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COMPUTER SCIENCE

Mathematical Modeling of Melting Tungsten Exposed to Pulsed Laser Beam

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Abstract—Melting in a tungsten plate exposed to a pulsed thermal load is numerically simulated by solving the two-phase Stefan problem. The free boundary is ignored during the calculation, since the numerical model is based on Samarskii's approach. Calculations in axially symmetric geometry show that about a quarter of the incident energy is consumed by evaporation in the center of the melt region. This is five times more than estimates based on the solution of the one-dimensional heat equation. When tungsten evaporation is taken into account, the calculated and experimental cooling surface temperatures and radii of the melt region are in good agreement. The results of the mathematical modeling confirm the existence of an evaporation cooling mode when the tungsten is heated by an electron beam to temperatures significantly higher than the melting point.

Keywords: mathematical modeling, tungsten evaporation, Stefan's problem

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1. INTRODUCTION

Modern systems for magnetic plasma confinement assume that peripheral plasma comes in contact with the wall. In a thermonuclear plasma, power fluxes at the surface are high enough [1] to have a large effect on the wall material. In the case of plasma instability development, the surfaces in contact can take a significant fraction of the confined plasma energy that is several orders of magnitude higher than energy fluxes in a steady state [2]. This can lead to the destruction of the first wall. Under sufficient heating, tungsten is heated to the ductile-brittle transition temperature and starts to deform irreversibly. With a further increase in the heat flux, oxides and entrapped gases are removed from the tungsten. It is well known that the evaporation rate begins to grow rapidly at the melting point. The melt moves along the surface under the action of the Ampere force on currents flowing through the melt.

There are three popular methods for emulating heat fluxes in a supposed thermonuclear reactor: laser [3], plasma [4], and an electron beam [5]. They do not agree in a full extent with influences that will occur in thermonuclear reactors. It is important to describe the influences of the various sources for their comparison and understanding the range of applicability of generalizations of experimental results. Tungsten as a plasma-facing material is studied at Kurchatov Institute, National Research Nuclear University "MEPhI," Lomonosov Moscow State University, the Ioffe Physical-Technical Institute, Private Institution Project Center ITER, the Troitsk Institute of Innovative and Thermonuclear Research, etc. The research conducted at the Budker Institute of Nuclear Physics of the Siberian Branch of the Russian Academy of Sciences (BINP SB RAS) focuses on surface erosion in a tungsten sample exposed to a pulsed laser or electron beam. Experiments are conducted at the Beam of Electrons for materials Test Applications (BETA) facility created at the BINP SB RAS [6]. Evaporation of substance is an important process influencing pulsed thermal heating above the melting point, which is expected in the case of plasma instability development in future thermonuclear installations.

A model of tungsten heating and melting is based on the solution of the two-phase Stefan problem for temperature calculation in the sample domain. The Stefan problem is a classical one in computational mathematics, which is still of interest [7]. The position and velocity of the phase boundary depends on nonlinear coefficients. At the free melt–wall interface, the temperature is continuous, while the heat flux is discontinuous due to the absorption or release of a given amount of heat. The novelty and complexity of the solution to the problem is explained by the necessity for a valid treatment of the nonlinear boundary condi-

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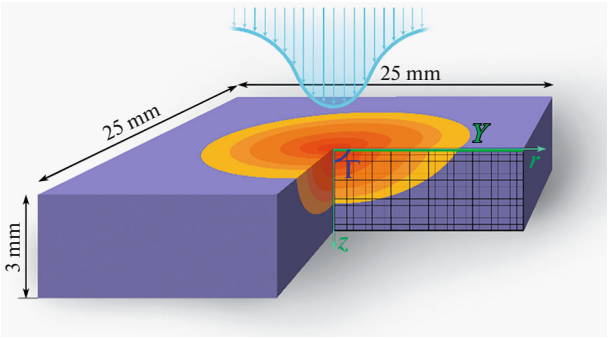

Fig. 1. Experimental setup: the computational domain in the plate's cross section (green), heating region (yellow), melt region (red) with a free boundary Γ, and the laser pulse (blue).

tions describing the heating and evaporation of the material.

