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Tuning Phase Errors of a Superconducting Undulator

P. V. Kanonik[a, *], **V. A. Shkaruba**[a, b], **A. A. Volkov**[a, b], **A. I. Erokhin**[a], **A. V. Zorin**[a], **F. P. Kazantsev**[a], **N. A. Mezentsev**[a, b], **O. A. Tarasenko**[a], **S. V. Khrushchev**[a, b], **and V. M. Tsukanov**[a, b]

[a] *Budker Institute of Nuclear Physics, Siberian Branch, Russian Academy of Sciences, Novosibirsk, 630090 Russia*

[b] *Siberian Circular Photon Source Synchrotron Radiation Facility, Boreskov Institute of Catalysis, Siberian Branch, Russian Academy of Sciences, Koltsovo, 630559 Russia*

**e-mail: kanpoul@mail.ru*

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Abstract—The authors describe a way of tuning the magnetic field of a superconducting undulator with neutral poles developed at the Budker Institute of Nuclear Physics, along with a mathematical apparatus for calculating additional power currents for tuning a magnetic field. The field and orbit inside the undulator are tuned by primary undulator windings arranged into separate groups and powered by additional currents. The tuning circuit is tested, and theoretical and experimental data on the measured magnetic field and calculated phase errors are compared. Spectra of synchrotron irradiation before and after tuning are calculated with the SPECTRA software.

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INTRODUCTION

Synchrotron radiation (SR) is currently an important tool for studying the properties of substances. It is generated when a beam of electrons moves in a magnetic field. Sources of SRs include multipole insertion based on superconducting technologies. An advantage of these sources is that they greatly increase the spectral brightness of the radiation flux with no appreciable changes in the ring parameters. Such devices are subdivided into two types: wigglers and undulators. The magnetic field in wigglers and undulators is characterized by sign-variable periodic structures in the longitudinal direction with parameters of deflection $K = 0.934B\lambda$, where K/γ is the maximum angle of deflection of the orbit of an electron in the magnetic structure, B is the amplitude of magnetic field (T), λ is the period (cm), and γ is the relativistic electron factor. The periodic structure is considered to be an undulator if $K < 3$, and a wiggler if $K > 3$.

A qualitative property of an undulator's magnetic field is its phase error. It is important to reduce the root-mean-square deviation of the phase error in order to increase the spectral brightness of the source.

The Budker Institute of Nuclear Physics is developing a superconducting undulator for the Diamond Light Source (DLS) national synchrotron facility in Great Britain. According to the customer's requirements, the root-mean-square deviation of the local phase error must be no more than 3°. To meet these requirements, research on achieving the required phase error began in 2017. The procedure for winding coil was first upgraded by adding a core strap to improve the order of winding layers. The coils were then sorted using a micrometer. Finally, a system of magnetic field tuning consisting of 24 sources of current was developed that can add power to the coils both individually and in groups.

UNDULATOR

Our superconducting undulator is characterized by a maximum field in a median plane of 1.2 T composed of 120 periods. A period is created on an iron pole with winding (an active pole) and an iron pole without winding (a neutral pole). The length of an undulator period is 1.56 cm. A special yoke with grooves for the active and neutral poles is used to improve the accuracy of a coil's position. Two such yokes are then installed in front of each other, joined, and placed into a cryostat.

The upper part of the magnet is displaced relative to the lower part by a half of period. Each active pole is therefore located opposite the neutral pole. The structure with the neutral pole operates with half the number of coils and better accuracy of positioning.

The active pole is 11 mm wide, 146 mm long, and 13.9 mm tall. The active pole's winding is made of Nb-Ti wire 0.55 mm in diameter on an iron core. A winding is 7 layers wide, with 12 or 11 turns in a layer. The total number of turns in the main poles is 81. The neutral pole is 4.2 mm wide and ~100 mm long. These parameters of active and neutral poles were optimized to ensure the maximum magnetic field in the median

Table 1. Parameters of our superconducting undulator

Parameter	Value	Unit of measurement
Magnetic length	2	m
Total length of the undulator	2486	mm
Period	15.6	mm
Field on axis	>1.2	T
K (parameter of undulator strength)	>1.7367	—
Minimum aperture	6	mm (vertical)
	60	mm (horizontal)
First integral of the field	<±0.5	G m
Second integral of the field	<±1	G m^2

plane and meet the requirements for the homogeneity of the magnetic field.

