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Natural Explanation of Recent Results on $e^+e^- \to \Lambda\bar{\Lambda}$

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We show that the recent experimental data on the cross section of the process $e^+e^- \to \Lambda\bar{\Lambda}$ near the threshold can be perfectly explained by the final-state interaction of Λ and $\bar{\Lambda}$. The enhancement of the cross section is related to the existence of low-energy real or virtual state in the corresponding potential. We present a simple analytical formula that fits the experimental data very well.

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Recently, new experimental data have appeared on the cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ annihilation near the threshold [1]. These data are consistent with the results of previous works [2–4], but have much higher accuracy. All these results demonstrate a strong energy dependence of the cross section near the threshold. A similar phenomenon has been observed in such processes as $e^+e^- \to p\bar{p}$ [5–12], $e^+e^- \to n\bar{n}$ [13–15], $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ [16], [17], $e^+e^- \to B\bar{B}$ [18], and others. In all these cases the shapes of near-threshold resonances differ significantly from the standard Breit–Wigner parameterization. The origin of the phenomenon is naturally explained by the strong interaction of produced particles near the threshold (the so-called final-state interaction). Since a typical value of the corresponding potential is rather large (hundreds of MeV), existence of either low-energy bound state or virtual state is possible. In the latter case a small deepening of the potential well leads to appearance of a real low-energy bound state. In both cases, the value of the wave function (or its derivative) inside the potential well significantly exceeds the value of the wave function without the final-state interaction. As a result, the energy dependence of the wave function inside the potential well is very strong. Since quarks in e^+e^- annihilation are produced at small distances of the order of $1/\sqrt{s}$, a strong energy dependence of the cross section is determined solely by the energy dependence of the wave function of produced pair of hadrons at small distances. Such a natural approach made it possible to describe well the energy dependence of almost all known near-threshold resonances (see [19–25] and references therein).

The annihilation $e^+e^- \to \Lambda\bar{\Lambda}$ near the threshold is the simplest for investigation. This is due to the fact that the $\Lambda\bar{\Lambda}$ system has a fixed isotopic spin $I = 0$, and the pair is produced mainly in the state with an angular momentum $l = 0$ (the contribution of state with $l = 2$ can be neglected). Moreover, there is no Coulomb interaction between Λ and $\bar{\Lambda}$. Our analysis shows that the imaginary part of the optical potential of $\Lambda\bar{\Lambda}$ interaction, which takes into account the possibility of annihilation of $\Lambda\bar{\Lambda}$ pair into mesons, has only little effect on the cross section. Therefore, we neglect the imaginary part of the potential. Finally, we describe the cross section of the process $e^+e^- \to \Lambda\bar{\Lambda}$ by a simple analytical formula (see [25] and references therein for more details):

$$\sigma = \frac{2\pi\beta\alpha^2}{s} g^2 F_D^2(s) |\psi(0)|^2, \tag{1}$$

where $\beta = k/M_\Lambda$ is the baryon velocity, $k = \sqrt{M_\Lambda E}$, $s = (2M_\Lambda + E)^2$, E is the kinetic energy of the pair, $F_D(s) = \left(1 - s/\Lambda^2\right)^{-2}$ is the dipole form factor, and Λ is some parameter close to 1 GeV. The factor g is related to the probability of pair production at small distance $\sim 1/\sqrt{s}$ and can be considered as a constant independent of energy. In Eq. (1), $\psi(0)$ is the wave function of $\Lambda\bar{\Lambda}$ pair at $r = 0$.

The cross section (1) is enhanced by the factor $|\psi(0)|^2 \gg 1$ if there is a loosely bound state or a virtual state of $\Lambda\bar{\Lambda}$ pair. In both cases the modulus of scatter-

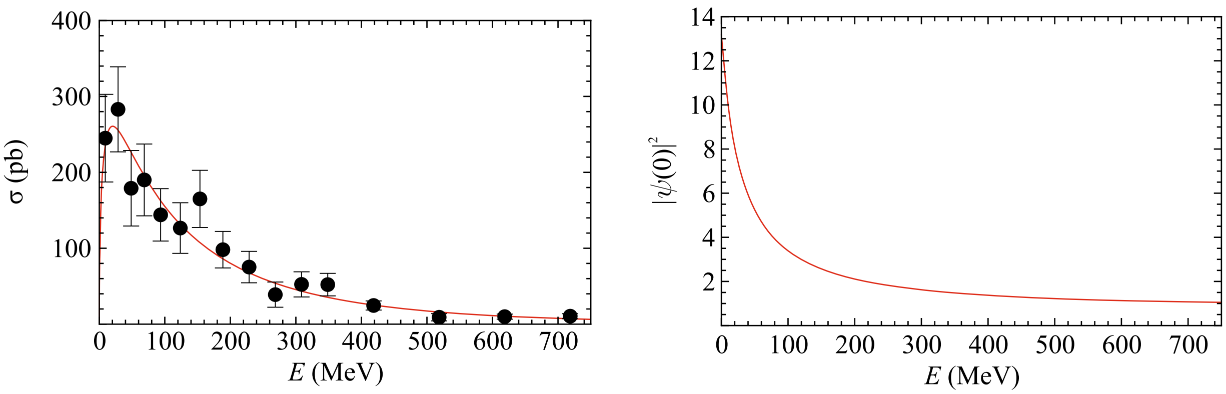

Fig. 1. (Color online) Cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ annihilation (left) and the enhancement factor $|\psi(0)|^2$ (right) as the functions of energy E. The parameters of the potential are $U_0 = 584$ MeV and $R = 0.45$ fm. Experimental data are taken from [1].

