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PHYSICS OF ELEMENTARY PARTICLES AND ATOMIC NUCLEI. THEORY

Regge Cuts in QCD

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Abstract—At high energies and limited transverse momenta, the amplitudes of QCD processes in the leading logarithmic approximation (LLA) are determined by the Regge pole with gluon quantum numbers and a negative signature. This property, called gluon reggeization, is extremely important for a theoretical description of high-energy processes in QCD. In particular, it underlies the derivation of the Balitsky—Fadin—Kuraev—Lipatov (BFKL) equation. The pole Regge form is also preserved in the next-to-leading logarithmic approximation (NLLA). However, this form is violated in higher approximations. It is natural to assume that it is violated by the contributions of the Regge cuts. The structure of these cuts and its difference from the structure of cuts in the old (before QCD) theory of complex angular momenta is discussed.

Keywords: gluon Reggeization, BFKL equation, Regge cuts

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1. INTRODUCTION

QCD can be called a unique theory due to the reggeization of all its elementary particles—quarks and gluons—in perturbation theory. This remarkable property is extremely important and is widely used for the theoretical description of high-energy processes. The reggeization of the gluon is especially important, since gluon exchanges in cross channels provide cross sections that do not decrease with energy. In particular, gluon reggeization underlies the derivation of the Balitsky—Fadin—Kuraev—Lipatov (BFKL) equation. In the original version, this equation was obtained [1–3] in non-Abelian theories with spontaneously broken symmetry in which gauge bosons acquire a mass that ensures the absence of infrared singularities. Subsequently, the applicability of this equation was shown [4] in QCD, with massless gauge bosons—gluons.

For elastic scattering processes $A + B \to A' + B'$, reggeation means that, in the kinematic region $s \simeq -u \to \infty$, t is fixed (i.e., does not grow from s), $s = (p_A + p_B)^2$, $u = (p_A - p_{B'})^2$ and $t = (p_A - p_{A'})^2$; scattering amplitudes with gluon quantum numbers in the t -channel and negative signature (symmetry with respect to $s \leftrightarrow u$) are shown as Fig. 1 and are written as

$$\mathscr{A}_{AB}^{A'B'} = \Gamma_{A'A}^{c}\left[(-s-t)^{j(t)} - (s-t)^{j(t)}\right]\Gamma_{B'B}^{c}, \qquad (1)$$

where $\Gamma_{P'P}^{c}$ is particle-particle-reggeon vertices or scattering vertices, c is the color index of the Reggeized gluon, and $j(t) = 1 + \omega(t)$ is its trajectory. Reggeization also means the factorized form of inelastic amplitudes in multi-Regge kinematics (MRK). In the MRK, all particles in the final state are bounded (not growing with s) transverse momenta and are combined into jets with a fixed invariant mass of each jet and large (growing with s) invariant masses of any pair of jets. This kinematics makes the main contribution to the cross sections of QCD processes at high energies $\sqrt{s}$. In the LLA, each jet can have only one gluon. In the NLLA, it is necessary to take into account the production of quark-antiquark ($Q\bar{Q}$) and two-gluon (GG) jets.

The creation amplitudes of n jets J_i with momenta k_i are presented in the form 2, and their real parts have a simple factorized form

$$\Re\mathscr{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_ic_{i+1}}^{J_i}(q_i,q_{i+1})\left(\frac{s_i}{s_0}\right)^{\omega(t_i)}\frac{1}{t_i}\right) \frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_0}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}. \qquad (2)$$

Here, $\gamma_{c_ic_{i+1}}^{J_i}(q_i,q_{i+1})$ are jet production vertices J_i.

The pole Regge forms (1) and (2) are valid for amplitudes with gluon quantum numbers in cross-channels and a negative signature in both the leading logarithmic approximation (LLA) [5] and in the next one (NLLA) (see [6, 7] and links there). Their use in the conditions of s-channel unitarity for elastic scattering amplitudes leads to the BFKL equation.

