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Dynamic Aperture of Synchrotron with Electron Cooling

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Abstract—The heavy ion facility for technological applications is developed at the Budker Institute of Nuclear Physics, Siberian Branch, Russian Academy of Sciences. The booster synchrotron with electron cooling is one of the main parts of the facility. This work presents the beam dynamics simulation with betatron coupling and nonlinearities of the guided magnetic field. The transverse betatron coupling excited by electron cooling solenoid was compensated by the pair of skew quadrupole triplets with antisymmetric supply and located symmetrically relative to the solenoid center. The calculation of the vertical dispersion excited by the magnetic field of toroidal sections of the electron cooler was performed. The accelerator lattice was optimized for minimization of vertical dispersion. Another important factor of the beam dynamics is the tune dependence versus momentum deviation called chromaticity. To correct chromaticity, sextupole magnets are applied. On the other hand, sextupoles excite nonlinear resonances that can lead to significant limitation of the dynamic aperture. The dynamic aperture of the synchrotron was simulated by scanning of horizontal and vertical tunes in conditions of chromaticity and betatron coupling suppression. This method makes it possible to identify dangerous resonances. According to the calculations presented, a scheme with application of six sextupole families are developed. This scheme makes it possible to significantly reduce the influence of most dangerous resonances. The wide area of tunes with a fairly large dynamic aperture for particles with required momentum spread was found in simulations. Selection of the operating point in this particular area makes it possible to reduce the space charge effect on the dynamic aperture. At the chosen operating point, the influence of eddy currents and magnetic fields nonlinearities on the dynamic aperture was investigated.

Keywords: synchrotron, electron cooling, dynamic aperture

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INTRODUCTION

At the present time, electron cooling is widely applied at low and medium energy ion facilities for medical, scientific, and other applications, e.g., [1–5]. The Budker Institute of Nuclear Physics, Siberian Branch, Russian Academy of Sciences, is developing the ion acceleration facility for technological applications. The facility consists of two linear accelerators, the booster synchrotron with electron cooling and the main synchrotron. The accelerator facility is designed for ion beam generation in a wide range of species: from protons to bismuth ions.

The mirror-symmetric lattice of the booster synchrotron consists of two achromatic arches and pair of long dispersion-free drifts. A missing bend scheme is applied for dispersion suppression in drifts. The first drift contains an electron cooler and a pair of quadrupole triplets, which ensure quite large and equal beta functions in the electron cooling section. This makes it possible to reduce the angular spread of the ion beam in the interaction section with the electron beam, which leads to an increase in efficiency of electron cooling. The other drift space is intended for the placement of injection-extraction elements and RF cavity. A sketch of the booster synchrotron is shown in Fig. 1; the lattice functions are presented in Fig. 2. The main parameters of the booster synchrotron are listed in Table 1.

The single-turn injection in the vertical plane with horizontal outward beam displacement is applied. To

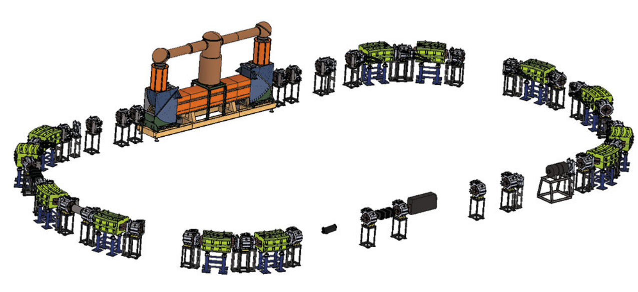

Fig. 1. Sketch of the booster synchrotron.

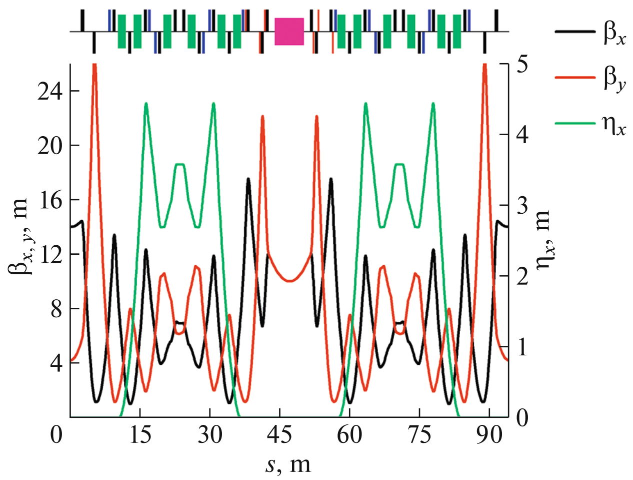

Fig. 2. Optical functions of the booster synchrotron.

enable the accumulation of high intensity beams, a well-known kick/pre-kick system is proposed. In this method, the new portion of beam is injected into free phase space that has appeared upon cooling the previous portions. The synchrotron has two extraction modes: the slow extraction at $\nu_x = n/3$ resonance with amplitude driven RF knockout and single-turn extraction at maximum energy.

