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High Energy Physics – Phenomenology

Meson production in J/ψ decays and $J/\psi \to N\bar{N}\gamma$ process

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A R T I C L E I N F O

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A B S T R A C T

It is shown that an account for the final-state interaction of real or virtual nucleon and antinucleon produced in the processes $J/\psi \to p\bar{p}\gamma$, $\psi(2S) \to p\bar{p}\gamma$, $J/\psi \to p\bar{p}\omega$, and $J/\psi \to 3\left(\pi^+\pi^-\right)\gamma$ near the threshold of $N\bar{N}$ pair production allows one to obtain self-consistent description of these processes. Predictions of our model are in good agreement with experimental data available. The proposed potential model also reproduces the corresponding partial cross sections of $p\bar{p}$ scattering.

1. Introduction

Invariant mass M of a nucleon-antinucleon pair $N\bar{N}$ in the decay $J/\psi \to N\bar{N} + A$, where $A = \gamma, \rho, \omega, \pi^0, \eta$, in the rest frame of J/ψ is determined by the energy E_A of particle A, $M^2 = m_{J/\psi}^2 - 2m_{J/\psi}E_A$. Therefore, measurement of E_A allows one to fix the value of M. Anomalous behavior of the decay probabilities has been observed in the processes $J/\psi \to p\bar{p} + A$ [1–7] and $J/\psi \to \text{mesons} + A$ [8,9], where the invariant mass of produced mesons is close to the double proton mass, $M \approx 2m_p$. Usually, this anomalous behavior is explained by the existence of a family of resonances $X(1835)$, $X(1840)$, $X(1870)$, and others. However all available experimental data can be well explained within the approach based on an account for the interaction of real or virtual nucleon and antinucleon produced in J/ψ decays (see [10–15] and references therein). The same approach explains successfully experimental data for the cross sections of processes $e^+e^- \to \text{mesons}$ near the threshold of real $N\bar{N}$ pair production (see [16–18]).

In the approach based on the account for the final-state interaction, a quark-antiquark pair is produced at small distances $r \sim 1/2m_p$ and then transforms into a nucleon-antinucleon pair at large distances $r \sim 1/\Lambda_{QCD}$ as a result of hadronization. For small relative velocity of N and $\bar{N}$, the $N\bar{N}$ interaction may significantly increase the modulus of wave function $|\psi(0)|$ (here $\psi(0)$ is the value of wave function of $N\bar{N}$ pair at distances $r \sim 1/\Lambda_{QCD}$). Firstly, this happens when there is a loosely bound $N\bar{N}$ state with the binding energy $|\varepsilon| \ll \left|\overline{U}\right|$, $\varepsilon < 0$, where $\overline{U}$ is the characteristic value of the potential $U(r)$ of $N\bar{N}$ interaction. Secondly, there is no loosely bound state, but a slight increase of the potential depth results in its appearance. We refer to the latter case as a virtual state with an energy $\varepsilon \ll \left|\overline{U}\right|$, $\varepsilon > 0$. In both cases, an energy ε is expressed in terms of the $N\bar{N}$ scattering length a, $|\varepsilon| = 1/m_p a^2$, where $|a|$ is much larger than the characteristic size R of the potential. Moreover, $a > 0$ in the case of a loosely bound state and $a < 0$ for a virtual state.

Frequently, to describe the production of near-threshold resonances, the Flatté approach is applied, which exploit scattering lengths as parameters [19]. However, it is shown in our previous works that the method of effective potentials of produced particles is more convenient. These potentials, of course, are not the real interaction potentials. However, for the production of near-threshold

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resonances these potentials reproduce the corresponding scattering amplitudes, and therefore the scattering lengths. It is this approach that we use in this work.

We refer to the production of a real $N\bar{N}$ pair as an elastic process. A produced virtual $N\bar{N}$ pair can annihilate into a system of mesons, we refer to such process as inelastic. The sum of probabilities of elastic and inelastic processes is the total probability. The inelastic processes are possible above the threshold of real $N\bar{N}$ pair production as well as below this threshold (due to annihilation of a virtual pair). Therefore, the anomalous behavior of the probabilities of decays $J/\psi \to N\bar{N} + A \to \text{mesons} + A$ are determined by the energy dependence of the probability of virtual $N\bar{N}$ production.

