Production of $D^{(*)}\bar{D}^{(*)}$ near the thresholds in e^+e^- annihilation

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(Received 9 April 2024; accepted 17 May 2024; published 11 June 2024)

It is shown that the nontrivial energy dependencies of $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ pair production cross sections in e^+e^- annihilation are well described within the approach based on an account of the final-state interaction of produced particles. This statement is valid for the production of charged and neutral particles. Interaction of $D^{(*)}$ and $\bar{D}^{(*)}$ is taken into account using the effective potential method. Its applicability is based on the fact that for near-threshold resonance the characteristic width of peak in the wave function is much larger than the interaction radius. The transition amplitudes between all three channels play an important role in the description of cross sections. These transitions are possible since all channels have the same quantum numbers $J^{PC} = 1^{--}$.

DOI: 10.1103/PhysRevD.109.114015

I. INTRODUCTION

Currently, many dozens of resonances, having very nontrivial energy dependence of the cross sections of processes, have been discovered: $e^+e^- \to p\bar{p}$ [1–8], $e^+e^- \to n\bar{n}$ [9–11], $e^+e^- \to \Lambda\bar{\Lambda}$ [12–15], $e^+e^- \to \Lambda_c\bar{\Lambda}_c$ [16–18], $e^+e^- \to B\bar{B}$ [19,20], and others. In these processes, the widths of resonances are of the order of the distances to the thresholds of particle production, into which resonances mainly decay. In addition, the cross sections of production of light particles in the vicinity of near-threshold resonances and the probabilities of heavy particle decays into certain channels also demonstrate a nontrivial energy dependence. For instance, such energy dependence is observed in the processes $e^+e^- \to 6\pi$ [6,21–23], $e^+e^- \to K^+K^-\pi^+\pi^-$ [6,24,25], $J/\psi \to \gamma\eta'\pi^+\pi^-$ [26], and $J/\psi \to 3(\pi^+\pi^-)\gamma$ [27]. Despite the current availability of a fairly large amount of experimental data, the debate about the nature of near-threshold resonances is still ongoing.

Natural explanation of near-threshold resonances is based on account for the interaction of produced particles. In this approach, resonances arise in two cases (see, e.g., [28,29] and references therein). In the first case, there is a bound state with the binding energy much less than the characteristic value of the interaction potential (about several hundreds of MeV). In the second case, there is no loosely bound state but a slight increase in the depth of potential leads to appearance of such state (this is the so-called virtual level). In both cases, at scattering of produced hadrons on each other, the modulus of scattering length significantly exceeds the characteristic potential size (of the order of 1 fm). At the same time, the wave function at small distances calculated with account for the interaction of produced hadrons has characteristic value much larger than that without account for the interaction. The ratio of squares of the modules of corresponding wave functions for the relative angular momentum $l = 0$ (or their derivatives for $l = 1$) is the amplification factor, which can be very large. As a result, resonant structures arise in the particle production cross section. Currently, more and more scientists are coming to the conclusion that taking into account the interaction in the final state is of crucial importance for the correct description of cross sections in the near-threshold region (see, e.g., [30] and references therein).

The description of final-state interaction becomes noticeably more complicated, when there are several near-threshold resonances with the same quantum numbers and thresholds located close to each other. As a result, nonzero transition amplitudes between resonances arise, which leads to a significant distortion of the resonance shape. In our recent work [29], we have discussed various cases of coupled channels, where each channel is either loosely bound or virtual state. Moreover, it is shown in Ref. [29] that the account for the final-state interaction allows one to successfully describe the $B^{(*)}\bar{B}^{(*)}$ production near the thresholds in e^+e^- annihilation. Similar results for the system of $B^{(*)}\bar{B}^{(*)}$ mesons were obtained in Ref. [31] using the K-matrix approach.