More detailed properties of the undulator are summarized in Table 1 and [5, 6].

PHASE ERRORS

Due to the curvature of the trajectory and difference between the speed of an electron and the speed of light, there is a delay between the emitted waves (which at each half period should be constant) when an electron moves along the undulator's axis. Phase errors occur due to inaccuracies in manufacturing the magnet that reduce wave coherence.

The delay phase between an electron and a light wave is calculated numerically using Eqs. (1)–(3). The delay phase in a half period is calculated according to Eq. (4). The root-mean-square of these differences is local phase error (5), and the root mean square of function (6) is cumulative phase error (7):

$$\varphi(z) = \frac{2\pi}{\lambda_{\text{rad}}}\left(\frac{z}{2\gamma^2} - \frac{1}{2}\int_0^z x'^2 dz\right), \quad (1)$$

$$x'(z) = \frac{e}{\gamma mc}\int_0^z B(z)\,dz, \quad (2)$$

$$\lambda_{\text{rad}} = \frac{\lambda_0}{2\gamma^2}\left(1 + \frac{K^2}{2}\right), \quad (3)$$

where x' is the function of the beam's angle of inclination relative to the undulator's axis, B is the undulator's magnetic field, z is a coordinate, m is the electron weight, c is the speed of light, γ is the electron beam Lorentz factor, λ_{rad} is the radiation's wavelength, λ_0 is the undulator's period, and K is the parameter of the undulator's strength:

$$\psi_i = \varphi(z_{i+1}) - \varphi(z_i), \quad (4)$$

$$\sigma_\psi = \sqrt{\frac{\sum_{n=0}^{N}\left(\psi_n - \overline{\psi}\right)^2}{N}}, \quad (5)$$

$$\phi_i = \sum_{n=0}^{n=i}(\psi_n - i\overline{\psi}), \quad (6)$$

$$\sigma_\phi = \sqrt{\frac{\sum_{n=0}^{N}\left(\phi_n\right)^2}{N}}, \quad (7)$$

where N is the number of undulator poles, i is the number of a pole, z_i is the zero coordinate of the undulator's magnetic field, and $\overline{\psi}$ is the average delay phase for all half periods (4).

Phase errors have an especially strong effect on obtaining high frequency undulator harmonics. It was shown in [3] that the ratio of the maxima of the spectral brightness of undulator SR with a phase error to those of the spectral brightness with no phase error is determined by coefficient R:

$$R = \exp\left(-n^2\sigma_\phi^2\right) \quad (8)$$

where n is the number of the harmonic and σ_ϕ is the cumulative phase error.

It was shown in [1] that Eq. (8) in some cases underestimates the undulator's spectral brightness. The author of [2] derived an equation that describes the influence of phase errors and considers all possible beam sizes and different values of cumulative phase errors. He also proved this equation is ultimately reduced to Eq. (8) in the beam limit of high sizes, where σ_ψ is used instead of σ_ϕ

In estimating the effect phase errors have on an undulator's spectrum, it is useful to consider different phase errors caused by the size of the beam itself. According to Eq. (8), a local phase error of 3° is sufficient to obtain 15–16 harmonics on existing SR sources of the third generation. In light of the beam parameters of the SKIF synchrotron under development, it is recommended to consider a cumulative phase error of 3° when tuning the undulator.

TUNING SYSTEM

Two steps are needed when tuning the magnetic field of a superconducting undulator. The required angle of entering the undulator is first preset, and the parasitic vertical component of the field is eliminated. This is done with special additional coils and a stretched wire [8], or magnetic measurements made by the Hall sensor.