2. FORMULATION OF THE PROBLEM

A tungsten plate placed in a vacuum is heated by an intense pulsed laser beam (Fig. 1). Since the plate heats up to a depth of several hundred microns in such a short time, the computational domain is the cross section of the sample, i.e., a region of 2 × 3 mm. The power density $W(t, r)$ over the heated surface has a nearly normal distribution along the radius r, which motivates the formulation of the problem in axially symmetric geometry:

$$W(t,r) = W_{\max}(t)\cdot\exp(-A\cdot r^2),\quad A=0.03088523. \quad (1)$$

At every time step of the numerical simulation, $W_{\max}(t)$ is taken from a file of experimental data. The distribution of the heating power density over the surface is measured using X-ray visualization [8]. Electrons with energy of 80–90 keV heat the material in a layer that is thin as compared with the characteristic depth of heating of the material. The temperature field in the sample is computed using the Stefan problem

$$\begin{cases} c(T)\rho(T)\dfrac{\partial T}{\partial t} = \dfrac{1}{r}\dfrac{\partial}{\partial r}r\lambda(T)\dfrac{\partial T}{\partial r} + \dfrac{\partial}{\partial z}\lambda(T)\dfrac{\partial T}{\partial z}, \\ (n,\nabla T)\big|_{\gamma} = \dfrac{W(t,r)-N(T)}{\lambda(T)}, \quad (n,\nabla T)\big|_{\Omega-\gamma} = 0, \\ [T]\big|_{\Gamma} = 0, \quad \left[\lambda(T)\dfrac{\partial T}{\partial t}\right]\Big|_{\Gamma} = L_m v_n, \quad T\big|_{t=0} = T_0, \end{cases} \quad (2)$$

where Ω is the entire surface of the sample, γ is the heated surface, Γ is the melt–metal interface, $T(t,r,z)$ is the temperature, $c(T)$ is the specific heat, $\rho(T)$ is the density, $\lambda(T)$ is the thermal conductivity, $W(t,r)$ is the power density on the surface γ, $N(T)$ is the evaporation rate, n is the normal to the surface, T_0 is the initial temperature, and v_n is the velocity of the phase boundary. During heating, the tungsten begins to evaporate, so the heat flux on the surface is reduced. The evaporation at the boundary is taken into account using the given resulting energy flux $W_*(t,r) = W(t,r) - N\left(T\big|_{\gamma}\right)$. Here, $W(t,r)$ is specified by distribution (1) and the evaporation rate $N\left(T\big|_{\gamma}\right)$ is specified in terms of the power loss L_e and the mass evaporation rate $\frac{1}{S}\frac{dm}{dt}$:

$$N\left(T\big|_{\gamma}\right) = L_e\cdot\frac{1}{S}\frac{dm}{dt}, \quad \frac{1}{S}\frac{dm}{dt} = P\left(T\big|_{\gamma}\right)\sqrt{\frac{M}{2\pi R T\big|_{\gamma}}}. \quad (3)$$

Here, $L_e = 4.482\times10^{12}\ \dfrac{\text{W}\,\mu\text{s}}{\text{kg}}$ and the saturated vapor pressure is given by

$$P\left(T\big|_{\gamma}\right) = \exp\left(26.19104 - \frac{83971.3\ \text{K}}{T\big|_{\gamma}}\right)\frac{\text{kg}}{\text{mm}^2\,\mu\text{s}}.$$

Thus, the power loss due to evaporation (Fig. 2a) and, hence, the resulting energy flux on the heated surface can be determined via the surface temperature. The loss of the material due to evaporation is not taken into account in the model. The computation continues until the last measurement of the surface temperature is made.

3. SOLUTION METHOD

To take into account the melting process, the phase transition enthalpy L_m is introduced into the thermal energy equation:

$$\begin{gathered}(c(T)\rho(T) + L_m\delta(T,\varepsilon))\frac{\partial T}{\partial t} = \operatorname{div}(\lambda(T)\operatorname{grad}T), \\ \delta(T,\varepsilon) = \begin{cases}\dfrac{1}{2\varepsilon}, & |T-T_m|\le\varepsilon \\ 0, & |T-T_m|>\varepsilon,\end{cases}\end{gathered} \quad (5)$$

where $[-\varepsilon\ \varepsilon]$ is a smoothing interval, $\varepsilon = 5$ K, the melting point is $T_m = 3695$ K, and the phase transition

