi_0} \right), \quad (6)$$

where

$$f(W) = \frac{\pi \beta}{\sin \pi \beta} \left(\frac{W^2}{M^2 - W^2 - iM\Gamma} \right)^{1-\beta}, \quad (7)$$

$$\delta = \frac{3}{4} \beta + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right) + \beta^2 \left(\frac{37}{96} - \frac{\pi^2}{12} - \frac{1}{36} \ln \frac{W}{m_e} \right), \quad (8)$$

β is defined in Eq. (5).

Above R is the hadron-to-muon cross section ratio out of the resonance peak, Π_0 is the vacuum polarization with the resonance contribution excluded, $\Gamma_{ee} = \Gamma_{ee}^{(0)}/|1 - \Pi_0|^2$ is the physical value of the electron width, $\tilde{\Gamma}_h$ is some effective value of the hadronic partial width, and λ is the parameter introduced in Ref. [22] to characterize the strength of the interference effects.

Due to the interference of electromagnetic and strong decays of the resonance, $\tilde{\Gamma}_h$ differs from the true hadronic partial width Γ_h , but the value of $\tilde{\Gamma}_h$ is not of first importance to the mass determination. The problem is discussed in details in Ref. [20]. The following result was obtained for λ :

$$\lambda = \sqrt{\frac{R \mathcal{B}_{ee}}{\mathcal{B}_h}} + \sqrt{\frac{1}{\mathcal{B}_h}} \sum_m \sqrt{b_m \mathcal{B}_m^{(s)}} \langle \cos \phi_m \rangle. \quad (9)$$

Table 1

Corrections to the published mass values compared to the uncertainties declared in the papers (keV).

Υ -state	$\Upsilon(1S)$		$\Upsilon(2S)$		$\Upsilon(3S)$
	CESR	VEPP-4	DORIS	VEPP-4	VEPP-4
σ_W (MeV)	3.2	4.5	8.1	5.3	5.4
Electron mass	-82	-2	-87	-2	-2
Radiative corrections	-81		-181		
Interference	-71	-112	-168	-105	-130
Resonance shape calculation	+375				
Total	+141	-2	-430	-107	-132
Shift with correct fit	+143	-113	-436	-106	-131
Declared uncertainty:	systematic	70	40	100	<200
	total	130	100	400	500

The sum in Eq. (9) is performed over all hadronic modes, \mathcal{B}_{ee} and \mathcal{B}_h are the resonance decay probabilities to e^+e^- -pairs and hadrons, respectively, $\mathcal{B}_m^{(s)} = \Gamma_m^{(s)}/\Gamma$ is related to the strong contribution to the decay mode m , process, ϕ_m is its phase relative to the electromagnetic contribution and $b_m = R_m/R$ is the branching fraction of the corresponding continuum. The angle brackets denote averaging over the decay products phase space.

Following Ref. [23], we assumed that the relative phases of the strong and electromagnetic amplitudes in hundreds of different decay modes are not correlated and take all possible values. The intensive cancellation takes place in the sum, thus the second term of Eq. (9) can be neglected compared to the first one, which is about 0.31, 0.27 and 0.29 for $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$, respectively. The same value of λ follows from naive parton model in which $q\bar{q}$, ggg and $gg\gamma$ decay modes are considered. For J/ψ -meson the measurement gave $\lambda_{J/\psi} = 0.45 \pm 0.07 \pm 0.04$ at the expected value of 0.39 [23]. Scaling this result to $\Upsilon(nS)$, one obtains the mass uncertainty estimate of about $4.10^{-3}\sigma_W$. The correction grows with the value of the energy spread, see Table 1. Studying of the interference effect is one of the goals for the new experiment planned at VEPP-4M.

5. Reanalysis of CUSB data

During the $\Upsilon(1S)$ mass measurement at the CESR collider with the CUSB detector, 22 runs were recorded, which were jointed in 11 data points for the fit. The beam energy during runs were determined using the value of bend magnetic field measured with NMR according to linear relation $E = A(B - B_0) + C$, where B is a measured NMR value and B_0 is some reference one. The constants A and C were obtained by fitting of 10 measurements of the beam energy with the resonance depolarization method.

The data point number, the NMR value, the number of hadronic events, the integrated luminosity and the cross section for each run were presented at Table I of Ref. [9].