The main effects that influence the transverse beam dynamics in the booster synchrotron are the linear betatron coupling excited by the solenoid of cooling section, chromatic sextupoles, nonlinearities of magnets, eddy currents in the walls of vacuum chamber during acceleration, and space charge phenomena.

Table 1. Main parameters of booster synchrotron

Parameter	Value	
Species	p	$^{209}Bi^{41+}$
Injection energy, MeV/nucleon	7.5	4
Extraction energy, MeV/nucleon	7.5–700	4–36
Revolution frequency, MHz	0.4–2.612	0.261–0.861
Magnetic field, T	0.142–1.6	
Repetition rate, Hz	1	
Circumference, m	94.1	
Max. β_x/β_y, m	15/25.6	
Max. η_x, m	4.25	
Betatron tunes, ν_x/ν_y	3.328/3.18	
Momentum compaction factor α	0.1153	
Transition energy	2.94	
Natural chromaticity, x/y	−3.08/3.25	
Acceptance, x/y	25/8.5 π cm mrad	
Momentum spread $\delta = \Delta p/p$, %	0.5	
Booster dimensions, m	34.7 × 19.75	

The longitudinal magnetic field of the cooling section solenoid excites the transverse betatron coupling. The betatron coupling leads to a decrease in the beam dynamic stability, mismatching of optical functions, shift of tunes, and excitation of vertical dispersion; therefore, coupling compensation is required.

Sextupole magnets located in arcs and intended for the control of linear chromaticity can excite nonlinear resonances, leading to limitations of dynamic aperture. Nonlinear fringe fields of the main magnetic elements can also lead to the excitation of nonlinear resonances and, as a consequence, to additional limitations of dynamic aperture.

Eddy currents are induced during the acceleration in the walls of the dipole vacuum chamber. The magnetic fields of the eddy currents generally contain dipole and sextupole harmonics, whose values are proportional to slope of magnetic field ramp. Therefore, the influence of eddy currents on the transverse dynamic aperture is most significant at the injection energy.

One of the main tasks is researching the space charge effects on beam dynamics. This development is the most complex and its consideration is planned in the future, taking into account the results presented in this paper.

BETATRON COUPLING COMPENSATION

The electron cooling method is based on the conjugation of ion and electron beams, which move with the same average velocity [6]. Thus, the interaction between beams is for the most part effective, which leads to equalization of their temperatures, i.e., cooling of the "hot" ion beam due to Coulomb collisions with the gas of "cold" electrons. The cooling occurs in a fairly strong longitudinal magnetic field of the solenoid, which increases the frictional force significantly. The cooling process continues until the temperature of the protons at the center of mass becomes equal to the effective temperature of the magnetized electrons.

The electron cooling is applied in the booster synchrotron to achieve low transverse emittances, low momentum spread, and high intensity. The 3D model of the electron cooling system is shown in Fig. 3. The main parameters of the cooler are presented in Table 2. The electron cooling system contains an electron gun, collector, accelerating and decelerating sections, and cooling section solenoid and toroids. The toroidal sections are used for conjugation and disjunction of electron and ion beams. Thus, the combination of the cooling solenoid longitudinal field and toroidal fields influences the beam particles. To provide the integral compensation of vertical magnetic field, horizontal dipole correctors are installed at the edges of the

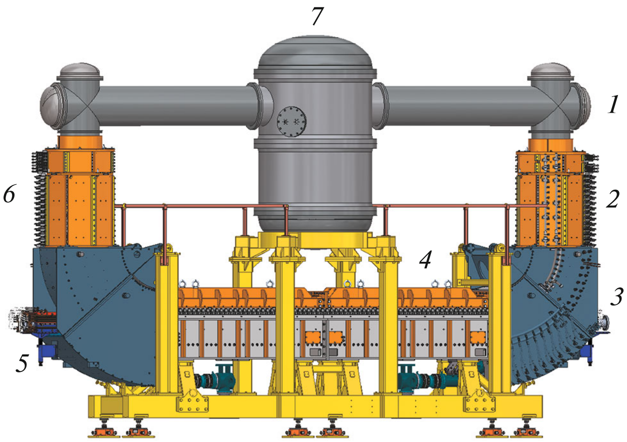

Fig. 3. 3D model of electron cooling system. (*1*) Electron gun, (*2*) accelerating section, (*3*) toroidal section, (*4*) cooling section solenoid, (*5*) dipole corrector, (*6*) decelerator section, (*7*) high voltage terminal.