In this work, the processes $e^+e^- \to D^{(*)}\bar{D}^{(*)}$ near the thresholds are discussed. Our approach is based on account

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for the final-state interaction in the case of coupled channels. Certainly, our information on the interaction potential between D mesons is very limited. However, it is not necessary to know these potentials very precisely. As already mentioned, the characteristic size of a peak in the wave function of produced $D^{(*)}\bar{D}^{(*)}$ system near the threshold is much larger than the characteristic size of the potential. Therefore, specific shapes of the potentials are not important. They can be parametrized in any convenient way by a few parameters. The numerical values of parameters are obtained by comparison of theoretical predictions and experimental data.

II. DESCRIPTION OF THE MODEL

Pairs $D\bar{D}$, $D\bar{D}^*$, and $D^*\bar{D}^*$ are produced in e^+e^- annihilation in the states with quantum numbers $J^{PC} = 1^{--}$. In this case, the relative angular momentum of produced particles is $l = 1$. Due to C-parity conservation, the total spin S of $D^*\bar{D}^*$ pair can be either $S = 0$ or $S = 2$. In our paper we discuss the total cross section for the production of these states with different spins. At small distances $r \sim 1/\sqrt{s}$, where $\sqrt{s}$ is the total energy of electron and positron in the center-of-mass frame, a hadronic system is produced as $c\bar{c}$ pair and, therefore, has isospin $I = 0$. However, at large distances $r \gtrsim 1/\Lambda_{\mathrm{QCD}}$ the difference in masses of charged and neutral D mesons (D^* mesons), as well as the Coulomb interaction between charged particles, leads to violation of isospin invariance. Thus, we have six states with $C = -1$: $\Psi_1 = D^0\bar{D}^0$, $\Psi_2 = D^+D^-$, $\Psi_3 = (D^0\bar{D}^{*0} + \bar{D}^0 D^{*0})/\sqrt{2}$, $\Psi_4 = (D^+D^{*-} + D^-D^{*+})/\sqrt{2}$, $\Psi_5 = D^{*0}\bar{D}^{*0}$, and $\Psi_6 = D^{*+}D^{*-}$. Taking into account violation of isospin invariance, we conclude that it is necessary to solve the six-channel problem. The threshold of Ψ_1 state production is 3730 MeV. We will count the remaining thresholds Δ_i from this value. Therefore, $\Delta_1 = 0$, $\Delta_2 = 9.6$ MeV, $\Delta_3 = 142$ MeV, $\Delta_4 = 150$ MeV, $\Delta_5 = 284$ MeV, and $\Delta_6 = 291$ MeV.

The radial Schrödinger equation, which describes our six-channel system, has the form

$$\left(p_r^2 + M_D\mathcal{V} + \frac{l(l+1)}{r^2} - \mathcal{K}^2\right)\mathbf{\Psi}(r) = 0, \qquad (\mathcal{K}^2)_{ij} = \delta_{ij}k_i^2,$$
$$\mathcal{V} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{12} & V_{22} & V_{23} \\ V_{13} & V_{23} & V_{33} \end{pmatrix}, \tag{1}$$

where $(-p_r^2)$ is the radial part of the Laplacian, $k_i = \sqrt{M_D(E - \Delta_i)}$, $M_D = 1865$ MeV is the D^0 mass, E is the energy of a system counted from the threshold of $D^0\bar{D}^0$ production, and $l = 1$. The wave function

$$\mathbf{\Psi}(r) = \left(\psi_1(r), \ldots, \psi_6(r)\right)^T$$

consists of radial parts $\psi_i(r)$ of wave functions of states Ψ_i, index T denotes transposition. The matrices V_{ij} are symmetric blocks of dimension 2×2 having the form

$$V_{ij} = \begin{pmatrix} U_{ij}^{(0)}(r) - U_{ij}^{(1)}(r) & -2U_{ij}^{(1)}(r) \\ -2U_{ij}^{(1)}(r) & U_{ij}^{(0)}(r) - U_{ij}^{(1)}(r) \end{pmatrix}, \tag{2}$$

