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PHYSICS AND TECHNIQUE OF ACCELERATORS

Spontaneous Emission from Distributed Optical Klystron

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Abstract—In many cases, users of contemporary undulator-based x-ray sources exploit the relatively narrow spectral width of the undulator radiation in the forward direction. In this paper, we discuss the use of a distributed optical klystron, i.e., an optical klystron with several magnetic bunchers, to minimize the linewidth of the undulator radiation. We compare this approach with the common use of high-order harmonics of the undulator radiation.

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INTRODUCTION

In many cases, users of contemporary undulator-based x-ray sources exploit the relatively narrow spectral width of the undulator radiation in the forward direction. In this paper, we discuss the use of a distributed optical klystron (DOK) [1], i.e., an optical klystron with several magnetic bunchers, to obtain the minimum linewidth of the undulator radiation. We will compare this approach with the common use of high-order (5–9) harmonics of the undulator radiation.

For an undulator with N periods, the relative linewidth of a harmonic of the number n is about $1/(nN)$ [2–4]. This follows from the simple fact that the duration of a wave packet radiated by an electron in such undulator is $\Delta t = 2\pi N/\omega_1$, where ω_1 is the frequency of the first harmonic of the radiation. Then $(\Delta\omega)_{\text{FWHM}} \approx 5.6/\Delta t \approx 0.9\,\omega_1/N$. To simplify the analytic estimates, we will use the effective "r.m.s." linewidth $\sigma_\omega = (\Delta\omega)_{\text{FWHM}}/\sqrt{8\ln(2)} \approx \sqrt{1/7}\,\omega_1/N$.

One of the ways to narrow the spectral line is a decrease in the fundamental frequency ω_1 and the use of a high harmonic of the number n. Then the r.m.s. spectral width

$$\frac{\sigma_\omega}{\omega} \approx \sqrt{\frac{1}{7(nN)^2} + \left(2\frac{\sigma_\gamma}{\gamma}\right)^2 + 2\left(\frac{\gamma^2}{1+K^2/2}\right)^2\left(\sigma_{x'}^4 + \sigma_{y'}^4\right)}, \quad (1)$$

where γ is the electron Lorentz factor, σ_γ/γ is the relative energy spread, $\sigma_{x'}$ and $\sigma_{y'}$ is the horizontal and the vertical angular spread, respectively, and K is the undulator deflection parameter, is limited by the longitudinal velocity spread $\sqrt{(\sigma_\gamma/\gamma)^2\left(1+K^2/2\right)^2/\gamma^4 + \left(\sigma_{x'}^4 + \sigma_{y'}^4\right)/2}$. Equation (1) is valid for a planar undulator and a sufficiently high electron energy $\gamma \gg k_u K \sigma_y/\sigma_{x'}$ ($k_u = 2\pi/\lambda_u = \omega_1\left(1+K^2/2\right)/\left(2\gamma^2 c\right)$ is the wave number of the undulator, and σ_y is the vertical beam size), such that the influence of the undulator field inhomogeneity can be neglected (see details in [4–6]). Therefore, the lower limit of the spectrum width of Eq. (1) is $2\sigma_\gamma/\gamma$, which is about 10^{-3} in contemporary x-ray source projects. It can be achieved for $2\sqrt{7}nN > \gamma/\sigma_\gamma$, or $K > \gamma\sqrt{\lambda/(L\sigma_\gamma/\gamma)}$. For typical parameters, $K > 2$.

Another well-known way to increase the radiation pulse length is the use of a magnetic buncher (chicane) between undulator sections. A chicane is just a single-period undulator, which provides a significant delay of an electron due to its non-straight trajectory. Such an undulator system is called "optical klystron" [5–7]. The most interesting option for our purpose is a DOK, which has several undulators and chicanes. The trajectory in a DOK is shown in Fig. 1.

RADIATION SPECTRUM

The radiation field of such undulator system can be written as

$$E_x(t) = \sum_{m=1}^{M} E_m(t - T_m), \quad (2)$$

where M is the number of the undulator sections, E_m is the radiation field of the undulator section with the number m, and T_m is the delay of the radiation pulse of the undulator section with the number m. In the simplest case of M identical undulator sections with

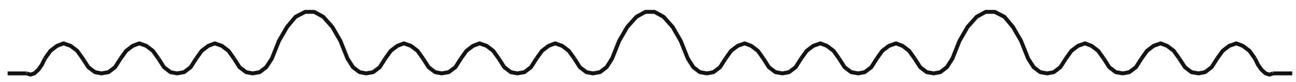

Fig. 1. Electron trajectory in DOK with $M = 4$ undulator sections and $M - 1 = 3$ chicanes. Number of periods at each undulator is $N_1 = 3$.