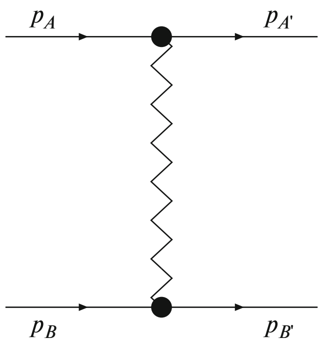

Fig. 1. Schematic representation of elastic scattering amplitudes.

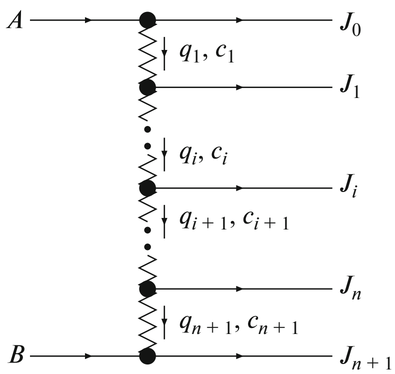

Fig. 2. Schematic representation of the amplitudes of jet production in MRKs.

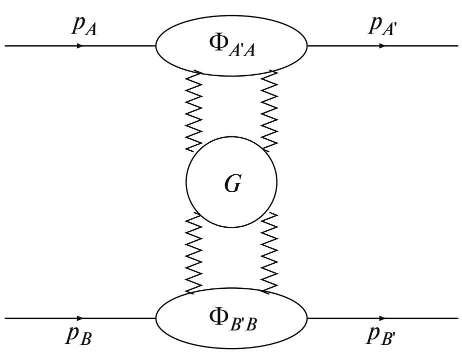

Fig. 3. Schematic representation s-channel jumps of elastic scattering amplitudes.

2. TWO-REGGEON CUTS IN QCD

It is well known that there is no consistent theory in which all singularities in j plane (plane of complex angular momentum) are moving poles. Poles in j plane necessarily generate cuts.

The primary Reggeon in QCD is the Reggeized gluon with the trajectory $j(t) = 1 + \omega(t)$ passing through 1 at $t = 0$. Since it has a negative signature, its contribution to the amplitudes is the main one. In amplitudes with a positive signature the real parts of the leading logarithms cancel out, so that in the LLA these amplitudes are purely imaginary. Note that we use the term LLA for them, as we do for amplitudes with a negative signature, although in each order of perturbation theory they have one less power of the large logarithm.

Ratios s-channel unitarity, together with the pole Regge form of elastic amplitudes and particle production amplitudes in MRKs, lead to the representation of s-channel discontinuites of elastic scattering amplitudes $A + B \to A' + B'$ (Fig. 3) in the form of a convolution

$$\Phi_{A'A} \otimes G \otimes \Phi_{B'B} \tag{3}$$

of impact factors $\Phi_{A'A}$ and $\Phi_{B'B}$ describing the transitions $A \to A'$ and $B \to B'$ with Green's function G of two interacting Reggeons (reggeized gluons),

$$\hat{\mathcal{G}} = e^{Y\hat{\mathfrak{R}}}.$$

Here $Y = \ln(s/s_0)$; s_0 is the energy scale, the choice of which is related to the determination of the impact factors; and $\hat{\mathfrak{R}}$ is the BFKL kernel, which determines the entire energy dependence of the scattering amplitudes. It is given as the sum

$$\hat{\mathfrak{R}} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{\mathfrak{R}}_r,$$

where $\omega_{1,2}$ are the trajectories of Reggeized gluons and $\hat{\mathfrak{R}}_r$ is the so-called real part of the kernel, which is expressed through the product of vertices $\gamma^{J_i}_{c_i c_{i+1}}(q_i, q_{i+1})$ production of jets by reggeons. In the NLLA, along with the production of one gluon, the production of two-gluon and quark-antiquark jets should be taken into account, so that

$$\hat{\mathfrak{R}}_r = \hat{\mathfrak{R}}_G + \hat{\mathfrak{R}}_{Q\bar{Q}} + \hat{\mathfrak{R}}_{GG}.$$