ing length a of Λ and $\bar{\Lambda}$ is large compared to the characteristic radius R of $\Lambda\bar{\Lambda}$ interaction potential, $|a| \gg R$. For a loosely bound state a is positive and the binding energy is $\varepsilon = -1/M_\Lambda a^2$. For a virtual state a is negative and the energy of virtual state is defined as $\varepsilon = 1/M_\Lambda a^2$. In both cases $|\varepsilon|$ is much smaller than the characteristic depth of the potential well. The energy dependence of $|\psi(0)|^2$ for near-threshold resonances is more or less universal and is determined by the scattering length a and the effective radius of interaction [26]. Therefore, one can use any convenient form of potential $U(r)$ for description of near-threshold resonances.

In the present paper we parametrize the potential as $U(r) = -U_0\theta(R - r)$. For this potential, the energy dependence of $|\psi(0)|^2$ is well-known (see, e.g., [26]):

$$|\psi(0)|^2 = \frac{q^2}{q^2\cos^2(qR) + k^2\sin^2(qR)}, \quad q = \sqrt{M_\Lambda(E + U_0)}. \tag{2}$$

Near the threshold $k \ll q$ and the cross section (1) is enhanced if

$$q_0R \approx \pi\left(n + \frac{1}{2}\right) + \delta, \quad |\delta| \ll 1, \tag{3}$$

where $q_0 = \sqrt{M_\Lambda U_0}$, and n is an integer. For $|\delta| \ll 1$, the scattering length is $a = 1/q_0\delta$, where $\delta > 0$ for the bound state, and $\delta < 0$ for the virtual state.

By means of Eq. (3) the expression (2) can be simplified:

$$|\psi(0)|^2 \approx \frac{\gamma U_0}{(E + \varepsilon_0)^2 + \gamma E}, \quad \gamma = 4\kappa^2 U_0, \quad \varepsilon_0 = 2\delta\kappa U_0, \quad \kappa = \frac{1}{\pi(n + 1/2)}. \tag{4}$$

The corresponding energy dependence of the cross section (1) is equivalent to the Flatté formula [27], which is expressed in terms of the scattering length and the effective radius r_0 of interaction. Note that for the rectangular potential well $r_0 = R$. One can easily verify that the precise and approximate formulas for the cross section are in good agreement with each other for $|\delta| \ll 1$ and $E \lesssim \varepsilon_0 \ll U_0$. Note that $|\varepsilon_0| \gg |\varepsilon|$ for both bound and virtual states, namely $\varepsilon_0 \approx 2|\varepsilon|a/R$. However, the position of peak in the cross section, which is proportional to $\sqrt{E}|\psi(0)|^2$, is located at energy $E \approx |\varepsilon|$ for both bound and virtual states.

In Fig. 1, we show our predictions for the cross section compared to experimental data [1], as well as the enhancement factor $|\psi(0)|^2$. The parameters of the model are $U_0 = 584$ MeV, $R = 0.45$ fm, and $g = 0.2$. In the energy region under consideration the dependence of our predictions on the parameter Λ is very weak. To be specific, we set $\Lambda = 1$ GeV. Our model, giving $\chi^2/N_{\rm df} = 9.8/13$, provides a good description of experimental data [1]. Note that account for the enhancement factor $|\psi(0)|^2$ is of great importance for correct description of experimental data.

Within our model, we also predict a bound state with the binding energy $E_0 \approx -30$ MeV. Observation of this bound state would be very important. The results of [28–36] indicate the anomalous behavior of the cross sections $e^+e^- \to K^+K^-\pi^+\pi^-$, $e^+e^- \to 2(K^+K^-)$, $e^+e^- \to \phi K^+K^-$, and others at $\sqrt{s} \approx 2.2$ GeV (this value of s corresponds to $E \approx -30$ MeV). However, a more detailed study of this energy region is required.

In conclusion, the assumption of existence of a low-energy real or virtual state has allowed us to describe perfectly recent and previous experimental data for the cross section of $e^+e^- \to \Lambda\bar{\Lambda}$ annihilation near the threshold. Our model indicates possible existence of a bound $\Lambda\bar{\Lambda}$ state with energy $E \approx -30$ MeV.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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