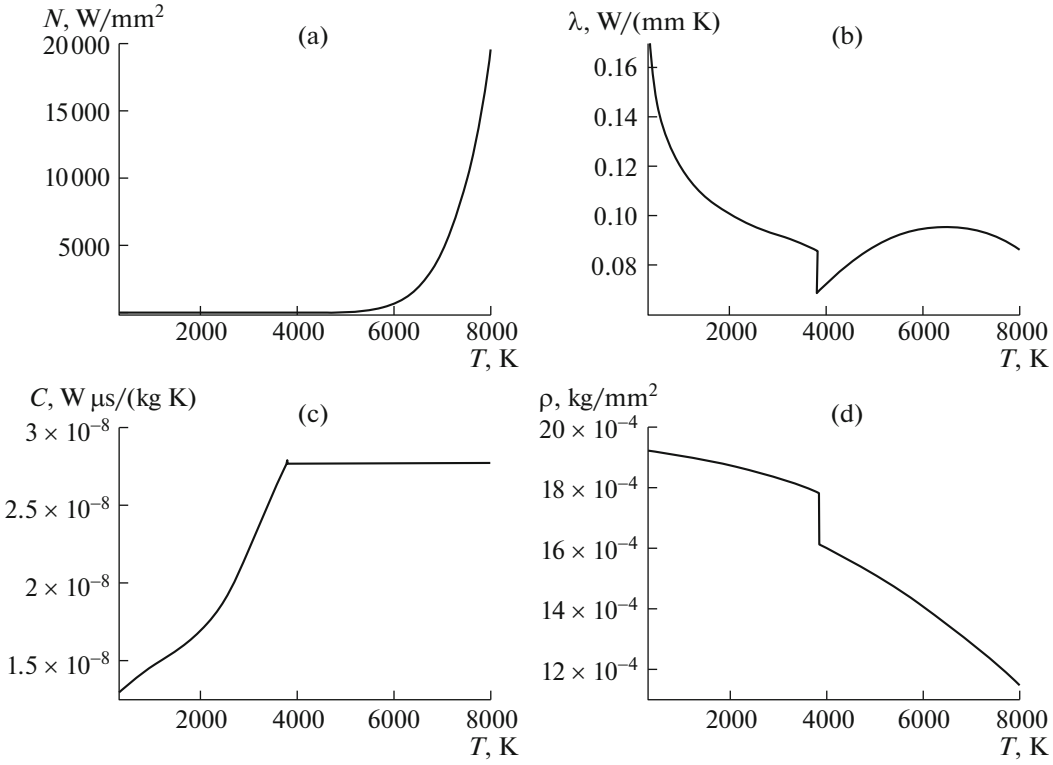

Fig. 2. Plots of (a) evaporation power loss, (b) thermal conductivity, (c) specific heat, and (d) density as functions of temperature.

enthalpy is $L_m = 51.1\times10^5 \dfrac{\text{W}\ \mu\text{s}}{\text{mm}^3}$. The derivation of Eq. (5) and the justification of the surface temperature radiation neglected from the energy balance can be found in more detail in [9]. The numerical implementation of Eq. (5) is based on the stabilizing correction scheme [10].

Experimental data and analytical estimates show that the surface heats up to at most 8000 K in the regimes investigated on the BETA facility. This is explained by the exponential growth of the evaporation energy loss with increasing surface temperature. The power loss, thermal conductivity [11], specific heat [11], and density (Fig. 2) are used in the form of functions of the material temperature. These functions have discontinuous derivatives at the melting point T_m. The estimated thermal conductivity $\lambda(T)$ of liquid tungsten was taken from [12], [13] with a refined data approximation (Fig. 2b), which made it possible to perform the computations at temperature values higher than 6000 K.