$N_1 = N/M$ periods, M-1 identical chicanes, and $T_m = (m-1)T$, the spectral intensity is

$$I_\omega = I_1 \frac{\sin^2(M\omega T/2)}{\sin^2(\omega T/2)}, \quad (3)$$

where $I_1 = I_0 \sin^2(\pi N_1 \delta)/(\pi N_1 \delta)^2$ and I_0 is the spectral intensity and the maximum spectral intensity of the first harmonics, respectively, of the radiation from one undulator section and $\delta = (\omega - \omega_1)/\omega_1$. To describe the delay $T = 2\pi(N_1 + N_D)/\omega_1$, one can introduce the effective period number N_D [7] for a chicane. Equation (3) shows that the spectrum has peaks at $\omega_q = 2\pi q/T$ ($q \approx N_1 + N_D$ is an integer) with the relative FWHM width $2\pi/(M\omega_1 T) = [M(N_1 + N_D)]^{-1}$, i.e., the "finesse" (the ratio of the peak width to the distance between peaks) of the spectrum is $1/M$. That is why one needs several chicanes to provide significant separation of these spectral lines. It is worth noting that Eqs. (2) and (3) are exactly the same as for diffraction on a screen with M equidistant slits (diffraction grating) [8].

The peaks of the spectrum of Eq. (3) are widened by the electron energy spread and finite transverse emittances. The delay time for an electron with energy and angular deviations is

$$T = T_0 + \frac{dT}{d\gamma}\Delta\gamma + L\frac{x'^2 + y'^2}{2c}, \quad (4)$$

where L is the distance between the centers of neighboring undulator sections. Then the r. m. s. spectral width is

$$\frac{\sigma_\omega}{\omega_1} \approx \sqrt{\frac{1}{7}\left(\frac{2\pi}{MT\omega_1}\right)^2 + \left(\frac{1}{\omega_q}\frac{d\omega_q}{dT}\sigma_T\right)^2} = \sqrt{\frac{1}{7(N + MN_D)^2} + \left(\frac{\sigma_T}{T}\right)^2}, \quad (5)$$

where

$$\sigma_T^2 = \left(\frac{dT}{d\gamma}\right)^2 \sigma_\gamma^2 + \left(\frac{L}{2c}\right)^2 \times \left\langle \left(x'^2 - \langle x'^2 \rangle\right)^2 + \left(y'^2 - \langle y'^2 \rangle\right)^2 \right\rangle = \left(\frac{dT}{d\gamma}\right)^2 \sigma_\gamma^2 + L^2 \frac{\sigma_{x'}^4 + \sigma_{y'}^4}{2c^2}. \quad (6)$$

Then, instead of Eq. (1),

$$\frac{\sigma_\omega}{\omega} \approx \sqrt{\frac{1}{7(N + MN_D)^2} + \left(\frac{1}{T}\frac{dT}{d\gamma}\right)^2 \sigma_\gamma^2 + L^2 \frac{\sigma_{x'}^4 + \sigma_{y'}^4}{2c^2T^2}}. \quad (7)$$

In the simplest case of magnetic chicanes, $T \propto 1/\gamma^2$ and Eq. (7) gives

$$\frac{\sigma_\omega}{\omega} \approx \sqrt{\frac{1}{7(N + MN_D)^2} + \left(2\frac{\sigma_\gamma}{\gamma}\right)^2 + \frac{L^2}{\lambda_1^2}\frac{\sigma_{x'}^4 + \sigma_{y'}^4}{2(N + MN_D)^2}}, \quad (8)$$

where $N = MN_1$ is the total number of undulator periods, and the minimum linewidth $2\sigma_\gamma/\gamma$ can be achieved at $N + MN_D \gg \left(2\sqrt{7}\sigma_\gamma/\gamma\right)^{-1}$ and a sufficiently low angular spread. On the other hand, one can say that for $N + MN_D < (10\sigma_\gamma/\gamma)^{-1}$ it is possible to neglect the spectrum widening and obtain the spectrum of Eq. (3), which is convenient to represent as

$$I_\omega = I_0 \frac{\sin^2(\pi N_1 \delta)}{(\pi N_1 \delta)^2} \frac{\sin^2[\pi M(N_1 + N_D)(1+\delta)]}{\sin^2[\pi(N_1 + N_D)(1+\delta)]} \quad (9)$$

with the FWHM linewidth of a peak equal to $1/(N + MN_D) > 10\sigma_\gamma/\gamma$.

EXAMPLE AND POSSIBLE IMPLEMENTATIONS

For example, for $\sigma_\gamma/\gamma = 5 \times 10^{-4}$, $N + MN_D < 200$. Taking $N = 100$ and $M = 4$, one can choose $N_D = 25$. The normalized spectral intensity $F(\delta) = I_\omega/(M^2 I_0)$ of radiation in the forward direction near the frequency ω_1 of the fundamental harmonic of a DOK with these parameters is shown in Fig. 2.