where the diagonal potentials correspond to the transitions without change of particle electric charges, and off-diagonal ones describe processes with charge exchange. These potentials contain contributions from isoscalar and isovector exchange, $U_{ij}^{(0)}(r)$ and $U_{ij}^{(1)}(r)$, respectively. All potentials can be parameterized as

$$U_{ij}^{(I)}(r) = u_{ij}^{(I)}\theta(a_{ij}^{(I)} - r). \tag{3}$$

Here $\theta(x)$ is the Heaviside function, $u_{ij}^{(I)}$ and $a_{ij}^{(I)}$ are some constants that are found from comparison of theoretical predictions with experimental data.

The equation (1) has six linearly independent regular at origin solutions,

$$\mathbf{\Psi}^{(m)} = \left(\psi_1^{(m)}(r), \ldots, \psi_6^{(m)}(r)\right)^T, \quad m = 1, \ldots, 6. \tag{4}$$

Each solution is determined by the asymptotic behavior at $r \to \infty$,

$$\mathbf{\Psi}^{(m)} = \frac{1}{2ik_m r}\left(S_1^{(m)}\chi_1^+, \ldots, S_m^{(m)}\chi_m^+ - \chi_m^-, \ldots, S_6^{(m)}\chi_6^+\right)^T,$$
$$\chi_i^\pm = \exp\left[\pm i(k_i r - \pi/2)\right], \tag{5}$$

where $S_i^{(m)}$ are some coefficients. The cross sections $\sigma^{(m)}$ of pair production in the states Ψ_m have the form

$$\sigma^{(m)} = \frac{2\pi\beta_m\alpha^2}{s}\left|\sum_{i=1}^{6} g_i\dot{\psi}_i^{(m)}(0)\right|^2. \tag{6}$$

Here $\beta_m = k_m/M_D$, g_i are some constants that determine the production of corresponding states at small distances, $\dot{\psi}_i^{(m)}(r) = \partial/\partial r\psi_i^{(m)}(r)$. Since an isoscalar state is produced at small distances, then $g_1 = g_2$, $g_3 = g_4$, and $g_5 = g_6$.

III. RESULTS

In Refs. [32–38] detailed experimental data on cross sections $\sigma^{(m)}$ have been obtained for all m. Parameters $u_{ij}^{(I)}$, $a_{ij}^{(I)}$, and g_i of our model are determined by comparing predictions with all experimental data listed above. We analyze data for energies E up to 450 MeV since, on the one hand, we want to cover the range of thresholds of all six

TABLE I. Parameters of interaction potentials defined by Eqs. (2) and (3).

	Isoscalar exchange		Isovector exchange	
	$u^{(0)}$ (MeV)	$a^{(0)}$ (fm)	$u^{(1)}$ (MeV)	$a^{(1)}$ (fm)
V_{11}	−233.2	1.432	56.5	1.925
V_{22}	−104	1.61	184.6	0.932
V_{33}	−18.4	2.198	129.5	1.263
V_{12}	143.5	1.708	5.9	2.671
V_{13}	43	1.73	−100.9	0.443
V_{23}	−22.5	1.821	−13.6	1.425

channels, and on the other hand, we use a nonrelativistic model and cannot consider too high energies.

The parameters of the model are obtained using the χ^2 minimization method. The values of parameters that provide the best agreement with experiment are given in Table I. Constants g_i, that determine production of different states at small distances, have been also considered as fitting parameters. Their values are $g_1 = g_2 = 0.069$, $g_3 = g_4 = 0.003 + 0.169i$, and $g_5 = g_6 = 0.429 - 0.156i$. As a result of fitting, we have obtained $\chi^2/N_{\rm df} = 302/275 = 1.1$, where $N_{\rm df}$ is the number of degrees of freedom. The latter equals to the difference between the number of experimental points and the number of parameters in the model.