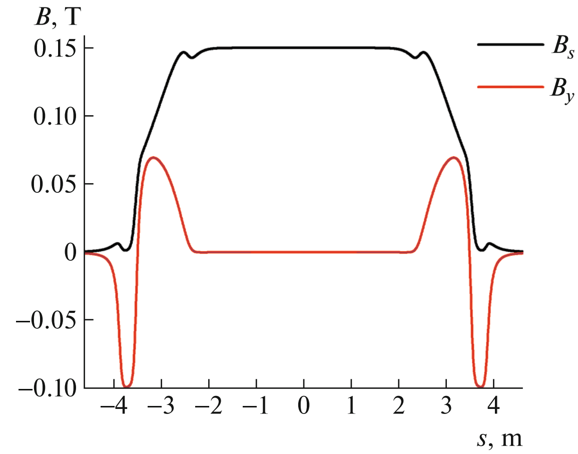

Fig. 4. Magnetic field profile in electron cooler.

cooler. The total calculated magnetic fields on the ion beam path are presented in Fig. 4.

The magnetic field in the cooling section should have highly homogeneity (10^{-4}–10^{-5}) to provide highly efficient cooling. Conservation of the required field quality at fast ramping of solenoid current during beam acceleration is very difficult technically. Thus the magnetic field of the cooling solenoid is constant in the acceleration cycle (for protons, $B = 5 \times 10^{-2}$ T; for heavy ions, $B = 0.15$ T). The longitudinal magnetic field of the cooler excites transverse betatron coupling. The coupling coefficient is changed in the acceleration cycle and has the maximum value near the proton injection energy. Moreover, the longitudinal solenoid field induces a fairly strong additional focusing, which leads to the shift of tunes and mismatching of optics. An optimal solution is to localize coupling in the dispersion-free drift to exclude generation of vertical dispersion and conserve optics in arcs.

The problem of transverse coupling compensating excited by the solenoid is discussed in detail in [7, 8]. In most of the cases considered, betatron coupling is rather small. The transverse betatron coupling in the solenoid can be characterized by the Larmor angle of ion rotation. In the considered solenoid, the maximum Larmor angle $\varphi_L = \frac{\int Bdl}{2HR} = 24°$ is achieved at the injection energy for protons. In other words, the betatron coupling in our case cannot be assumed to be small. Several schemes of couple compensation were considered: scheme with anti-solenoids, scheme with magic skew quadrupoles, and scheme with a pair of skew quadrupole triplets.

First of all, the scheme with anti-solenoids was considered owing to its simplicity and widespread use [4]. The anti-solenoids should be located in a close neighborhood to the solenoid to provide an effective compensation, and the total integral of the longitudinal magnetic field should be equal to zero. However, the placement of anti-solenoids requires an additional space between quadrupoles in the cooling section. This leads to a significant growth of beta functions, which decreases the booster acceptance. Therefore, the anti-solenoid scheme is not efficient in the considered case. Moreover, anti-solenoids are quite bulky and have a significant energy consumption.

The Steffen scheme [8], often called the “magic” skew quadrupole scheme, was proposed and implemented at DESY (1982). This well-known scheme uses a pair of skew quadrupoles and is applied in colliders to compensate the betatron coupling excited by the detector solenoid, e.g., [9]. This scheme assumes the smallness of Larmor angle in the solenoid. However, the linear coupling can be compensated, but the optics of cooling drift cannot be matched with arcs. Moreover, skew quadrupoles should be located at so called “magic” places, whose position should be changed depending on the energy. Thus, the “magic” skew quadrupole scheme is not effective in considered case.

The disadvantages of the “magic” skew quadrupoles scheme can be eliminated by adding two pairs of skew quadrupoles. In this case, the pair of skew quadrupole triplets should be located symmetrically relative to the solenoid center and antisymmetric supply.