The quantum numbers of a nucleon-antinucleon pair are determined by that of particle A and also by the quantum numbers of mesonic system. For instance, the main contribution to the probabilities of $J/\psi \to N\bar{N}\gamma(\rho,\omega)$ decays near the $N\bar{N}$ production threshold is given by the production of $N\bar{N}$ pair with the angular momentum $l = 0$, total spin $S = 0$, and charge parity $C = +1$. Then, the isospin of $N\bar{N}$ pair is $I = 1$ in the decay with ρ meson production, isospin $I = 0$ with ω meson production, and the isospin I is not fixed in the decay $J/\psi \to N\bar{N}\gamma$. In the process $J/\psi \to (6\pi)\gamma$ the G-parity of 6π state is $G = +1$, and the C-parity of $N\bar{N}$ pair in the process $J/\psi \to N\bar{N}\gamma$ is $C = +1$. Therefore, the contribution to the probability of the decay $J/\psi \to (6\pi)\gamma$ is given by virtual $N\bar{N}$ pair with the isospin $I = 0$. Let's now consider the process $J/\psi \to (6\pi)\pi^0$, where 6π are produced through an intermediate $N\bar{N}$ state. In this case, the C-parity of $N\bar{N}$ pair is $C = -1$, total spin $S = 1$, and the isospin of the pair is $I = 1$, as well as the isospin of produced 6π system. Therefore, the effective nucleon-antinucleon potentials in the processes $J/\psi \to N\bar{N}\gamma$ and $J/\psi \to N\bar{N}\pi^0$ are different, and the probabilities of the corresponding processes are also different. This is the reason why a large number of resonances X has been introduced for interpretation of anomalous behavior of probabilities in various processes with the meson production.

In the present paper, using the approach based on the account for the final-state interaction, we perform a self-consistent description of the processes $J/\psi \to p\bar{p}\gamma$, $\psi(2S) \to p\bar{p}\gamma$, $J/\psi \to p\bar{p}\omega$, and $J/\psi \to 3\left(\pi^+\pi^-\right)\gamma$. We show that the effective potential approach successfully reproduces the nontrivial structure of all these cross sections around the threshold. The natural explanation of the sharp dip in the energy dependence of $J/\psi \to 3\left(\pi^+\pi^-\right)\gamma$ decay probability [9] is given.

2. Theoretical approach

In Ref. [12] we successfully described the experimental data for the energy dependence of $J/\psi \to p\bar{p}\gamma(\omega)$ decays probabilities. Therefore, we can predict the behavior of $J/\psi \to (6\pi)\gamma$ decay probability near the threshold of $N\bar{N}$ pair production. The probability of the process $J/\psi \to p\bar{p}\gamma$ can be written as (see Ref. [12])

$$\frac{d\Gamma_{p\bar{p}\gamma}}{dM} = \frac{p\,k^3}{2^4\,3\pi^3 m_{J/\psi}^4}\left|\mathcal{G}_{\gamma 0}\,\psi^{(0)}(0) + \mathcal{G}_{\gamma 1}\,\psi^{(1)}(0)\right|^2,$$
$$k = \frac{m_{J/\psi}^2 - M^2}{2m_{J/\psi}}, \qquad p = \sqrt{m_p E}, \qquad E = M - 2m_p. \tag{1}$$

Here k is the photon momentum in the J/ψ rest frame, p is the nucleon momentum in the $N\bar{N}$ center of mass frame, $\mathcal{G}_{\gamma 0}$ and $\mathcal{G}_{\gamma 1}$ are some energy-independent constants related to the amplitudes of $N\bar{N}\gamma$ state production at small distances. The functions $\psi^{(I)}(r)$ are the regular solutions of the radial Schrödinger equations for $N\bar{N}$ pair with the corresponding isospin I. The probability of $J/\psi \to p\bar{p}\omega$ decay reads (see Ref. [12])

$$\frac{d\Gamma_{p\bar{p}\omega}}{dM} = \frac{p\,p_\omega^3}{2^4\,3\pi^3 m_{J/\psi}^4}\left|\mathcal{G}_\omega \psi^{(0)}(0)\right|^2, \qquad p_\omega = \sqrt{\varepsilon_\omega^2 - m_\omega^2}, \qquad \varepsilon_\omega = \frac{m_{J/\psi}^2 + m_\omega^2 - M^2}{2m_{J/\psi}}, \tag{2}$$

where $\mathcal{G}_\omega$ is some constant.