To determine the parameters of potentials, the following procedure was used. First, some random values of the model parameters were taken and the value of χ^2 was calculated using standard formulas with experimental errors considered as uncorrelated quantities. Then, the parameters were variated to minimize χ^2. The variation continued until the procedure converged to a stable value of χ^2. The minimization of χ^2 was repeated many times with different initial values of parameters. As a result, several sets of parameters were found that provided sufficiently small values of χ^2. Although the obtained sets of parameters were noticeably different, the energy dependences of the cross sections for all sets of parameters were very similar to each other. Table I shows one of these sets of parameters that provided $\chi^2/N_{\rm df} = 1.1$. The situation is very similar to the description of quarkonium spectra using potential models [39]. There are a large number of potentials with completely different analytical forms that well reproduce the spectrum of quarkonium and their leptonic widths. That is why it does not make much sense

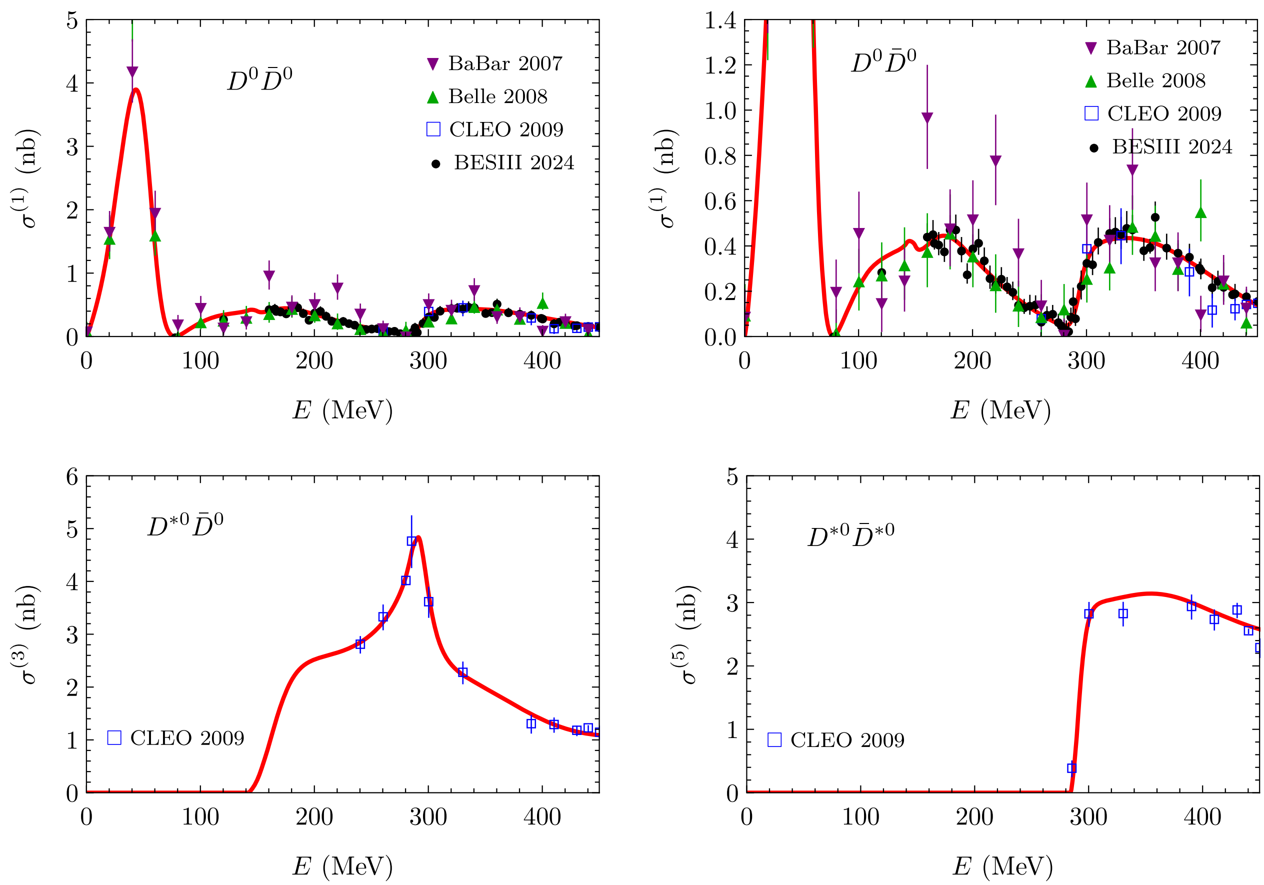

FIG. 1. Energy dependence of the cross sections for the production of neutral particles. Experimental data are taken from Refs. [32–35,38].