4. SIMULATION RESULTS

The computations have shown that the sample heating model sufficiently accurately reflects the behavior of the solution (Fig. 3a) taking into account the diagnosis features. When the surface cooling due to evaporation is taken into account at a sufficiently high heating power density, the temperature to which the material heats up is reduced considerably. In this case, the tungsten loses energy together with the evaporating material. As was shown in [14], the evaporation energy loss S in the center of the plate is about 26% of the pulsed heating energy:

$$S = \int_0^\infty\int_0^\infty N\left(T|_\gamma\right) r dt dr \Big/ \int_0^\infty\int_0^\infty W(t,r) r dt dr. \qquad (6)$$

On the BETA facility, four consecutive photographs of the sample surface were taken during cooling after pulsed heating [6], [15]. The radius of the melt region was measured at times corresponding to the camera exposure starting from electron beam heating with an exposure duration of 10 μs. The numerical results agree with the experimental data (Fig. 3b) in the early stage of cooling, but later the numerically computed radius of the melt region is much larger than the observed one. Moreover, in this case, according to formula (3), the temperature computed in the center of the heated domain yields impossibly high values of the power loss $N\left(T|_\gamma\right)$, which exceed the surface heating power density $W(t,r)$. After the surface cooling due to evaporation was taken into account in the numerical simulation, the temperature ceased to exceed the maximum permissible value and the calcu-

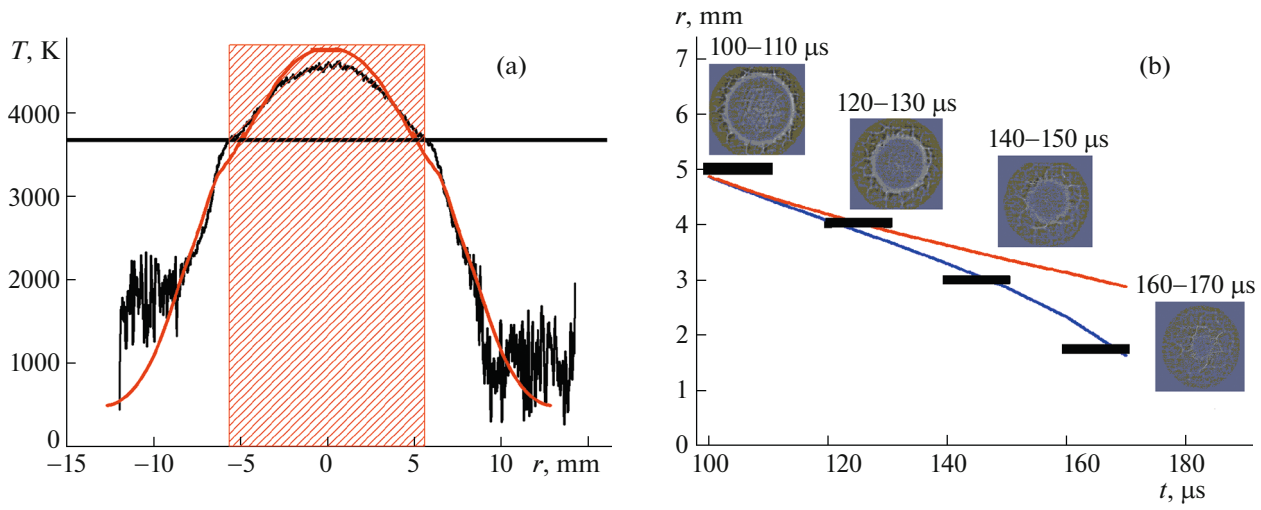

Fig. 3. (a) Temperature distribution over the surface: numerical results (red), experimental data (black curve), melting point (black line), and the melt region (red hatching). (b) The radius of the melt region as a function of time: experimental data (black segments) and numerical results without (red) and with (blue) cooling due to evaporation taken into account.

lated dependence of the melt region radius agreed with the experimental data.

5. CONCLUSIONS

The mathematical modeling of melting in a tungsten sample exposed to pulsed thermal loading has confirmed the existence of an evaporation cooling mode when the tungsten is heated by an electron beam to temperatures significantly higher than the melting point. In this regime, the evaporation rate becomes comparable with the incident power, but the thickness of the evaporated material is insufficient for vapor shielding, so the surface temperature is stabilized. The numerical solution of the two-phase Stefan problem in axially symmetric geometry showed that about a quarter of the incident energy is consumed by evaporation in the center of the melt region. The computed and experimental surface temperatures and melt region radii were compared. They were found to agree well when tungsten evaporation is taken into account. As a further development of the model, we intend to solve an equation for hear fluxes, to determine boundary conditions for tungsten vapors using molecular models, and, eventually, to solve the system of gas dynamics equations in a self-consistent electric field. This will allow us to describe the motion of the melt under the influence of hear fluxes, vapor ionization, gas condensation into droplets, as well as instabilities leading to fluid separation from the surface.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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