The total probability $\Gamma_{\text{tot}}^{(0)}$ of $J/\psi \to N\bar{N} + \gamma$ and $J/\psi \to N\bar{N} + \gamma \to \text{mesons} + \gamma$ processes, in which real or virtual $N\bar{N}$ pair has the isospin $I = 0$, is expressed via the Green's function $\mathcal{D}^{(0)}(r, r'|E)$ of the radial Schrödinger equation for $N\bar{N}$ pair with quantum numbers $l = 0$, $S = 0$, and $I = 0$ (see Ref. [12])

$$\frac{d\Gamma_{\text{tot}}^{(0)}}{dM} = -\frac{\left|\mathcal{G}_{\gamma 0}\right|^2 k^3}{2^4\,3\pi^3 m_p m_{J/\psi}^4}\,\text{Im}\,\mathcal{D}^{(0)}(0, 0|E). \tag{3}$$

It is known that, due to the unitarity relation, the total probability Γ_{tot} of production of a state with given quantum numbers is expressed via the imaginary part of the corresponding polarization operator (see, e.g., [20], Eq. (113.5)). However, in the nonrelativistic approximation the polarization operator is proportional to the Green's function $\mathcal{D}(r, r'|E)$ (see Ref. [21]).

The probability of elastic process $J/\psi \to N\bar{N}\gamma$, where $N\bar{N}$ has $I = 0$, reads

$$\frac{d\Gamma_{\text{el}}^{(0)}}{dM} = \frac{\left|\mathcal{G}_{\gamma 0}\right|^2 p\,k^3}{2^4\,3\pi^3 m_{J/\psi}^4}\left|\psi^{(0)}(0)\right|^2. \tag{4}$$

The probability $d\Gamma_{\text{inel}}^{(0)}/dM$ of inelastic decays $J/\psi \to \text{mesons} + \gamma$, in which the system of mesons has $I = 0$, is

$$\frac{d\Gamma^{(0)}_{\text{inel}}}{dM} = \frac{d\Gamma^{(0)}_{\text{tot}}}{dM} - \frac{d\Gamma^{(0)}_{\text{el}}}{dM}\,. \tag{5}$$

Note that the effects of isotopic invariance violation (the proton and neutron mass difference and the Coulomb interaction of proton and antiproton) only slightly affect $d\Gamma^{(0)}_{\text{inel}}/dM$.

3. Results

In a recent work [9], the distribution $d\Gamma_{6\pi}/dM$ over the invariant mass of $3\left(\pi^+\pi^-\right)$ in the decay $J/\psi \to 3\left(\pi^+\pi^-\right)\gamma$ has been measured with high accuracy. To describe the probability of this process, it is necessary to take into account both the contribution of virtual $N\bar{N}$ pair with $I = 0$ in an intermediate state and the contributions of mechanisms not related to the annihilation of a virtual $N\bar{N}$ pair. The energy dependence of latter contributions is a smooth function of M near the threshold of real $N\bar{N}$ pair production, while the former contribution depends strongly on M in the near-threshold region. A smooth dependence on M of contributions, which are not related to the virtual $N\bar{N}$ pair production, can be approximated using few parameters [9].

Eq. (5) predicts the sum of probabilities of all inelastic processes, and the production of $3\left(\pi^+\pi^-\right)$ system is only one of possible channels. However, it is natural to assume that the annihilation amplitude of $N\bar{N}$ pair into mesons weakly depends on M near the threshold, and it can be considered a constant. Therefore, the contribution of virtual $N\bar{N}$ annihilation to the probability of $J/\psi \to 3\left(\pi^+\pi^-\right)\gamma$ decay is proportional to $d\Gamma^{(0)}_{\text{inel}}/dM$. This assumption was fully justified when describing the anomalous behavior of meson production cross sections in e^+e^- annihilation near the threshold of real $N\bar{N}$ production [12,17,18].

Using the experimental data [9] we found that the contribution of background processes in the near-threshold region can be approximated with good accuracy by a linear function of energy E. As a result, we describe the distribution $d\Gamma_{6\pi}/dM$ by the formula

$$\frac{d\Gamma_{6\pi}}{dM} = a + b\,E + c\,\frac{d\Gamma^{(0)}_{\text{inel}}}{dM}\,, \tag{6}$$

where a, b and c are some parameters that have been determined by comparison of our predictions with experimental data.