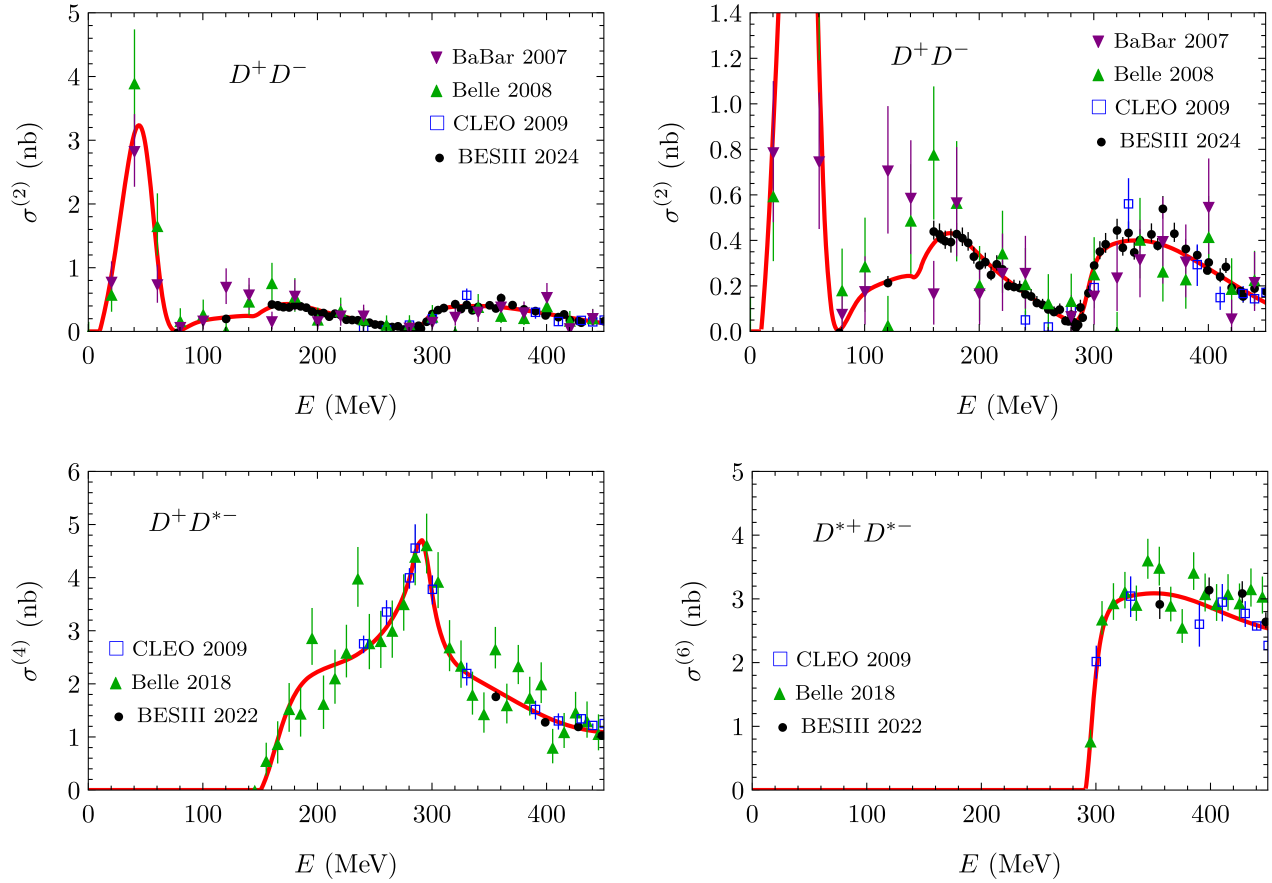

FIG. 2. Energy dependence of the cross sections for production of charged particles. Experimental data are taken from Refs. [32–38].