Table 2. Main parameters of electron cooling system

Electron beam energy, keV	To 350
Total length, m	8
Cooling section length, m	4.8
Effective solenoid length, m	6.51
Magnetic field, T	0.05–0.15
Vacuum chamber aperture, mm	∅150

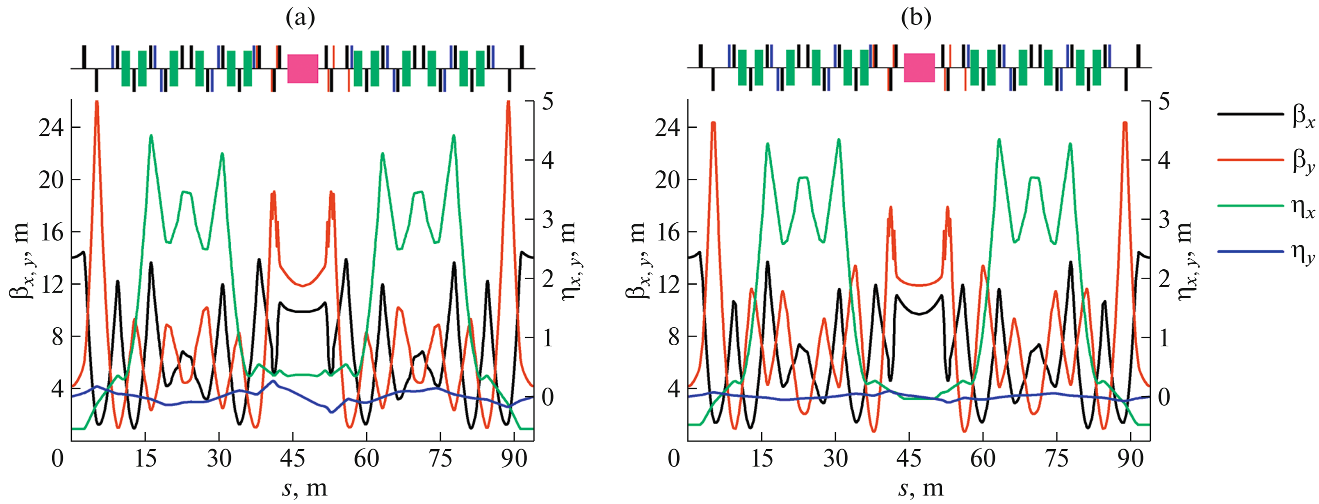

Fig. 5. Lattice functions of booster synchrotron (a) only with betatron coupling compensation, (b) with coupling compensation and verical dispersion suppression.

This decision makes it possible to localize the betatron coupling in the electron cooler drift and conserve the optics in arcs. According to the calculations the skew quadrupole with effective length $L = 0.153$ m and maximum field gradient $G = 2.2$ T/m was designed. Moreover, the behavior of the current for skew quadrupoles and quadrupoles in the electron cooling section during the acceleration was determined. The vertical magnetic field generated by toroids leads to the excitation of horizontal and vertical dispersions, even in the case of compensated betatron coupling. The lattice functions with coupling compensation for protons at injection are shown in Fig. 5a. By tuning of the booster optics, the excited dispersion in the cooling section can be reduced by a factor of approximately 5 with coupling compensation, as shown in Fig. 5b.

DYNAMIC APERTURE SIMULATION

The area of stable motion of circulating particles in lattice with nonlinearities called as dynamic aperture is an important characteristic of accelerator. In general case the strength of nonlinearities depends on amplitudes of betatron and synchrotron motion. In the present work the transverse dynamic aperture is considered, without taking into account synchrotron motion. The following proton beam parameters were used in the calculations: momentum spread during injection $\delta = \pm 5 \times 10^{-3}$, during adiabatic capture $\delta = \pm 14 \times 10^{-3}$, at maximum energy $\delta = \pm 1.24 \times 10^{-3}$. The calculated dynamic aperture is compared with the geometric aperture of elliptical vacuum chamber with size of 150 × 65 mm.

Initially, two sextupole families located symmetrically in the arcs at places with maximum dispersion and maximum split of beta functions were supposed to be applied to control the chromaticity. In addition, two resonance sextupoles are located symmetrically in the injection-extraction drift, which are applied for slow extraction on a third-order resonance. All sextupoles have the same design and the total number of magnets is 10.

The dynamic aperture was calculated for particles with different momentum deviation at the operating point for slow extraction $\nu_x = 3.283$ and $\nu_y = 3.18$ (see Fig. 6). In this case, two sextupole families compensate natural chromaticity, resonance sextupoles are turned off, and the coupling is localized in the cooling section. The dynamic aperture in this configuration is smaller than the geometric one, as can be seen in Fig. 6. Moreover, stability region for particles with negative momentum deviation is decreased. The obtained result was expected, because for injection and acceleration another operating point is needed, located not far from the extraction tune, for simple retuning of the optics before extraction.