In order to determine $d\Gamma^{(0)}_{\text{inel}}/dM$ it is necessary to find the parameters of $N\bar{N}$ interaction potential with quantum numbers $l = 0$, $S = 0$, and $I = 0$. We have used the experimental data on the production of $p\bar{p}$ in $J/\psi \to p\bar{p}\gamma$, $\psi(2S) \to p\bar{p}\gamma$ and $J/\psi \to p\bar{p}\omega$ decays [3–7], and also the results of partial-wave analysis of elastic and inelastic $p\bar{p}$ scattering data performed by the Nijmegen group [22]. In processes $J/\psi \to p\bar{p}\gamma$ and $\psi(2S) \to p\bar{p}\gamma$, $p\bar{p}$ pairs are produced both with $I = 0$ and $I = 1$ (see Eq. (1)). In the process $J/\psi \to p\bar{p}\omega$, the $p\bar{p}$ pair is produced with $I = 0$ (see Eq. (2)). However, the experimental data for this decay are limited as compared to the decay $J/\psi \to p\bar{p}\gamma$. Therefore, we have used the whole set of experimental data listed above to better fix the parameters of the potential. As a result, we found not only the effective interaction potential of $N\bar{N}$ with $I = 0$, but also with $I = 1$.

As shown in Refs. [23,24], the behavior of cross sections in the near-threshold region is determined by a small number of parameters (scattering lengths, effective ranges of interaction). Therefore, one can use any convenient parameterization of the effective potentials, which reproduces the required values of these parameters. We have used the parameterization of $N\bar{N}$ interaction potentials in states with $l = 0$, $S = 0$, and $I = 0, 1$ in the form of rectangular wells

$$U^{(I)}(r) = \left(V^{(I)} - i\,W^{(I)}\right)\cdot \theta(R^{(I)} - r)\,, \tag{7}$$

where $V^{(I)}$, $W^{(I)}$ and $R^{(I)}$ — are some parameters, and $\theta(x)$ is the Heaviside function. The optical potential $U^{(I)}(r)$ contains an imaginary part that accounts for annihilation of $N\bar{N}$ pair into mesons. For such a parameterization one can obtain the analytical form of the wave functions $\psi^{(I)}(r)$ and the Green's functions $\mathcal{D}^{(I)}(r, r'|E)$. We have (see Ref. [24])

$$\begin{aligned}
&\psi^{(I)}(0) = \frac{q\,e^{-ipR^{(I)}}}{q\cos\left(qR^{(I)}\right) - ip\sin\left(qR^{(I)}\right)}\,,\\
&\operatorname{Im}\mathcal{D}^{(I)}(0,0|E) = \operatorname{Im}\left[q\,\frac{q\sin\left(qR^{(I)}\right) + ip\cos\left(qR^{(I)}\right)}{q\cos\left(qR^{(I)}\right) - ip\sin\left(qR^{(I)}\right)}\right],\\
&q = \sqrt{m_p\left(E - V^{(I)} + i\,W^{(I)}\right)}\,.
\end{aligned} \tag{8}$$

The values of potential parameters, that provide the best fit of the experimental data, are given in the Table 1. Fig. 1 shows the comparison of our predictions with experimental data for $J/\psi \to p\bar{p}\gamma$, $\psi(2S) \to p\bar{p}\gamma$, and $J/\psi \to p\bar{p}\omega$ decays. In all cases, good agreement between the predictions and the experimental data is evident. We have checked that our model is consistent with the results of partial-wave analysis of $p\bar{p}$ scattering data performed by the Nijmegen group [22]. The agreement of our predictions for the scattering cross sections with that of the Nijmegen group is as good as in our previous paper [18].

The Green's function $\mathcal{D}^{(0)}(r, r'|E)$ for quantum numbers $l = 0$, $S = 0$, $I = 0$ has poles in the complex energy plane at $E = E^{(0)}_R$. For $W^{(0)} = 0$ we obtain $E^{(0)}_R = -2\,\text{MeV}$, which correspond to subthreshold resonance. For $W^{(0)} = 114\,\text{MeV}$ we have $E^{(0)}_R = (36 - 57i)$ MeV, that corresponds to the unstable bound state, see Ref. [25]. Relatively large imaginary part of $E^{(0)}_R$ explains absence of a pronounced peak in the probability of $J/\psi \to p\bar{p}\omega$ decay.

Using the obtained potentials and Eq. (6), we compare our predictions with the recent experimental data [9]. To obtain $d\Gamma_{6\pi}/dM$ one should multiply the number of J/ψ decays into $3(\pi^+\pi^-)\gamma$ observed in Ref. [9] by the ratio $\Gamma_{J/\psi}/N_{J/\psi}$, where $\Gamma_{J/\psi} = 92.6\,\text{keV}$

Table 1
Parameters of potentials (7) of $N\bar{N}$ interaction in the states with isospin $I = 0, 1$.