Representation (3) corresponds to the exchange of two interacting Reggeons (reggeized gluons) in the t-channel. It is valid for all possible color states. It is clear that, in the general case, a two-Reggeon state cannot be a Regge pole; that is, a cut must already appear in the LLA. However, for an octet (the adjoint representation of the color group in QCD), a miracle happens: two interacting reggeons again give the reggeon. This miracle is provided by the bootstrap equations for the impact factors and the BFKL kernel in the adjoint representation (see [6] and the links provided there). It must be said that, since only gluons contribute to the kernel in the LLA, it turns out that in this approximation there is a degeneracy in signature, and the positive signature parts of the amplitudes of the t-channel with the adjoint representations of the color group are given by the contribution of the Regge pole with the same trajectory, that is, with the trajectory of the Reggeized gluon. This is no longer the case with the NLLA. In other representations, Regge cuts appear already in the LLA. In particular, the rightmost singularity in a colorless channel is a fixed branch point (located in the LLA at $\omega_P = 4N_c \frac{\alpha_s}{\pi}\ln 2$), so that the so-called BFKL pomeron is a cut.

3. PECULIARITY OF REGGE CUTS IN QCD PERTURBATION THEORY

There is a significant difference between the Regge cuts in perturbative QCD and in the former (before QCD) theory of complex angular momenta. This is due to different ideas about Regge poles. In the traditional theory of complex angular momenta, a pole is associated with an infinite series of ladder diagrams shown in Fig. 4. Based on the fact that this series gives the scattering amplitude $A(s,t) \sim s^{\alpha(t)}\ln s$, corresponding to the contribution of the Regge pole with the trajectory $j = \alpha(t)$ and considering the two-particle intermediate state in the relation s-channel unitarity for the amplitude corresponding to diagram in Fig. 5, D. Amati, S. Fubini, and A. Stangellini [8, 9] came to the conclusion about the existence of a Regge cut with a moving with t branch point passing through $2\alpha(0) - 1$ at $t = 0$ (it is worth noting that they considered their consideration not a proof of the existence of the cut, but only an argument in its favor).

Subsequently, their proposed mechanism for the formation of cuts was criticized. In work [10], the possibility of canceling the contributions of two and three-particle states in the unitarity relation (subsequently, such a cancellation was demonstrated in [11]) and the absence of a cut in the total contribution of the diagram in Fig. 5 was pointed out. A similar conclusion was made by S. Mandelstam [12] based on t-channel unitarity.

In [13] the presence of cuts was shown in the contributions of more complex diagrams, such as those shown in Fig. 6, in which there is no such cancellation. The essential circumstance here is that these diagrams are nonplanar.

Since then, AFS-type diagrams have been rejected and it has been believed that only nonplanar diagrams can lead to Regge cuts.

However, the situation with cuts in perturbative QCD is completely different. It differs as early as in the formulation of the problem.

In the old theory of complex angular momenta, the main subject of study was the asymptotic behavior of the amplitudes at high energies and fixed momentum transfers.

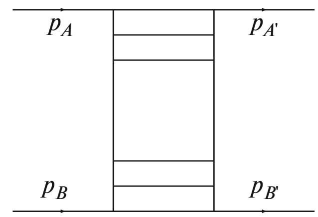

Fig. 4. Traditional representation of the Regge pole.

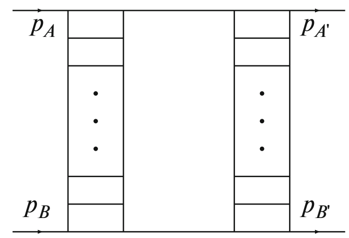

Fig. 5. First (AFS) diagrammatic representation of the Regge cut.