For designation of a new operating point of the booster synchrotron the scanning of vertical and horizontal actions that limit the region of stable motion was calculated (see Fig. 7). The scan was performed near the operating point for extraction. During the scanning process, the optics was tuned, maintaining the basic ring properties: symmetry of the optics; achromaticity of the arcs; the horizontal beta functions close to vertical in the cooling section; the value and angle of slope of the horizontal beta function on the injection section; limitations of maximum values for beta functions in every accelerator section, i.e., maintaining booster acceptance. In addition, the betatron coupling and chromaticity were compensated at every operating point. However (see in Fig. 7), the scanning cleared up the presence of large number of resonances leading to the losses of particles. Moreover, the islands of stable motion are not wide enough, which is neces-

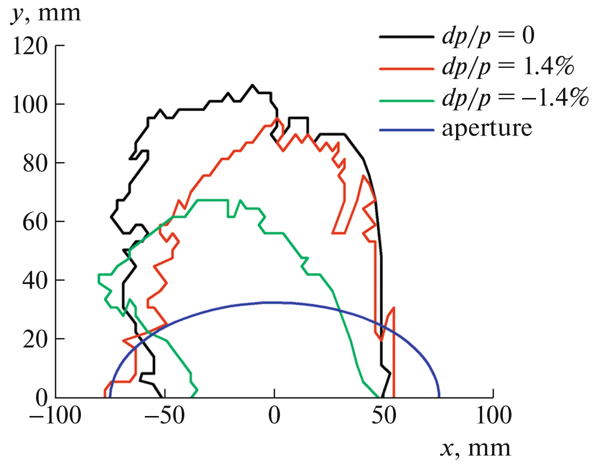

Fig. 6. Dynamic aperture with two sextupole families.

sary owing to the large shift of tunes caused by space charge effects. These results can be explained by non-optimal location of chromatic sextupoles that leads to excitation of dangerous low-order resonances. Their classification is presented in Table 3.

Calculation of the dynamic aperture was simulated with the MAD-X PTC library (polymorphic tracking code) application [10].

SUPPRESSION OF DANGEROUS RESONANCES

To ensure a sufficiently large area of stable motion in tune space, it was proposed to develop a special scheme for suppression of most dangerous resonances excited by chromatic sextupoles. This decision was accepted because of the inability of another placement for chromatic sextupoles in arcs. The suppression of resonances was performed as follows: chromatic sextupoles were separated into four families; two additional families were organized from a pair of resonance sextupoles and a pair of additional sextupoles installed at the ends of the cooling section. Thus, the scheme contains six sextupole families, and every family includes two sextupoles conserving the ring symmetry. In the proposed scheme, it was necessary to abandon the dividing of families by functions; i.e., chromatic sextupoles are needed both for chromaticity correction and for suppression the resonances.

Suppression of the resonances should be performed in a wide range of tunes owing to a significant space charge tune shift. Thus, it was decided to uniformly reduce the amplitudes of the dangerous resonances.

According to [11], the Hamiltonian of the isolated resonance $n_x\nu_x + n_y\nu_y = n$ for small detuning $\varepsilon = n_x\nu_x + n_y\nu_y - l$ can be written in action-angle variables with the time variable θ (azimuthal angle) as follows:

$$H(J_x, J_y, \psi_x, \psi_y, \theta) = J_x\nu_x + J_y\nu_y + \frac{1}{2}\sum_N \sum_{i_1+i_2+i_3+i_4=N} h_{i_1i_2i_3i_4} J_x^{(i_1+i_2)/2} J_y^{(i_3+i_4)/2} e^{iw(\psi_x,\psi_y,\theta)}, \quad (1)$$

where $w = (i_1 - i_2)\psi_x + (i_3 - i_4)\psi_y - l\theta$, and $|w| \ll 1$.