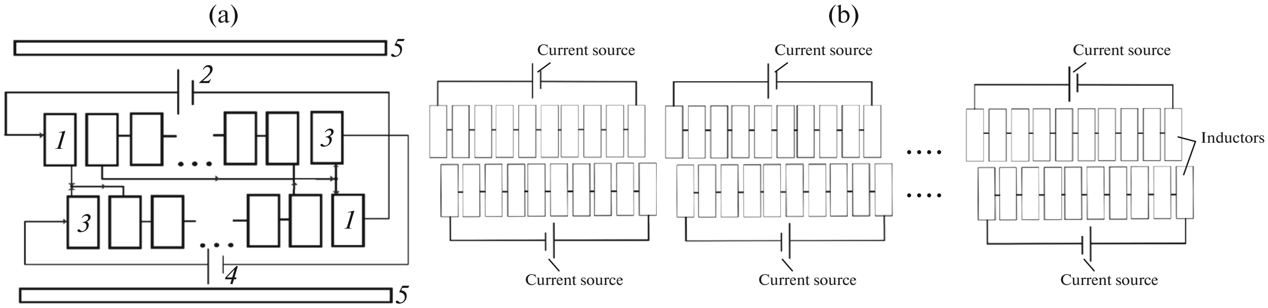

Fig. 1. (a) Schematic view of the undulator and the switching on of currents: (*1*, *3*) edge tuning coils, (*2*, *4*) current sources, (*5*) large coils compensating for the constant vertical component of the magnetic field. (b) Schematic view of the switching on of additional sources of current to arranged poles.

The undulator's power system contains two sources of 300 A current. One powers the first coil; the other, the second coil. The currents are then combined, and the cumulative current passes though the remaining coils. The angles of entering the undulator can be varied by tuning the ratio of the current sources (Fig. 1a).

With neutral poles, the molecular currents create an additional parasitic vertical component of the magnetic field that is compensated for by large windings positioned above each coil (Fig. 1a).

The second step of tuning follows when the required angle of entry is set and the parasitic component of magnetic field is eliminated. To reduce scattering of the amplitudes of the magnetic field and flatten the orbit of the beam inside the undulator, the coils are subdivided into 24 groups of 10 pieces and powered by additional sources of current (Fig. 1b).

Mathematical calculations of tuning currents are based on a preliminarily measured magnetic field in a horizontal cryostat using a HGT 1050 Hall sensor.

A set of additional currents is calculated on the basis of a function that describes the addition of a magnetic field from one coil to the field of the undulator when a current is applied to the former. This function was obtained by modeling an undulator in the Mathcad software and approximated analytically.

The design of the magnet is such that the upper and the lower groups of coils are displaced relative to each other by a half period (Fig. 1b). The trajectory of the beam therefore does not change when the currents in oppositely positioned coils are equal, but it does when they are different. The additional current then consists of two parts: one used to bring the average field of the group closer to that of the undulator, and one used to flatten the trajectory of the electron beam inside the undulator.

The first addition is calculated with an approximated function of adding a magnetic field from one coil. The second is calculated according to least squares (9). Below is a matrix whose solution gives the second addition of currents (10):

$$\sum\left(X_i' - \alpha_1 U_{1i} - \alpha_2 U_{2i} - \ldots - \alpha_{12} U_{12i}\right)^2 \to \min, \quad (9)$$

$$\cdot\begin{pmatrix}\alpha_1\\ \alpha_2\\ \vdots\\ \alpha_{12}\end{pmatrix} = \begin{pmatrix}\sum U_{1i}U_{1i} & \sum U_{2i}U_{1i} & \ldots & \sum U_{12i}U_{1i}\\ \sum U_{1i}U_{2i} & \sum U_{2i}U_{2i} & & \sum U_{12i}U_{2i}\\ \vdots & \vdots & \ddots & \vdots\\ \sum U_{1i}U_{12i} & \sum U_{2i}U_{12i} & \cdots & \sum U_{12i}U_{12i}\end{pmatrix}^{-1}\begin{pmatrix}\sum X_i' U_{1i}\\ \sum X_i' U_{2i}\\ \vdots\\ \sum X_i' U_{12i}\end{pmatrix}, \quad (10)$$

where X_i' is the first integral of the undulator up to the *i*th coordinate with no tuning by currents, and U_{ji} is an addition to the first integral of the *j*th group from a current of 1 ampere without allowing for the field from other groups. Coefficients α_j are values of the second group of additional currents for each group.