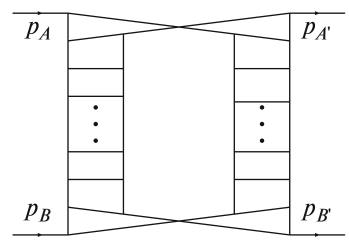

Fig. 6. Diagrammatic representation of the Regge section with the Mandelstam cross.

In perturbative QCD, the leading terms are calculated in each order of a series of perturbations, generally speaking, without any idea of what their summation will lead to. The results are interpreted as the contributions of the Regge poles and cuts. The expansion in terms of the coupling constant of the total contribution of the poles and cuts to the amplitudes of various processes should coincide with the results of direct perturbation theory calculations.

The most important difference between QCD and the former theory of complex angular momenta, which manifests itself in the structure of the cuts, is the structure of the reggeons. Reggeized gluons in QCD start from a single-gluon state, while reggeons in the former theory do not have single-particle t-channel states. Because of this, the statement that only nonplanar diagrams can lead to Regge cuts becomes incorrect. This is clear from the example of the BFKL pomeron. In the lower (g^4) order, it is a two-gluon t-channel exchange, which is obviously represented by planar diagrams. Of course, it can be assumed that, in the colorless channel, along with the branch point, there is also a pole lying to the left of the branch point, and planar diagrams are responsible for precisely this pole and the cut responsible for the asymptotics of the sections is associated with nonplanar diagrams. However, this is contrary to $\omega_P \sim N_c$, while the contribution of nonplanar diagrams must be suppressed by the number of colors. Thus, the BFKL pomeron is an AFS rather than a Mandelstam cut, although, of course, it is neither one nor the other. In contrast to them, the BFKL pomeron is a bound state of interacting Reggeons (their interaction is described by the BFKL kernel), while in both the AFS and Mandelstam patterns the Reggeons forming the cut do not interact. As a result, in these pictures, the cut is moving, while the BFKL pomeron is a fixed branching point. Thus, even what little is known about cuts in the old theory of complex angular momenta is not applicable to QCD.

4. THREE-REGGEON CUTS

It is worth noting, however, that the discussion of two-reggeon cuts in QCD is based on some solid foundation called gluon reggeization. The validity of the pole forms (1) and (2) of amplitudes with gluon quantum numbers and a negative signature in the Regge and multi-Regge kinematics has been proven in the LLA and NLLA. In these approximations, they determine s-channel discontinuites of elastic scattering amplitudes (and, due to analyticity, the amplitudes themselves) for any possible color states in the t-channel. Thus, all elastic scattering amplitudes are given by the exchange in the t-channel by two interacting Reggeized gluons. Two-reggeon cuts arise as a result of such an exchange and can be investigated in this approximation using the BFKL equation.

The situation with three-reggeon cuts is much more complicated, although the Bartels–Kvechinsky–Prashalovich equation (BKP) [14], [15] for an odderon (a colorless state with a positive signature that differs from a pomeron in C-parity) has been known for a long time. This equation (as well as its generalizations to multireggeon states in the t-channel) was obtained under the assumption of pairwise interaction of Reggeons, described by the BFKL kernel. This assumption looks quite natural, but, as far as I know, there is no rigorous derivation of the BKP equation or its verification in perturbation theory.

Recall that, in each order of perturbation theory, amplitudes with a negative signature are leading, so they play a special role in the BFKL approach. In the LLA and NLLA, these amplitudes have a pole Regge form (1), (2). However, in the next approximation (NNLLA), this form is violated. The violation of pole factorization was first noticed in [16] when considering nonlogarithmic terms in two-loop amplitudes gg, gq and qq scattering. In three loops, factorization is already violated by terms with the first degree $\ln s$. Their consideration was carried out in the works [17–19] using the infrared factorization method.