For estimation, terms without dependence on the angular variable or, in other words, giving only a tune shift can be dropped. The coupling localization in the region without sextupoles and the absence of skew sextupoles (the last is important only for determination of dangerous resonances) make it possible to significantly simplify the problem. Thus, for dangerous resonances, the Hamiltonian can be rewritten as follows:

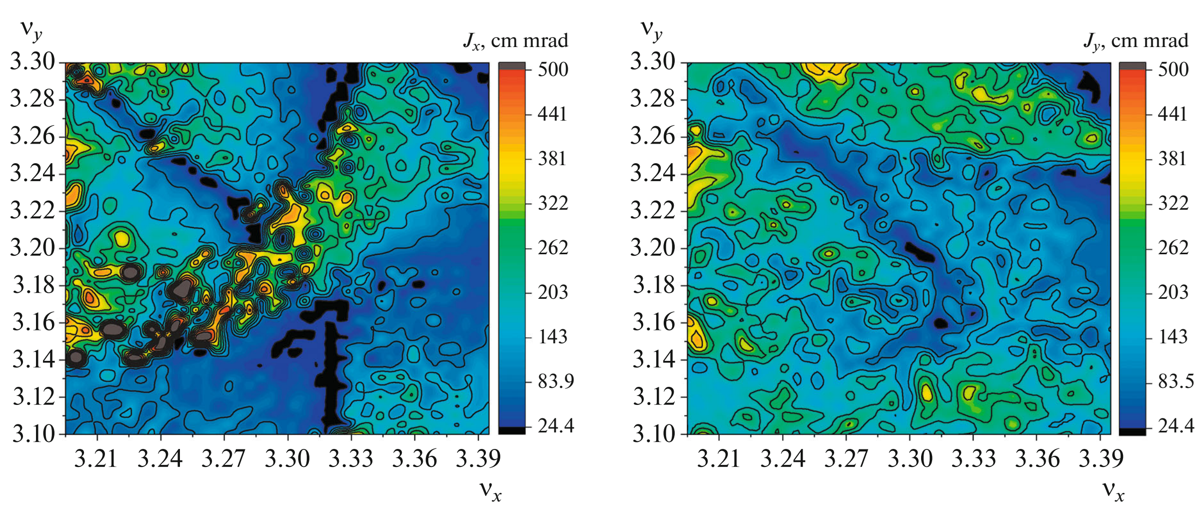

Fig. 7. Color map of horizontal (left) and vertical (right) tune scan with two sextupole families.

Table 3. Dangerous resonances

Perturbation theory order	Resonances		
1	$3\nu_x$	$2\nu_y + \nu_x$	
2	$2\nu_x + 2\nu_y$	$4\nu_x$	$4\nu_y$

Table 4. Restrictions for the harmonics of dangerous resonances

Resonance	Harmonic	Limit
$3\nu_x$	$\lvert h_{3000}\rvert$	1.5 $\text{m}^{-1/2}$
$2\nu_y + \nu_x$	$\lvert h_{1020}\rvert$	8 $\text{m}^{-1/2}$
$2\nu_x + 2\nu_y$	$\lvert h_{2020}\rvert$	0.8 m^{-1}
$4\nu_x$	$\lvert h_{4000}\rvert$	75 m^{-1}
$4\nu_y$	$\lvert h_{0004}\rvert$	83 m^{-1}

$$H\left(J_x, J_y, \psi_x, \psi_y, \theta\right) = J_x\nu_x + J_y\nu_y + \left|h_{i_1i_2i_3i_4}\right| J_x^{(i_1+i_2)/2} J_y^{(i_3+i_4)/2} \cos\left(w\left(\psi_x, \psi_y, \theta\right)\right). \tag{2}$$

Using the first integral of motion $J_x n_y + J_y n_x =$ const, the one-dimensional Hamiltonian can be expressed in variables J_x and w:

$$H\left(J_x, w\right) = J_x\varepsilon + (i_1 - i_2)\left(\frac{i_3 - i_4}{i_1 - i_2}\right)^{(i_3+i_4)/2} \left|h_{i_1i_2i_3i_4}\right| J_x^{\frac{N}{2}} \cos(w), \tag{3}$$

whereupon $N = 3$ for the first order of perturbation theory, and $N = 4$ for the second order.

Equation (1) was used for determination of stable motion islands for one-dimensional resonances, and Eq. (3) was used for the coupling resonances. Under the assumption that the area of stability for every operating point should be at least $J_x = J_y = 30$ cm mrad, restrictions on the harmonics of dangerous resonances presented in Table 4 were obtained. The suppression of resonances was performed by minimization of the harmonics using gradient descent with weights equal to the corresponding restrictions, conserving chromaticity compensation.

The separatrices of vertical and horizontal actions, which are dependent on tunes, were found (see Fig. 8), wherein the procedure of compensation of resonances, coupling, and chromaticity was performed at each tune. It can be seen that the proposed compensation scheme makes it possible to significantly increase the area of beam stability in a wide range of tunes.