	$U^{(0)}$	$U^{(1)}$
V (MeV)	−92	−24
W (MeV)	114	89
R (fm)	1.17	1.06

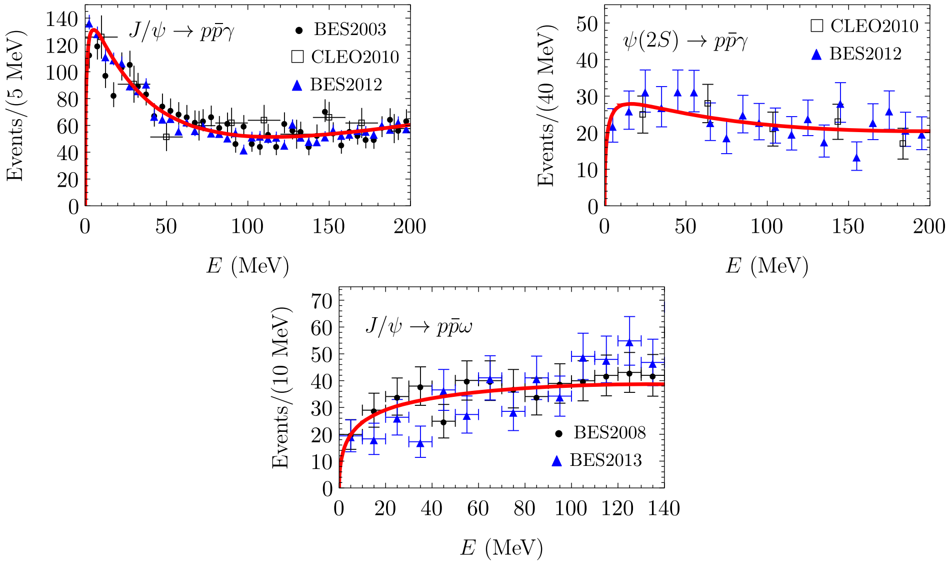

Fig. 1. Energy dependence of the probabilities of decays $J/\psi \to p\bar{p}\gamma$, $\psi(2S) \to p\bar{p}\gamma$, and $J/\psi \to p\bar{p}\omega$ in comparison with experimental data [3–7]. All graphs are normalized to the number of events in the earliest experiment.

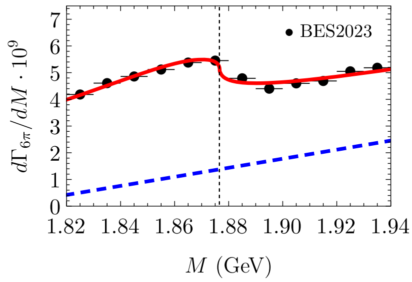

Fig. 2. The dependence of $J/\psi \to 3(\pi^+\pi^-)\gamma$ decay probability on the invariant mass M. The solid line is our predictions for $d\Gamma_{6\pi}/dM$, the dashed line is the background contribution. The vertical dotted line indicates the $N\bar{N}$ threshold. Experimental points are recalculated from Ref. [9].

is the total width of J/ψ meson and $N_{J/\psi} = 10087 \cdot 10^6$ is the total number of J/ψ events. Our comparison is shown in Fig. 2 for $a = 1.4 \cdot 10^{-9}$, $b = 0.017 \cdot 10^{-9}\,\text{MeV}^{-1}$, and $c = 7 \cdot 10^{-4}$. It is seen that our model successfully reproduces the nontrivial energy dependence of $J/\psi \to 3\left(\pi^+\pi^-\right)\gamma$ decay probability near the $N\bar{N}$ threshold.

4. Conclusion

A simple model, based on the account for the final-state interaction, is proposed for self-consistent description of the processes $J/\psi \to p\bar{p}\gamma$, $\psi(2S) \to p\bar{p}\gamma$, $J/\psi \to p\bar{p}\omega$ and $J/\psi \to 3\left(\pi^+\pi^-\right)\gamma$ near the threshold of $N\bar{N}$ pair production. It is shown that the nontrivial energy dependence of the probabilities of these processes is related to the interaction of real and virtual N and $\bar{N}$. The proposed potential model is also consistent with the results of partial-wave analysis of $p\bar{p}$ scattering data.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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