It is natural to assume that the violation of the pole Regge form is explained by the contributions of the three-Reggeon cuts. The first explanation using the diagrammatic approach was given in the works [20, 21]. In the two-loop approximation, the cut contribution comes from the Feynman diagrams with the exchange of three gluons in t-channels, which in this approximation represent reggeons. When calculating the three-loop contribution, we used the pair interaction of Reggeons described by the BFKL kernel. To separate the contributions of the pole and the cut, we used the difference in the energy dependence of these contributions.

It should be noted here that for amplitudes with a positive signature, in addition to the color octet in t-channel, there is also a singlet for quark-quark scattering and two mutually conjugate decuplets for gluon-gluon scattering. The cut contribution computed in the diagram approach for these representations [22] agrees with the results obtained by the infrared factorization method, which serves as an argument in favor of the chosen form of pairwise interaction between reggeons.

Soon after [20], there was a job [23], in which to calculate the contributions of the Regge cuts in the two- and three-loop approximation, the approach of Wilson lines was used, in which there is no connection between three-Regge cuts and Feynman diagrams. For representations other than the adjoint one the results obtained in [23] are consistent with those of the diagrammatic approach used in [20]; for the adjoint representation, where it is necessary to separate the contributions of the pole and the cut, they are different. In particular, in [23] a mixing of the pole and the cut is introduced, which is not present in [20]. In [23] there is a link to [20], but the results were not compared. Such a comparison and criticism of the [23] approach is contained in the works [24–27], in which also in the diagrammatic approach the cut contribution was calculated in four loops. One of the criticisms is that, due to the postulated in [23] color structure of the cut, its contribution to the adjoint representation of the color group is not suppressed for a large number of colors; that is, it does not disappear in the planar limit in the most extended supersymmetric Yang–Mills theory, which contradicts generally accepted ideas.

Recently, in the Wilson line approach, the contributions of the three-reggeon cuts to the elastic scattering amplitudes have been calculated in four loops [28], [29]. To avoid contradiction with the planar $N = 4$ SYM, the authors proposed a scheme for separating the contribution of the pole and the cut to the NLLA in all orders of perturbation theory, based on the assertion that the diagrams for Regge cuts are nonplanar. As was already discussed, this statement, which comes from the old theory of complex angular momenta, is inapplicable in QCD. In addition, such a separation of the pole and cut contributions is not gauge invariant, which is unacceptable in our opinion.

5. CONCLUSIONS

One of the most remarkable properties of QCD, which is extremely important for the theoretical description of processes at high energies, is the reggeization of the gluon. In particular, the derivation of the BFKL equation is based on the pole Regge form of the amplitudes with the adjoint representation of the color group in cross-channels and a negative signature. This form is valid in LLA and NLLA, but is violated in higher approximations. It is natural to assume that this violation is caused by the contributions of the Regge cuts, so their study, which is interesting in itself, is also important from the phenomenological point of view. Unfortunately, even the little that is known about Regge cuts in the old (pre-QCD) theory of complex angular momenta turns out to be inapplicable in QCD, precisely because of gluon Reggeization. This is clearly seen in the example of a pomeron that appears in QCD amplitudes with a positive signature as a colorless bound state of two interacting Reggeized gluons. Unlike the pomeron, which already appears in the LLA as a two-reggeon cut described by the BFKL equation, Regge cuts in amplitudes with a negative signature appear only in the NNLLA and are three-reggeon. When studying them, it is assumed that the pair interaction of Reggeons is described by the kernel of the BFKL equation. This assumption is confirmed by the comparison with the results of calculations of elastic scattering amplitudes performed by other methods with nonadjoint representations of the color group in the t-channel.

Currently, there are two approaches to determining the contributions of three-reggeon cuts, in which calculations are carried out up to four loops. These approaches are consistent for color group representations other than the adjoint, but give different results for the adjoint, where the problem of separating the pole and cut contributions arises.

CONFLICT OF INTEREST

The author declares that they have no conflicts of interest.

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