The dynamic aperture calculated for the scheme with six sextupole families at the operating point $\nu_x = 3.255$, $\nu_y = 3.2$ that was chosen for the injection and acceleration according to the calculations is shown in Fig. 9a. The calculated dynamic aperture became noticeably larger than the geometric one, which confirms the efficiency of the method.

Calculated nonlinearities of magnets and nonlinear fields of eddy currents induced in the vacuum chamber during acceleration were added to the simulation. The resulting dynamic aperture at the chosen operating point is shown in Fig. 9b. Separatrices of vertical and horizontal actions are shown in Fig. 10. It can be seen that additional calculated perturbations significantly influence the dynamics of particles with equilibrium energy. The resonance $\nu_x = n/3$ is excited and its interference with $2\nu_y - \nu_x = n$ becomes significant. Moreover, taking into account perturbations, the acceptance for particles with momentum deviation significantly decreases (see Fig. 9b). However, the above calculations show sufficient efficiency of the proposed scheme of suppression of the resonances,

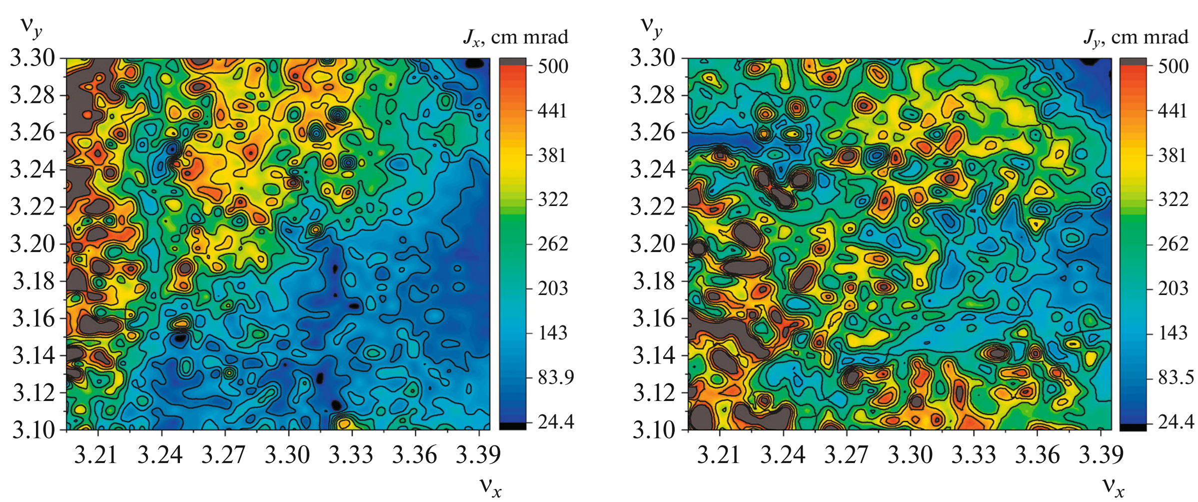

Fig. 8. Color map of horizontal (left) and vertical (right) tune scan with six sextupole families.