Depending on the calculated results, different sets of coils are powered by additional currents to reduce as much as possible the absolute value of scattering in the peaks of magnetic fields and flatten the trajectory of particles.

EXPERIMENTAL TESTS OF THE TUNING SYSTEM

The system for tuning an undulator's magnetic field is adjusted at this stage. Currents are calculated mathematically and supplied to the groups of coils. The magnetic field is measured experimentally in order to adjust the currents for the required number of times.

Results are given below. Figure 2a presents plots of additional currents for a field of 1.15 T. Figure 2b

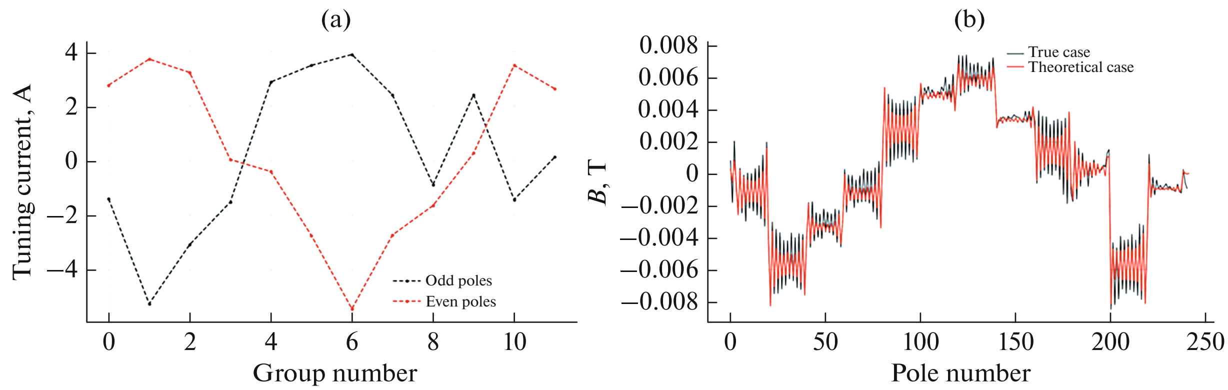

Fig. 2. (a) Additional currents for each group of undulator poles with a magnetic field of 1.15 T. (b) Difference between amplitudes of the magnetic field as a function of pole number before and after tuning, calculated theoretically and obtained experimentally.

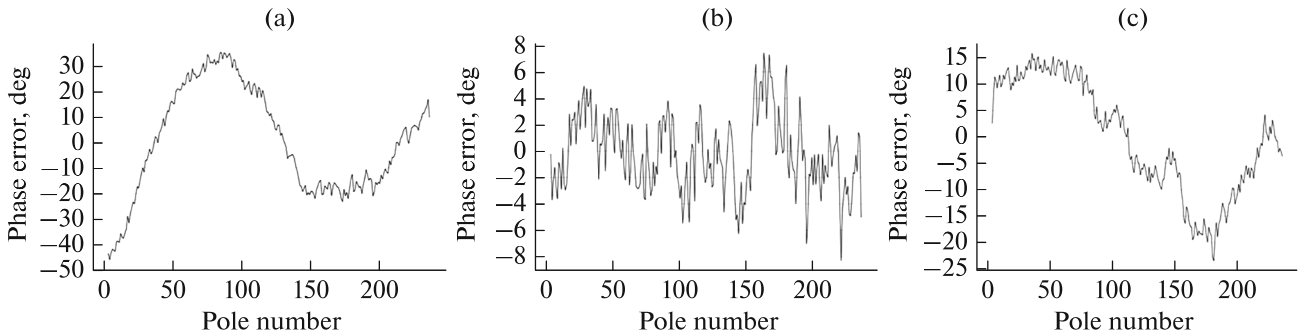

Fig. 3. Cumulative phase error as a function of pole number (a) before tuning, (b) as a consequence of theoretical tuning, and (c) experimental.