In Fig. 1 the data points calculated by us using Table I and published values of A , C and B_0 are compared with these extracted from Fig. 10 of Ref. [9] using the GIMP graphical editor. Both energies and cross sections of the points agree within the accuracy achieved with the editor. However, the curves of the fits performed using the same formulae and with the same value of the electron mass differ. We have checked our calculation comparing the results obtained in the zero-width approximation using two different implementations of the Weber function and those obtained using numerical convolution of Eq. (6) with the Gaussian energy spread. The three our results agree with each other thus we conclude that the mass value published in Ref. [9] is not fully correct and should be shifted by +0.375 MeV.

There is a question to Table I concerning assignment of the run 14 to the point 8, its NMR value is closer to those of the point 9.

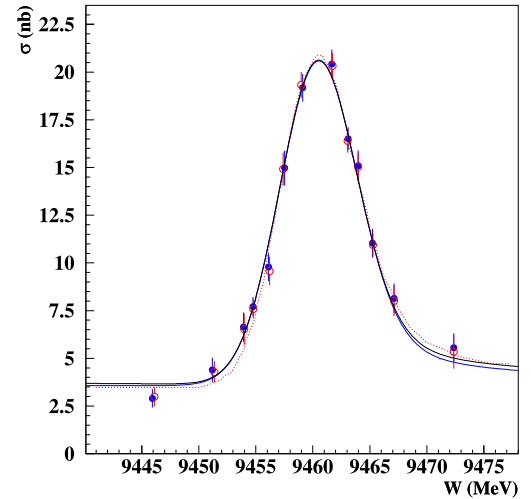


Fig. 1. The profile of the $\Upsilon(1S)$ as measured with the CUSB detector [9]. The red open circles and the dotted curve are digitized from Fig. 10 of the paper. The blue closed circles correspond to the data points from the published table. The blue dashed and black solid curves show our fit to Eq. (3) as in the original work and the fit according to Section 4, respectively.

This might be a misprint. Blue closed points in Fig. 1 are shown for the proper run-to-point assignment. The corresponding change of the mass value is negligible.

The fit of published data with Eq. (3) gives $\chi^2 = 8.32$ for 9 degrees of freedom. The correct fit according to Section 4 gives $\chi^2 = 7.45$, $P(\chi^2) = 0.59$. Note that the data taken from the plots are used only to ensure the absence of essential misprints in published data and to understand the reason of the problem.

6. Values of corrections to $\Upsilon(1S)$ – $\Upsilon(3S)$ masses

The corrections to mass values obtained in five experiments due to effects considered are presented in Table 1. The sum of corrections calculated separately is in a good agreement with the shift obtained with the correct fit. The uncertainty of reanalysis due to accuracy of the vacuum polarization data is 1–2 keV. The errors related to the digitization of journal plots are about 3% of total uncertainties of measurements and do not deteriorate the accuracy of resulting mass values.

7. Conclusion

The results of five experiments on the precision measurements of masses of narrow Υ states were reanalyzed to remove substantial drawbacks of original analyses. The following results were obtained:

$$M_{\Upsilon(1S)} = 9460.40 \pm 0.09 \pm 0.04 \text{ MeV (MD-1 [18])}.$$

$$M_{\Upsilon(1S)} = 9460.11 \pm 0.11 \pm 0.07 \text{ MeV (CUSB [9])}.$$

$$M_{\Upsilon(2S)} = 10023.4 \pm 0.5 \text{ MeV (MD-1 [11])}.$$

$$M_{\Upsilon(2S)} = 10022.7 \pm 0.4 \text{ MeV (ARGUS+CB [10])}.$$

$$M_{\Upsilon(3S)} = 10355.1 \pm 0.5 \text{ MeV (MD-1 [11])}.$$

The discrepancy between MD-1 and CUSB results on the $\Upsilon(1S)$ mass has been reduced from 3.3 to 1.8 standard deviations. The mean value of two experiments, calculated according the PDG rules with the scale factor of 1.8 is

$$M_{\Upsilon(1S)} = 9460.29 \pm 0.15 \text{ MeV}.$$

The uncertainty is reduced from 0.33 to 0.15 MeV. That is important for the precise determination of masses of wide Υ states in BELLE experiment at KEK and for other experiments.

We appeal the Particle Data Group to accept these mass values as it was done with the leptonic widths recalculated in Ref. [16].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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