to specify the uncertainties of values in Table I, since these values do not reflect real interaction potentials, but are only a way of describing experimental data for $D^{(*)}\bar{D}^{(*)}$ production cross sections.

Figures 1 and 2 show a comparison of our theoretical predictions with experimental data from Refs. [32–38]. It is seen that good agreement of predictions with experimental data is obtained over the entire energy range under consideration. In particular, recent data from Ref. [38] is perfectly described by our model. Few experimental points lie outside of our theoretical predictions, but this is related to the fact that these points have large experimental uncertainties and are not consistent with each other.

The cross sections of different D meson pair production have very nontrivial energy dependencies. There are many peaks of various shapes, as well as sharp gaps between them. Note that experimental data obtained for all six charged and neutral channels have high accuracy. Therefore, for simultaneous description of the cross sections of these processes, it is necessary to take into account all six channels and all possible transitions between them. All potentials (diagonal and off-diagonal, with charge exchange and without charge exchange) are important to obtained good agreement between theory and experiment.

Note that in the energy region under consideration there are also thresholds for the production of the states containing strange quarks, such as $D_s^+D_s^-$, and others. We suppose that the admixture of these states has small effect on the cross sections considered in the present paper due to the rather large mass of s quark. Evidently, the cross sections of $D_s^{(*)}D_s^{(*)}$ meson production should be considered separately. The influence of other channels, such as $D^+D^-\pi$, can be estimated using optical potentials containing imaginary part (see, e.g., Ref. [40] where the nucleon-antinucleon pair production was investigated). We have checked that a good agreement of our predictions with experimental data for the processes $e^+e^- \to D^{(*)}\bar{D}^{(*)}$ can be achieved without introducing an imaginary part to the optical potential.

It should be emphasized that the position of peaks in the corresponding cross sections and their shapes are determined not only by the diagonal potentials, but also, to a large extent, by the nonzero amplitudes of transitions between different channels. Therefore, it makes sense to talk not about poles and cuts in each specific channel, but about the analytical properties of the six-channel Green's function, which describes this system. Therefore, for example, peaks in the $D\bar{D}$ production cross sections cannot be associated with bound or virtual states in the $D\bar{D}$ system. The positions of these peaks and their shapes are the result of the collective interaction of all D mesons participating in the interaction. In the present work, we did not study the analytical properties of the six-channel Green's function, since in our approach this is not necessary to describe the cross sections.

IV. CONCLUSION

It is shown that the final-state interaction in the system of $D^{(*)}$ mesons explains the nontrivial energy dependence of the cross sections of $e^+e^- \to D^{(*)}\bar{D}^{(*)}$ annihilation. Interaction between $D^{(*)}$ mesons is described using the effective potentials. Their parameters are determined from comparison of experimental data with theoretical predictions in each channel. Good agreement is obtained for the cross sections of charged and neutral pair production. We emphasize again that, to obtain a good description of experimental data for the processes $e^+e^- \to D^{(*)}\bar{D}^{(*)}$, it is necessary to take into account all six channels simultaneously and all transitions between them.

Quite recently, a work [41] has appeared, where the cross sections of processes $e^+e^- \to D^{(*)}\bar{D}^{(*)}$ have been described using K-matrix approach. Although the approach of Ref. [41] differs significantly from ours and the experimental data averaged over isospin are used in the channels $D^*\bar{D}$ and $D^*\bar{D}^*$, the results of Ref. [41] are consistent with ours qualitatively and quantitatively.

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