According to Eq. (9), variation of N_D shifts the spectrum peaks. For example, the spectrum for the same undulator parameters except for $N_D = 24.5$ has two equal peaks (see Fig. 3).

In this example, the FWHM linewidth $1/(N + MN_D)$ is only twice less than the linewidth $1/N$ of regular undulator. For soft x-ray undulators with longer periods and smaller period numbers N, this ratio will be larger.

Chicanes can be easily implemented via upgrade of existing undulators. For example, a possible scheme of

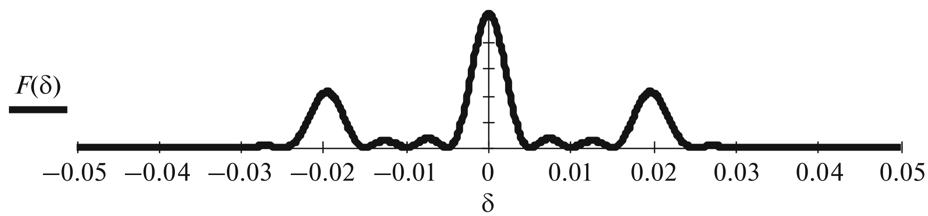

Fig. 2. Normalized spectral intensity of radiation in forward direction near frequency ω_1 of fundamental harmonic of DOK with $N = 100$, $M = 4$, and $N_D = 25$.

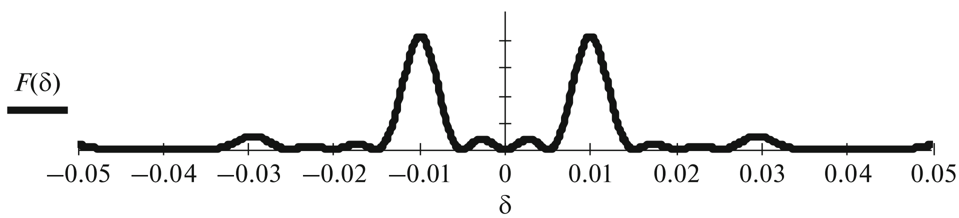

Fig. 3. The same as Fig. 2 except for $N_D = 24.5$.

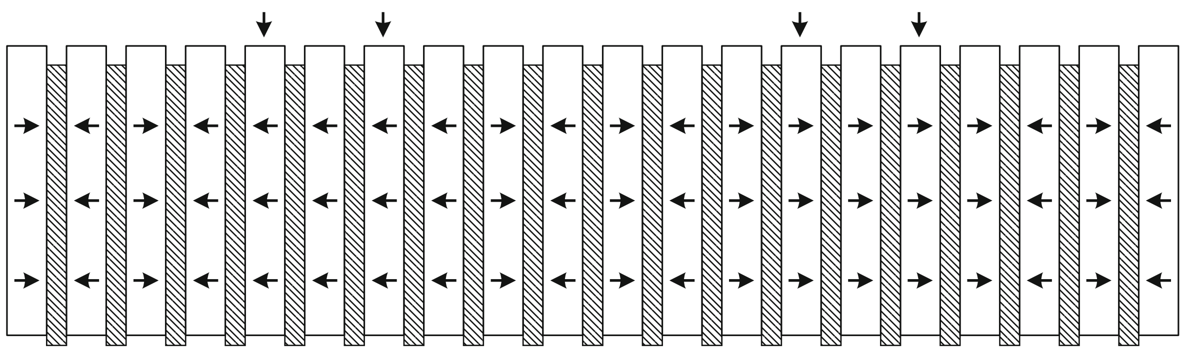

Fig. 4. Possible scheme of modification of hybrid permanent magnet undulator. Only upper part of undulator is shown. To make one chicane, four magnetic blocks (marked with vertical arrows) of upper part and four mirror-symmetric magnetic blocks of lower part are overturned.

modification of a hybrid permanent magnet undulator [9] is shown in Fig. 4.

The scheme relies on the simple fact that a variation in the magnetization of a couple (upper and lower) of magnetic blocks leads to a horizontal parallel shift of the trajectory. Therefore, the overturn of two successive even blocks provides a parallel shift, which is then compensated by the overturn of two successive odd blocks, which provides a bump to the trajectory.

CONCLUSIONS

In this paper, we discussed techniques to obtain narrow spectral lines in the undulator radiation of contemporary storage rings. It has been shown that the linewidth can be about 0.1%. It is worth noting that, in contrast to a high-harmonics undulator, a DOK can operate at lower amplitudes of magnetic field. Therefore, the total (spectrum-integrated) intensity of the DOK radiation at the same spectral intensity and linewidth is significantly less. Because of the problems of heating of specimens and optical components, this advantage is critical for modern high-brightness sources.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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