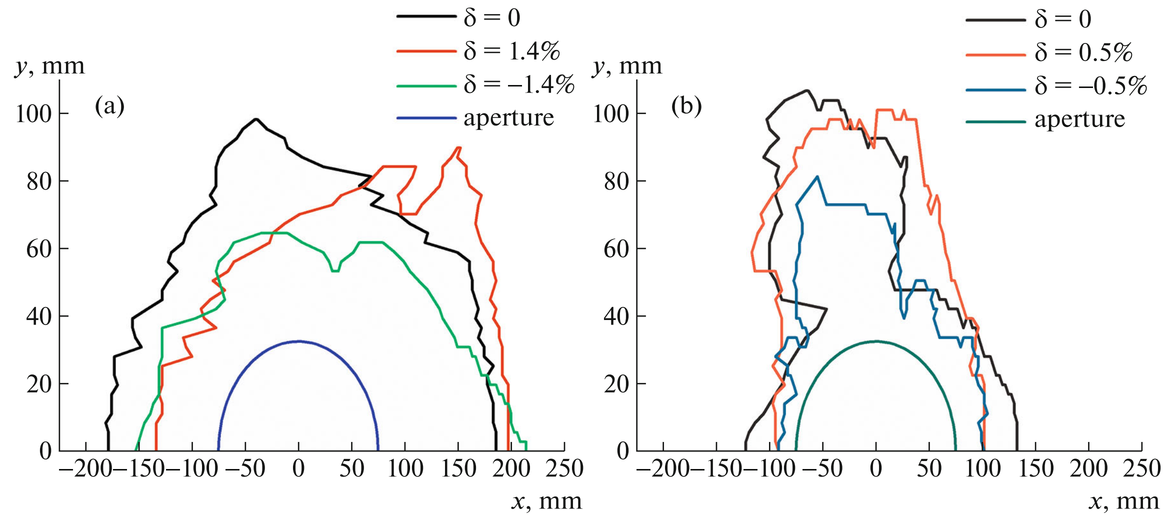

Fig. 9. Dynamic aperture with six sextupole families (a) without nonlinearities of magnets, (b) with calculated nonlinearities of magnets.

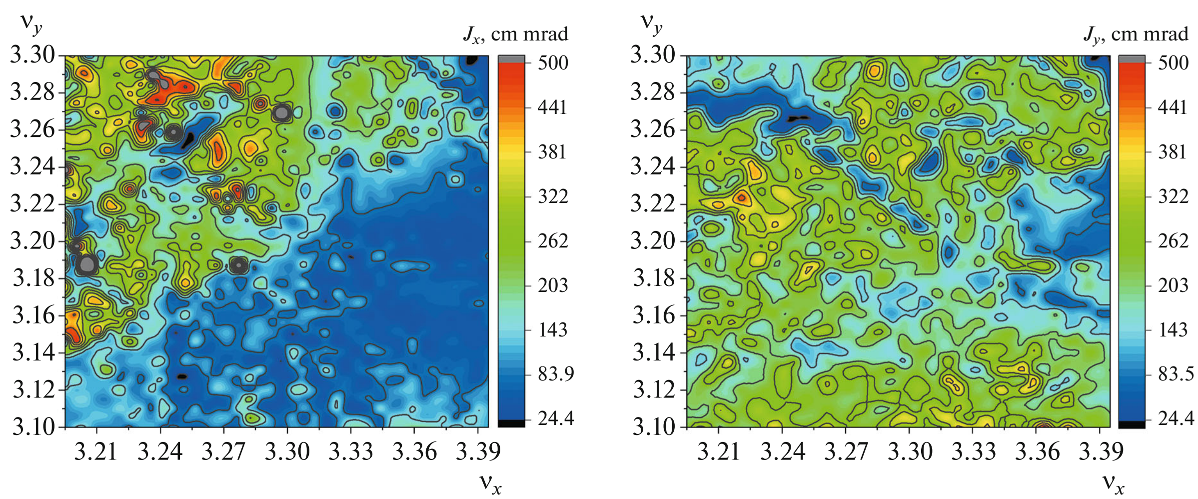

Fig. 10. Color map of horizontal (left) and vertical (right) tune scan with six sextupole families with nonlinearities of magnets and eddy currents (at injection energy).

and the calculated dynamic aperture is larger than the geometric one.

CONCLUSIONS

The presented work discusses the aspects affecting the transverse dynamics of the ion beam in the booster synchrotron with electron cooling, without including the space charge phenomena. The optimal scheme containing a pair of skew quadrupole triplets applied for localization of betatron coupling excited by longitudinal magnetic field of the electron cooling section was determined. The vertical dispersion excited by the magnetic field of toroids was calculated and minimized by lattice optimization.

As a result of the study of the dynamic aperture, it was found that the scheme with two sextupole families cannot provide the required dynamic aperture. The analysis of the dynamic aperture by scanning the dependence of actions on tunes was performed and dangerous resonances limiting the dynamic aperture were identified.

For a number of low-order resonances, the scheme with six sextupole families and uniform suppression of the resonances is proposed. The resonances were suppressed uniformly in order to maximize the dynamic aperture in a wide range of tunes near the working point. On the basis of the calculations, the operating point for injection and acceleration allowing simple retuning to slow extraction was determined. It was shown that the nonlinearities of the guided magnets and fields of eddy currents induced in the dipole vacuum chamber walls significantly influence the dynamics of particle with momentum deviation. However, these perturbations do not lead to reduction of the dynamic aperture less than the geometric one. Thus, the proposed resonance suppression scheme has shown sufficient efficiency.

In the future, it is necessary to perform calculations of the dynamic aperture taking into account measured nonlinearities of the guided magnets. In addition, another difficult problem that should be solved is the simulation of space charge influence on the low-energy ion beam dynamics.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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