shows plots of the difference between magnetic field amplitudes as a function of pole number before and after tuning, calculated theoretically and obtained experimentally. The root-mean-square deviation of the two plots is 0.001 T. It was found experimentally that it is the value of variations in measurements by the Hall sensor in one and the same magnetic field of the undulator.

Below, Fig. 3 presents plots of the cumulative phase error: (a) before tuning, (b) theoretical, and (c) experimental. The cumulative phase error was 20.8° and 10.1° before and after tuning, respectively. Even though the cumulative error was halved, it was not enough to reach 3°. The local phase error was reduced from 2.5 to 1.9, which coincided with the theoretical value. Figure 4 shows theoretical and experimental

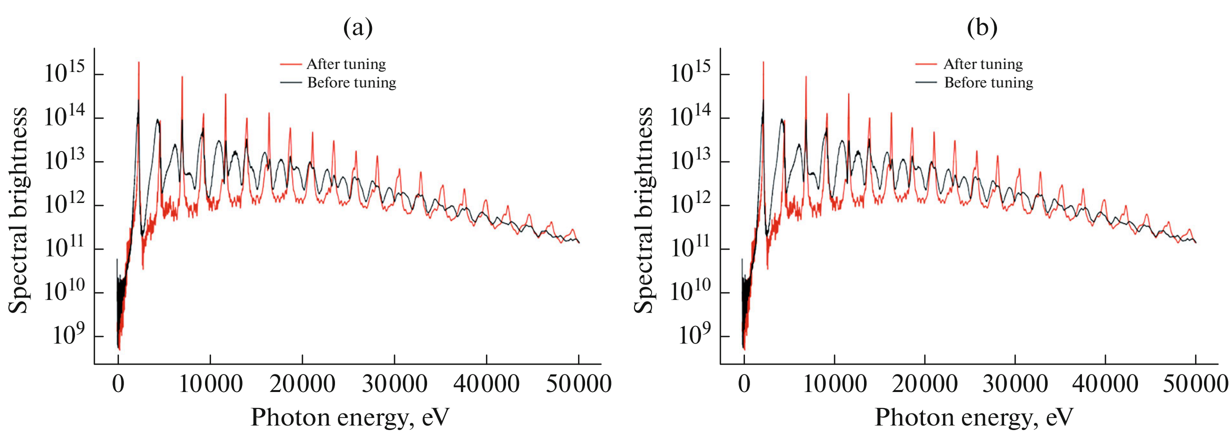

Fig. 4. Distribution of calculated spectral brightness before and after tuning: (a) theoretical and (b) experimental for beam parameters from Table 2.

Table 2. Parameters for calculating SR spectra in the SPECTRA software (ver. 11)

Current in accumulator *I*	400 mA
Electron energy *E*	3 GeV
Energy scatter in electron beam $\Delta E/E$	10^{-3}
Horizontal emittance of electron beam σ	5×10^{-9} m rad

plots of the distribution of calculated spectral brightness before and after tuning. The values of spectral brightness generally coincide with the theoretical calculations for beam sizes according to DLS (Table 2).

CONCLUSIONS

It was shown that group tuning by currents is effective. Using it substantially reduced local and cumulative phase errors. The former was 1.9° vs. 2.5°; the latter, 10° vs. 20.8°. On average, the calculated spectral brightness of the undulator also grew by 4.6 times in the first 10 peaks. Theoretical tuning brought the cumulative phase error to 3°, giving us an incentive to continue working in this direction. In the future, we will also need to reduce the parasitic component of the magnetic field and select tuning currents for all levels of the undulator's working magnetic field.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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